Chapter–1

Electric Charges and Fields

CBSE CLASS XII NOTES

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Electric Charges and Fields.

positive and negative charges are present in atoms with which every substances is made. All atoms are electrically neutral.

* What is an electric charge? Is it a scalar or Vector Quantity? Name its SI unit?

Electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects

→ Electric charge is a scalar Quantity

→ SI unit is Coulomb C

Eg; Proton (+e)
Electron (−e).

$e = 1.6 \times 10^{-19}$ Coulomb.

All atoms are electrically Neutral.

Electrostatics

Is the study of charges at rest. It deals with forces, fields and potentials arising from static charges.

* Scientist Dufey first confirmed there are two kinds of charges

* Benjamin Franklin assigned signs to charges.

Three ways of charging a body.

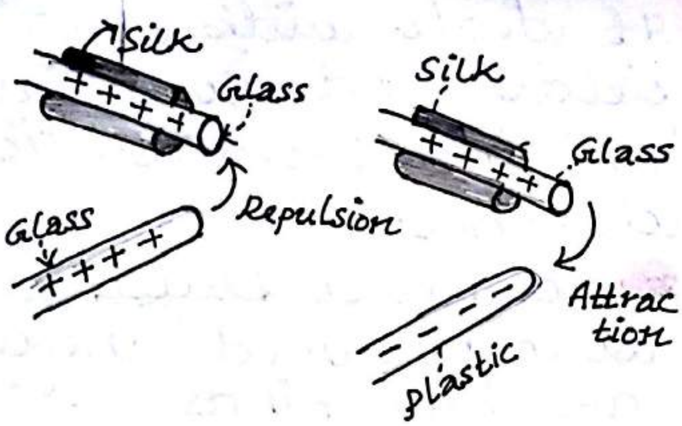
1, By Friction (rubbing)
2, Induction
3, Conduction.

1 Charging by Friction

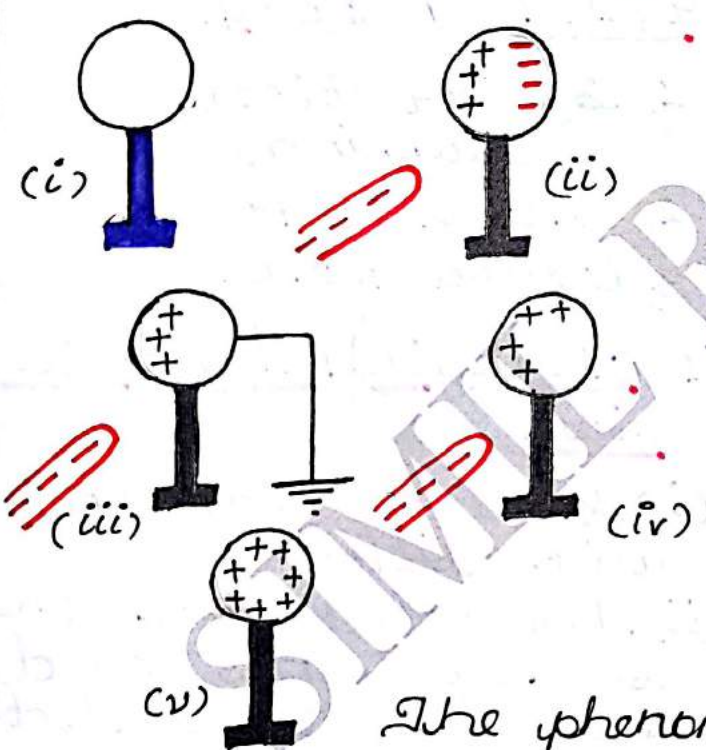
(i) Rub a glass rod with silk

(ii) The glass rod loses electrons and the silk gains electrons
[The body loosing electrons become +ve charge and body gaining ē ns become −ve charged]

(iii) Glass rod becomes +ve charged and silk becomes [−ve charged]



(2) Charging by Induction.



The phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farther end in the presence of near by charged body.

(i) Take an uncharged metallic sphere on an insulating stand

(ii) Bring -vely charged rod close to the metallic sphere. The electrons move away from the side facing the rod, leaving (+ve) charges on sphere, near the rod.

(iii) Earth the sphere on negative charged side.

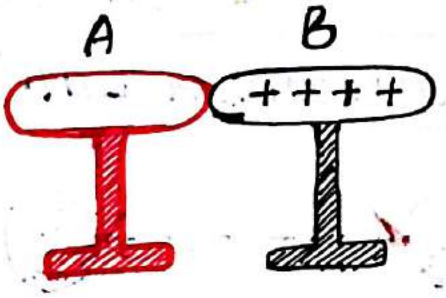
(iv) Remove earthing, remove rod also. Thus the +ve charges spread throughout the sphere uniformly.

(V) Thus uncharged body is charged by Induction.

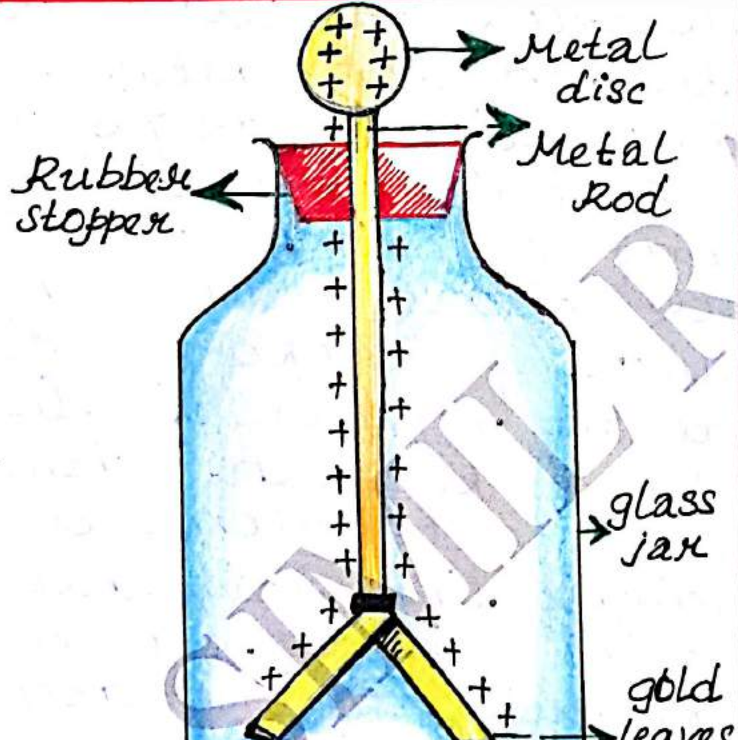
(3) Charging by Conduction

When an uncharged body is touched by a charged body, charges flow from the charged body to the uncharged body.

Thus the uncharged body can be charged by conduction.



GOLD LEAF ELECTROSCOPE



Using uncharged electroscope we are able to identify the existance of charge but not the polarity (+ve/-ve).

Gold leaf electroscope is used to check the nature of charge and gives the measure of the amount of charge on the body.

It consists of a metal rod with a metal disc attached at one end and metal leaves attached at the other end.

To avoid the current (air current) the arrangement is kept inside a glass jar.

Consider the electroscope which is +vely charged initially. Bring the given body near the +vely charged electroscope and keep the body in contact with metal disc of electroscope.

If the body has +ve charges, gold leaves will get diverged.

If the body has -ve charges, leaves will get collapsed.

The degree of divergence or

collapse gives the measure of the amount of charges on the body.

What are the basic properties of charge?

(1) Attractive and repulsive property.

like charges repel and unlike charges attract.

(2) Additivity of charges

The total electric charge on an object is equal to the algebraic sum of all charges on the object

$$q = q_1 + q_2 + q_3 + \dots + q_n$$

(3) conservation of charge.

The total charge of an isolated system is always conserved. charges can neither be created nor be destroyed.

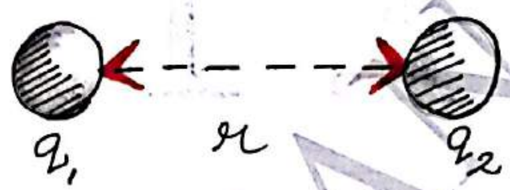
(4) Quantisation of charge.

Any charge in nature is an integral multiple of fundamental charge.

$$q = \pm ne$$

$q \rightarrow$ charge
$n \rightarrow$ no of charges.
$e - 1.6092 \times 10^{-19}\ C$

Coulombs Law



coulombs law states that the electrostatic force of interaction between two point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them and acts along straight line joining two charges. Force between two charges

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

where $k = \frac{1}{4\pi\varepsilon_0}$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ N^{-1} m^{-2} C^2$

Thus

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}$$

In vector form

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\cdot\frac{\vec{r}}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

Define S.I unit of charge? Define 1 C?

$$F = k\frac{q_1 q_2}{r^2}.$$

if $q_1 = q_2 = 1\,C$, $r = 1m$

so $F = 9\times10^9 \times \frac{1\times1}{1^2}$

$= \underline{9\times10^9 N}$

One coulomb is that charge which repels an equal and similar charge if a force of $9\times10^9 N$ when placed in a free space (air medium) at a distance of 1 m from it.

Relation between Coulomb and Ampere.

The coulomb is that quantity of charge which passes in one second through a cross section of conductor in which a constant current of one Ampere is flowing.

$$q = i\times t$$

$$i = \frac{q}{t}$$

$i \rightarrow$ current
$t \rightarrow$ time
$q \rightarrow$ charge.

Define relative permittivity of a medium?

we have

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \ldots (1)$$

Relative permittivity ε_r (or dielectric constant K) of a medium is defined as the ratio of the absolute permittivity of the medium to the absolute permittivity of free space

ie, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ $\quad \varepsilon = \varepsilon_0\varepsilon_r$

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \ldots (2)$$

$$F_m = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2} \quad \ldots (8)$$

(1) ÷ (2)

$$\frac{F_0}{F_m} = \frac{\varepsilon}{\varepsilon_0}$$

$$\Rightarrow \frac{F_0}{F_m} = \varepsilon_r$$

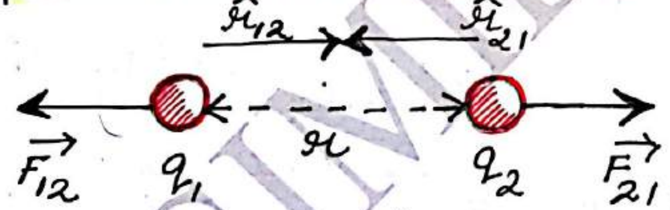
$\varepsilon_r \rightarrow$ relative permittivity of medium

ε – Absolute permittivity of medium

$\varepsilon_r \rightarrow$ has no unit.

Coulombs law in vector form

for like charges.



consider two like charges q_1 and q_2 seperated by distance 'r' in free space

The force on charge 'q_2' due to 'q_1' is

$$F_{21} = k \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$\hat{r}_{12} \rightarrow$ unit vector pointing from q_1 to q_2.

force of 'q_1' due to 'q_2' is

$$\vec{F}_{12} = k \frac{q_1 q_2}{r^2} \cdot \hat{r}_{21}$$

$$k = \frac{1}{4\pi\varepsilon_0}.$$

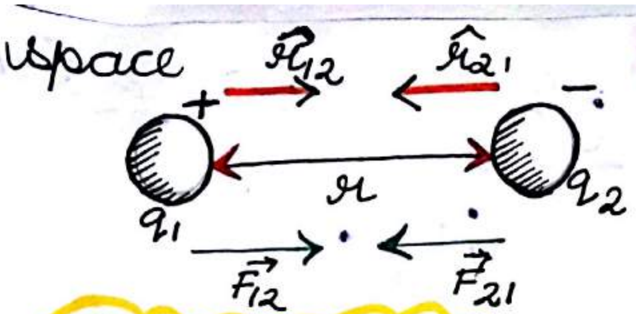
$\hat{r}_{21} \rightarrow$ unit vector pointing from 'q_2' to 'q_1'

since $\hat{r}_{12} = -\hat{r}_{21}$

$$\vec{F}_{12} = -\vec{F}_{21}$$

coulomb's law for unlike charges.

consider two unlike charges (q_1, q_2) seperated by a distance 'r' in free space



$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

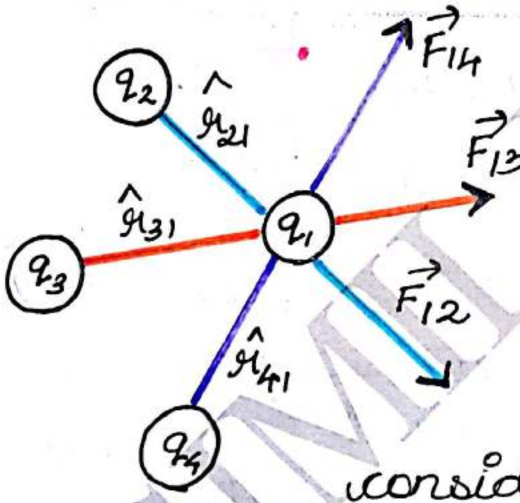
$$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

as $\hat{r}_{12} = -\hat{r}_{21}$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Principle of Superposition



consider a system of charges q_1, q_2, q_3, q_4 .... kept at different points whose position vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3$ etc. The total force on any charge say q_1 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$\vec{F}_1 = k\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2}\hat{r}_{41}\right]$$

The superposition principle states that the total force acting on a particular charge due to other charges present in the system is equal to vector sum of individual forces acting between the given charge and other charges.

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}}$$

Continues Charge Distribution

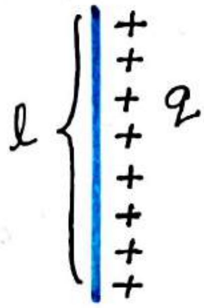
(i) linear charge distribution

(ii) Surface charge distribution

(iii) Volume charge distribution

(i) linear charge distribution (1D)

charges are uniformly distributed along the leng th of the wire.

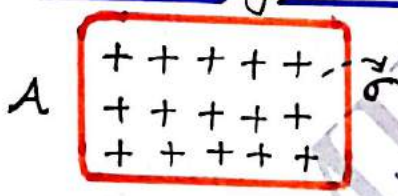


linear charge density = $\dfrac{\text{charge}}{\text{length}}$

$$\lambda = \frac{q}{l}$$

SI unit of λ is $\dfrac{C}{m}$ or Cm^{-1}.

(ii) Surface charge density (2D)

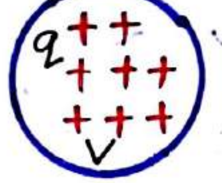


surface charge density = $\dfrac{\text{charge}}{\text{surface area}}$

$$\sigma = \frac{q}{A}$$

* SI unit of $\sigma = \dfrac{C}{m^2}$ or Cm^{-2}

(iii) Volume charge Density (3D)



volume charge density = $\dfrac{\text{charge}}{\text{volume}}$

$$\rho = \frac{q}{V}$$

* SI unit is $\dfrac{C}{m^3}$ or Cm^{-3}

Charge	Mass
(i) charge can be +ve or −ve.	(i) mass is always positive
(ii) charge is Quantised.	(ii) mass is not Quantised
(iii) charge is Independent of speed	(iii) mass varies with speed.
(iv) Force between charges are attractive or repulsive	(iv) Force b/w masses are always attractive.

Electric Field

The space around a charge in which its electrostatic Influence can be felt is called its electrostatic field.

Electric field Intensity [Field Strength]

The electric field intensity $\vec{E}$ at any point in an electrostatic field is defined as the force experienced on a unit positive test charge placed at that point.

If 'F' is the force experienced by a small positive charge 'q' at a point in an electric field 'E', then the Intensity of electric field at the point is

$$\vec{E} = \frac{\vec{F}}{q}$$ $$\vec{F} = \vec{E}q$$

In vector form $\vec{E}(r) = \frac{\vec{F}}{q}$

* Unit $\frac{N}{C}$ or NC^{-1}

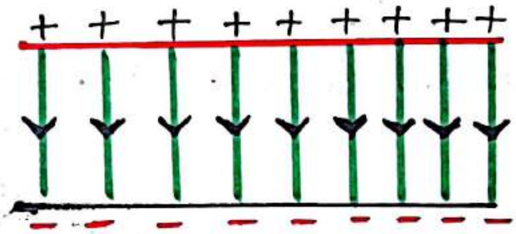
* Dimension = $\frac{MLT^{-2}}{AT}$

$= MLT^{-3}A^{-1}$

Uniform and non uniform Electric field.

If the intensity of $\vec{E}$ has same magnitude and same direction at every point in an electric field, it is a uniform electric field.

eg;



two conducting plates (+, −) kept parallel to each other and maintained a potential V.

$$E = \frac{V}{d}$$

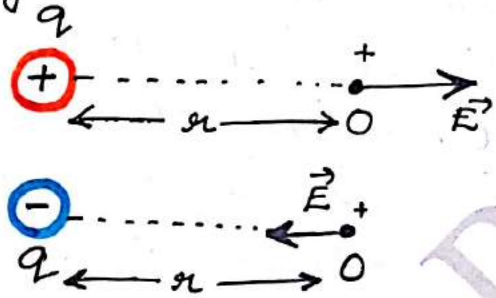
d → distance b/w the plates.

In a non uniform field, either the direction or the -

magnitude or both vary from point to point.

Electric field due to a point charge.

consider a point charge 'q' in free space. let 'O' be any point at a distance 'r' from the charge



Electrical Intensity $E = k\frac{q}{r^2}$

$k = \frac{1}{4\pi\varepsilon_0}$

direction

* away from the charge if charge is +ve
* Towards the charge if charge is -ve

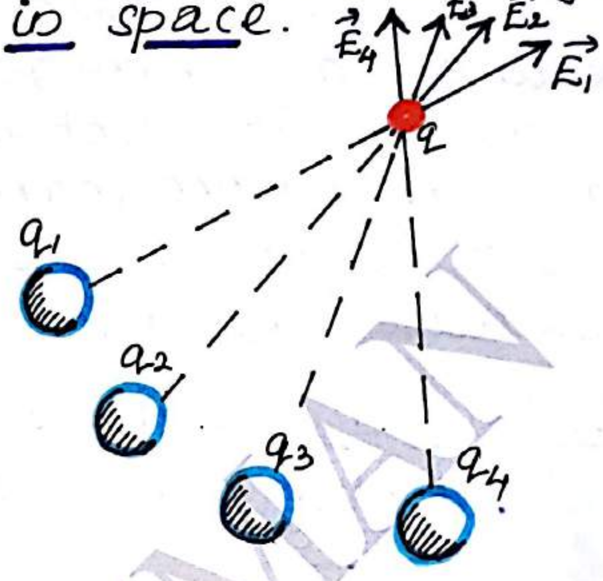
In vector form

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Electric field due to number of charges in space.



consider a number of point charges, $q_1, q_2, q_3 \cdots q_n$ in free space. The intensity of electric field at any point due to these charges is the vector sum of electric field intensities due to the individual charges at the point.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots \vec{E}_n$$

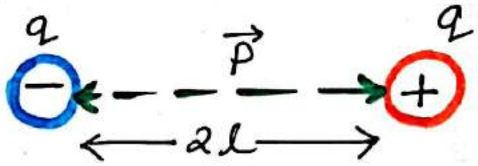
$$\vec{E} = k\sum_{i=1}^{i=n}\frac{q_i}{r_i^2}\hat{r}_i$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{r_i^2}\hat{r}_i$$

Electric Dipole and Dipole Moment.

A pair of equal and opposite point charges seperated by a distance is called an electric dipole.



If 'q' is the magnitude of each charge and '2l' is the distance between charges, the dipole moment

$$P = 2l \cdot q$$

It is a vector quantity whose direction along the axis of the dipole pointing from ⊖ve to ⊕ve

vector form $\vec{P} = 2lq \cdot \hat{P}$

* unit of dipole moment

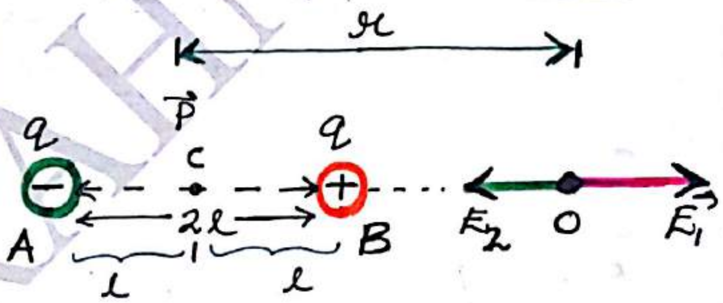
→ cm

* polar molecules like H_2O, HCl, NH_3 etc are electric dipoles
* H_2, O_2, CO_2 – do not posses dipole moment.

imp →

Intensity of Electric field Due to an Electric Dipole

(i) At a point on its Axis (Axial line)



consider an electric dipole of charge 'q' length '2l' and moment 'P'. Let 'O' be an point on its axial line at a distance 'r' from the centre 'C' of the dipole.

Electric Intensity at 'O' due to the charge +q at B

$E_1 = k \frac{q}{(r-l)^2}$

along BO.

* at 'O' due to $-q$ at A

$$E_2 = k \frac{q}{(r+l)^2}$$ along OA

$k = \frac{1}{4\pi\varepsilon_0}$

resultant Intensity at 'O' due to dipole

$$\boxed{E = E_1 - E_2}$$

$$E = kq\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$E = kq\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2(r+l)^2}\right]$$

$$E = kq\left[\frac{r^2+2lr+l^2-r^2+2lr-l^2}{[(r-l)(r+l)]^2}\right]$$

$$\vec{E} = kq\left[\frac{4lr}{(r^2-l^2)^2}\right]$$

since $2lq = P$

$$\vec{E} = k\,q\,\frac{2 \cdot 2lr}{(r^2-l^2)^2}$$

$$E = k\frac{2Pr}{(r^2-l^2)^2}$$ along CO

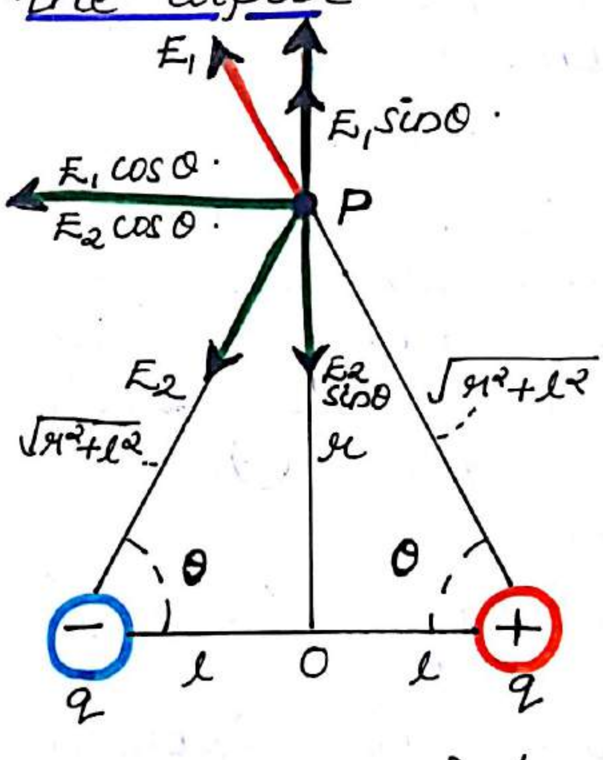
since $2l$ is small $l^2 << r^2$

$$\therefore \vec{E} = k\frac{2Pr}{r^4}$$

$$\vec{E} = k\frac{2P}{r^3}$$ along CO

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$$ since $k = \frac{1}{4\pi\varepsilon_0}$

(ii) At a point on the equatorial line of the dipole.



consider a point 'P' lying on the perpendicular

bisector of the line joining two charges (dipole) at a dist and 'r' from the midpoint 'O'. Let E_1 and E_2 be field intensities at P due to '+q' and '-q' charges
[E_1 is directed away (+q), E_2 is directed towards (-q)]

∴ The distance of charge (+q/-q) from point P. [pythagoras theorem]

$\sqrt{r^2+l^2}$

here
$r=\sqrt{r^2+l^2}$
$\therefore r^2=r^2+l^2$

magnitude of Electric field

$$E_1=E_2=k\frac{q}{(r^2+l^2)}$$

on resolving E_1 & E_2 along X and Y axes and adding their respective components
→ Y components cancel out
→ X components add up.

Resultant $\vec{E}$ points in the direction opposite to the dipole moment vector.

$$E=E_1\cos\theta+E_2\cos\theta$$
$$=2E_1\cos\theta \quad [\text{since } E_1=E_2]$$

$$E=2k\frac{q}{(r^2+l^2)}\times\frac{l}{(r^2+l^2)^{1/2}}$$

$\therefore \cos\theta=\frac{l}{\sqrt{r^2+l^2}}$

$\cos\theta=\frac{l}{(r^2+l^2)^{1/2}}$

$$E=\frac{2kql}{(r^2+l^2)^{1+1/2}}$$

$$E=\frac{2lq\,k}{(r^2+l^2)^{3/2}}$$

$\because 2lq=P$
$k=\frac{1}{4\pi\varepsilon_0}$

$$\boxed{E=\frac{1}{4\pi\varepsilon_0}\frac{P}{(r^2+l^2)^{3/2}}}$$

In vector form

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{(-\vec{P})}{(r^2+l^2)^{3/2}}$$

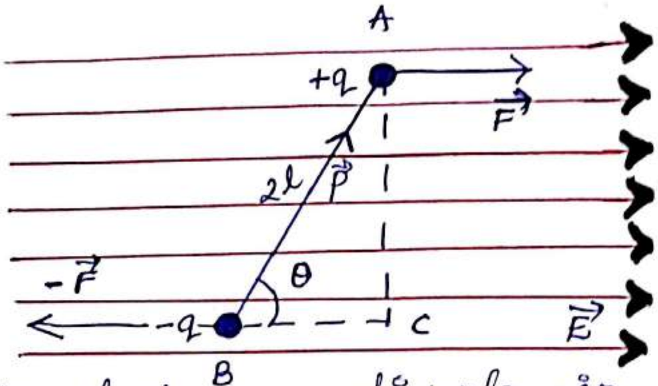
[since direction of $\vec{E}$ opposite to P]

special case
if $r>>l$ then $(r^2)^{3/2}=r^3$

$$\therefore E=\frac{1}{4\pi\varepsilon_0}\frac{(-\vec{P})}{r^3} \text{ or}$$

$$E=-k\frac{\vec{P}}{r^3}$$

* Dipole in an external electric field. Expression for torque acting on a dipole placed in a uniform electric field.



an electric dipole in a uniform electric field.

consider an electric dipole having dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ making an angle 'θ' with the direction of field.

Two charges experiences equal and opposite forces $\vec{F}$ and $-\vec{F}$ as shown $\left[\text{since } \vec{E} = \frac{F}{q} \quad \vec{F} = q\vec{E}\right]$

$\therefore$ Torque is given by

$$\tau = |\vec{F} \times \vec{AC}| = q\vec{E}\, 2l \sin\theta$$

$$\left\{ \begin{array}{l} \sin\theta = \frac{AC}{2l} \\ AC = 2l\sin\theta \end{array} \right.$$

$$\tau = \underline{q}\,\vec{E}\,\underline{2l} \sin\theta$$

$$\{\vec{p} = 2lq$$

$$\boxed{\begin{array}{l} \tau = p\vec{E}\sin\theta \\ \tau = \vec{p} \times \vec{E} \end{array}}$$

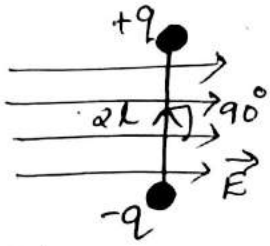
Note

(1) as two forces are equal and opposite, having different lines of action they form a couple (moment of couple — Torque)

(2) Direction of τ is $\perp r$ to the plane of paper or plane formed by $\vec{p}$ & $\vec{E}$.

Special cases [2012, 2009, 2008, 07, 06, 04 Delhi & Foreign]
D-2010

(i) The torque is maximum when $\tau = PE$ i.e., $\theta = 90^\circ$ [$\sin 90^\circ = 1$]

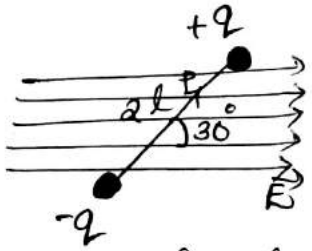


(ii) half the maximum value i.e.,

$\tau = \frac{PE}{2}$ when $\theta = 30^\circ$.
i.e., $\sin 30^\circ = 1/2$.

$\tau = PE \sin 30^\circ$

$\boxed{\tau = \frac{PE}{2}}$

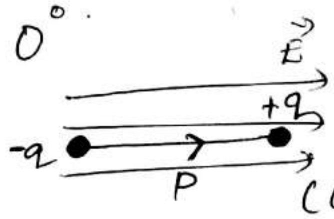


$\theta = 30^\circ$

(iii) for zero value of torque, $\sin \theta = 0$
[2010-Delhi]
$\theta = 0^\circ$.

$\tau = PE \sin 0^\circ$

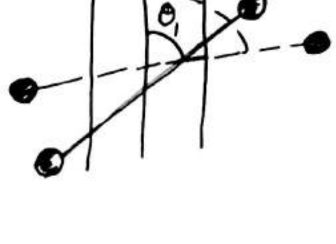
$\underline{\tau = 0}$



($\theta = 0^\circ$)
(stable equilibrium

$\theta = 180^\circ$ angle b/w $\vec{P}$ & $\vec{E}$ 180°
unstable equilibrium.

imp → **Work done in rotating a dipole in a uniform electric field.**



When a dipole of dipole moment $\vec{p}$; charge q and length $2l$ placed in a uniform electric field $\vec{E}$. at an angle θ. with the direction of field. The torque 'τ' acting on the dipole is given by $\boxed{\tau = PE\sin\theta.}$

The dipole is then turned through an additional small angle $d\theta$ against the torque. The work done is

$$\boxed{dw = \tau \times d\theta.}$$

$$\boxed{dw = PE\sin\theta \times d\theta.}$$

w.d in rotating dipole from θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta = pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}.$$

$$= PE\left[-\cos\theta_2 + \cos\theta_1\right]$$

$$W = PE\left[\cos\theta_1 - \cos\theta_2\right]$$

This work done is stored as potential Energy U in the system of dipole field and electric field.

$$U = PE\left[\cos\theta_1 - \cos\theta_2\right].$$

If a dipole is rotated from $\theta_1 = 90^\circ$ to $\theta_2 = \theta$.

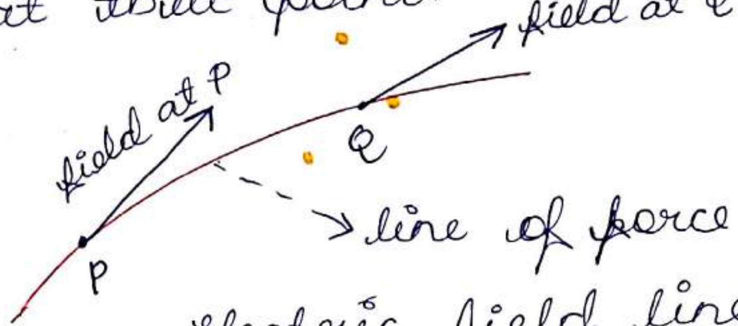
$$\therefore U = PE\left[\cos 90^\circ - \cos\theta\right]. \qquad \cos 90^\circ = 0.$$

$$\boxed{U = -pE\cos\theta.}$$

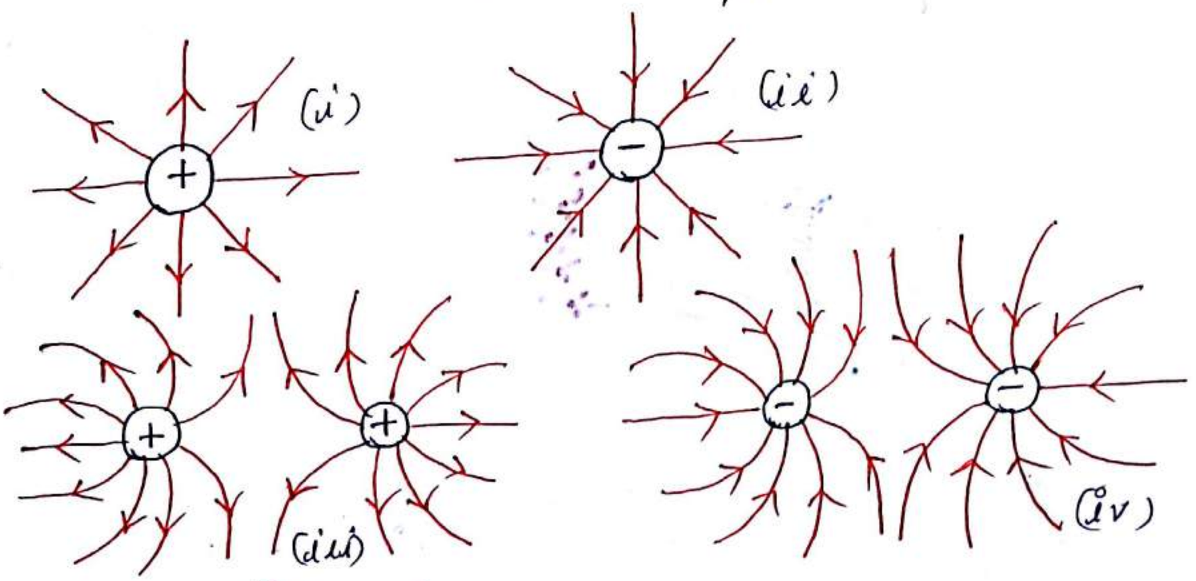
$\boxed{U = -\vec{p}.\vec{E}}$ → In vector form

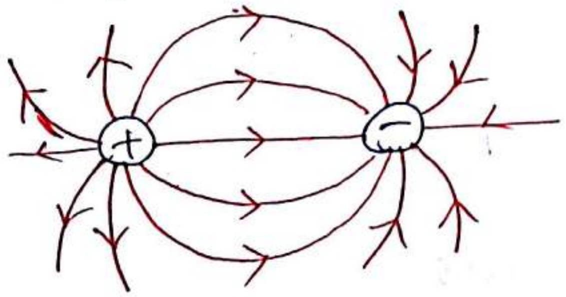
Electric lines of force.

A line of force is defined as a line, straight or curved, the tangent to which at any point gives the direction of the electric field intensity at that point.



electric field line





(v)

(i), (ii) → single charge

(iii) (iv) → like charges.

(v) → unlike charges.

properties of electric lines of force.

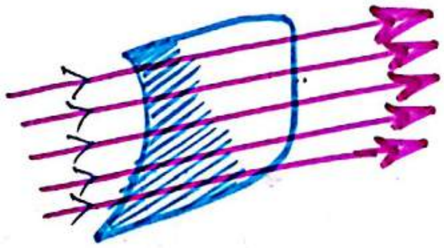
(1) All lines originates from a positive charge and terminate on a negative charge.

(2) Tangent to the line of force at any point gives the direction of $\vec{E}$ at that point.

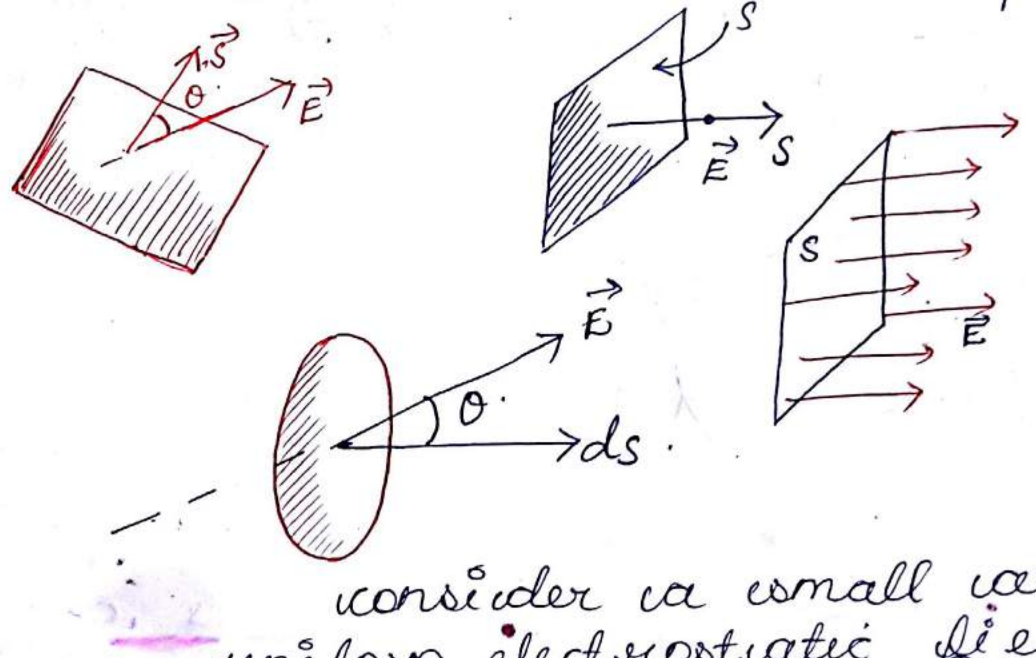
(3) lines of force never intersect.
(4) No lines of force exist inside a charge conductor.
(5) In a uniform electric field lines of force are parallel.
(6) They repel each other in case of two like charges and attract each other in case of two unlike charges.

Electric flux (ϕ)

Electric flux 'ϕ' through a surface is the total number of electric lines of force passing through the surface.



$\vec{E} \rightarrow$ Electric field
$S \rightarrow$ normal to the surface.



Consider a small area ds in a uniform electrostatic field $\vec{E}$. The flux passing through area is given by.

$$\boxed{d\phi = \vec{E}.\vec{ds}}$$

Field makes an angle 'θ' with the normal to the surface ∴ $\boxed{d\phi = E \cos\theta \, ds = Eds \cos\theta}$

In vector form

$d\phi = \vec{E}\cdot\vec{ds}$

Thus electric flux can be defined as the scalar product of the electric field Intensity and the area of the surface.

$\phi = \int_S \vec{E}\cdot\vec{ds}$ $\phi \rightarrow$ scalar Quantity

unit $= \frac{N}{C} \times m^2$.

$= NC^{-1}m^2$

or Vm

$\left\{ E = \frac{V}{d} \quad E = \frac{V}{m} \right.$ $\left\{ \frac{V}{m} \cdot m^2 = Vm \right.$

Note when $\theta = 0^\circ$ i.e. when

* surface is $\perp r$ to the field $\phi_{max} = EA$

* $\theta = 90^\circ$ (i.e. surface ~~not~~ parallel to the field $\phi_{min} = 0$

Gauss's theorem

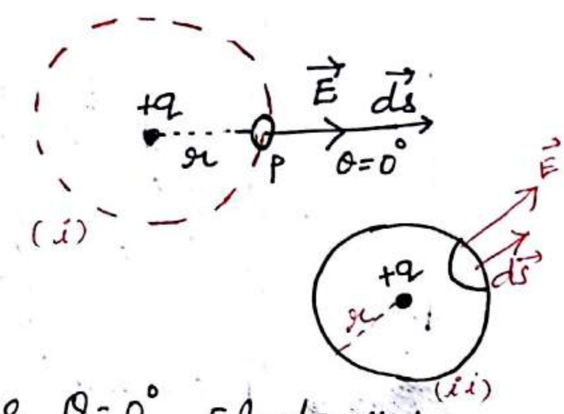
[imp 5 Marks]

Gauss's theorem states that the electric flux through any closed surface in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

$$\phi = \oint_S \vec{E}\cdot ds = \frac{q}{\varepsilon_0}$$

proof

consider a charge +q be placed at a point O. Imagine a spherical gaussian surface passing through the point P at a distance 'r' from O



since $\theta = 0^\circ$ E & ds ||el

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \oint_S E\,ds\cos 0^\circ$$

$$\phi = E\oint_S ds$$

$$\phi = E \times 4\pi r^2$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \times 4\pi r^2$$

$$\boxed{\therefore \phi = \frac{q}{\varepsilon_0}}$$

properties of gaussian surface.

(1) Any closed surface where gauss's theorem is applied is called gaussian surface

(2) It can have any shape.

(3) charges enclosed in the surface are only considered.

(4) If charges are outside the gaussian surface, electric flux $\phi = 0$.

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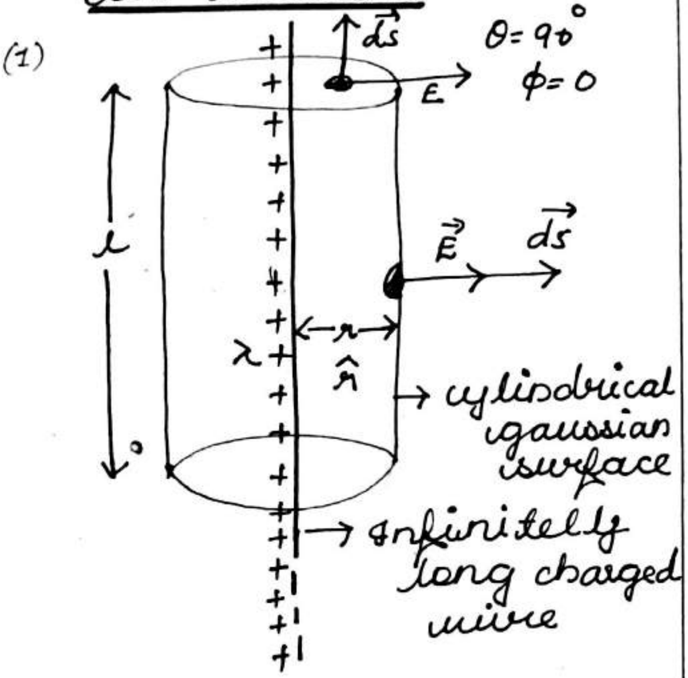
Applications of gauss's theorem

(1)



(1) Infinitely long charged wire

$E \to$ Electric field Intensity

$\vec{ds} \to$ normal to surface area

r & l $\to$ radius and length of gaussian surface.

Consider an infinitely long uniformly charged conductor of linear charge density λ.

Imagine a cylindrical gaussian surface around the wire.

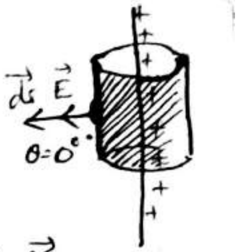
Circular faces

Electric flux is zero as $\vec{E}$ and $\vec{ds}$ are perpendicular to each other $\theta = 90^\circ$.

$$\phi = \oint_s E\,ds \cos 90^\circ \quad [\cos 90^\circ = 0]$$

$$\phi = 0$$

Curved faces



Electric flux

$$\phi = \oint_s E\,ds \cos 0^\circ$$

$$\phi = \oint_s \vec{E}\,\vec{ds} = \vec{E} \oint \vec{ds}$$

$$\phi = E \cdot 2\pi r l.$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

$$\boxed{q = \lambda l}$$

$$\frac{q}{\varepsilon_0} = E \cdot 2\pi r l$$

$$\frac{\lambda \cancel{l}}{\varepsilon_0} = \vec{E} \cdot 2\pi r \cancel{l}$$

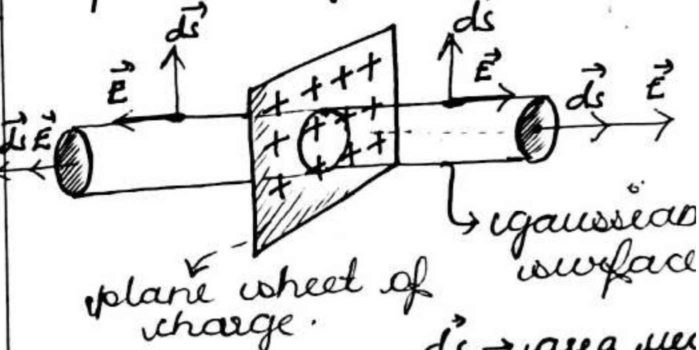
$$\boxed{\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}}$$

vector form

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\hat{r}}$$

☆ $r \uparrow E \downarrow$

(2) Infinite plane sheet of charge



$\vec{ds} \to$ area vector normal.

$\vec{E} \to$ Electric field Intensity

Consider a sheet of charge of density σ. Imagine a cylindrical gaussian surface passing through the sheet

→ area of gaussian surface = 2ds

→ charge enclosed $q = \sigma ds$

(a) Electric flux through curved surface

$[\theta = 90°]$

$$\phi = \oint_s E ds \cos 90° = 0$$

$$\therefore \phi = 0.$$

(b) At ends

$$\phi = \oint_s \vec{E} \cdot \vec{ds} \quad [\theta = 0°]$$

$$\phi = 2 E ds \qquad \left\{ \phi = \frac{q}{\varepsilon_0} \right\}$$

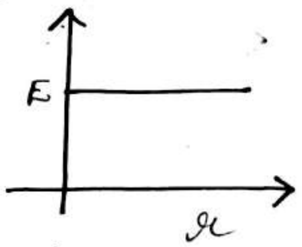
$$\frac{q}{\varepsilon_0} = 2 E ds.$$

$$\frac{\sigma \cancel{ds}}{\varepsilon_0} = 2 E \cancel{ds}.$$

$$\boxed{\therefore E = \frac{\sigma}{2\varepsilon_0}}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}$$



$\hat{n}$ → unit vector normal to the plane

★ E does not depends upon the distance

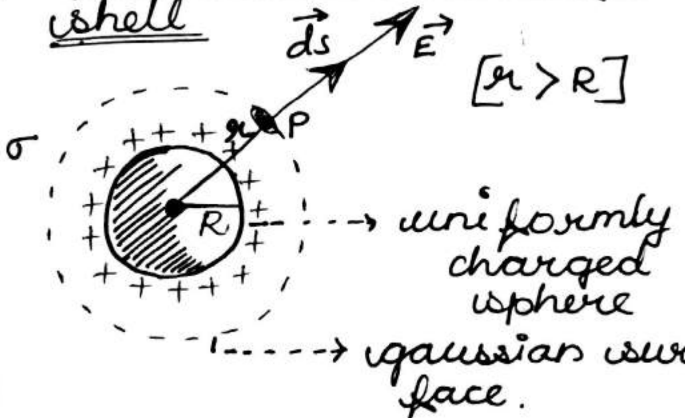
★★ Imp Using gausses theorem derive an expression for electric field Intensity at any point

(a) Outside

(b) Inside

(c) On the surface of uniformly charged thin spherical shell

(a) outside the charged shell



R → Radius of the sphere

P → point outside where E has to be calculated.

$$\phi = \oint_S \vec{E} \cdot \vec{ds} \qquad \theta = 0°, \ \cos\theta = 1$$

$$\therefore \frac{q}{\varepsilon_0} = \oint_s \vec{E} \cdot \vec{ds}$$

$$\frac{q}{\varepsilon_0} = \vec{E} \oint_s \vec{ds}$$

$$\frac{q}{\varepsilon_0} = E \cdot 4\pi r^2$$

$$\boxed{\therefore E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}$$

$\{ q = \sigma A$

$$\therefore E_{out} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma A}{r^2}$$

$$E_{out} = \frac{1}{4\pi\varepsilon_0} \sigma \frac{4\pi R^2}{r^2}.$$

$$\boxed{E_{out} = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

In vector form

$$\boxed{E_{out} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}}$$

(b) E Inside the shell.

[$r < R$]

charge 'q' is inside the gaussian surface is zero.

$$\phi = \oint_s \vec{E}\cdot\vec{ds}$$

$$\frac{q}{\varepsilon_0} = \oint_s \vec{E}\cdot\vec{ds}$$

$$q = 0 \quad \therefore \quad 0 = \oint_s \vec{E}\cdot\vec{ds}$$

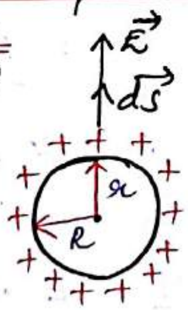
$$\Rightarrow \boxed{E = 0}$$

[$r = R$]

(c) E on the surface of the shell.

$$\phi = \oint_s \vec{E}\cdot\vec{ds}$$

$\theta = 0^\circ \quad \cos\theta = 1$



here $r = R$

$$\phi = E\oint_s ds = E\,4\pi R^2$$

$$\frac{q}{\varepsilon_0} = E\cdot 4\pi R^2 \qquad \left\{\phi = \frac{q}{\varepsilon_0}\right\}$$

$$\boxed{\therefore\ E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}}$$

In vector form

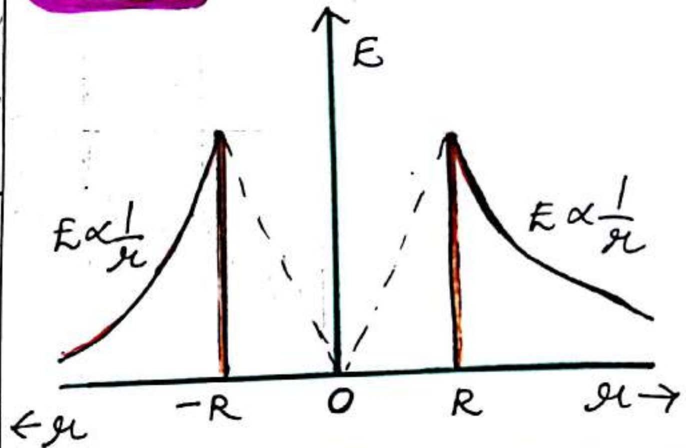
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\hat{r}}$$

$$\{q = \sigma A\}$$

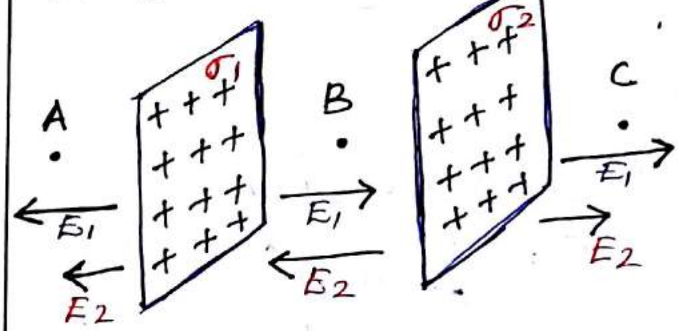
$$E = \frac{1}{4\pi\varepsilon_0}\frac{\sigma A}{R^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{R^2}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$



Electric field at points A, B & C. at points near parallel plane sheets of uniform charge density σ_1 & σ_2 such that $\sigma_1 > \sigma_2$



$\sigma_1 > \sigma_2$

at A (electric field)

$$E = E_1 + E_2$$

$$E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

$$E = \frac{1}{2\varepsilon_0}[\sigma_1 + \sigma_2] \text{ towards left.}$$

Electric field at B

$$E = E_1 - E_2$$

$$E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

$$E = \frac{1}{2\varepsilon_0}\left[\sigma_1 - \sigma_2\right]$$ right towards ~~left~~

Electric field at point C.

$$E = E_1 + E_2$$
$$= \frac{1}{2\varepsilon_0}\left[\sigma_1 + \sigma_2\right]$$

towards right

Special cases

** If both plates $\sigma_1 = \sigma_2 = \sigma$

(a) at A

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

(b) at B

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0.$$

(c) at C

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

** If $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$

(a) at A

$$\vec{E} = \frac{\sigma}{2\varepsilon_0} + \frac{(-\sigma)}{2\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(\sigma - \sigma) = 0$$

(b) at B

$$E = \frac{1}{2\varepsilon_0}\left[\sigma - (-\sigma)\right]$$
$$E = \frac{\sigma}{\varepsilon_0}$$

~~at C~~

(c) at C

$$E = \frac{1}{2\varepsilon_0}(\sigma - \sigma) = 0.$$
$$E = 0.$$

Numericals and Past papers

(1)

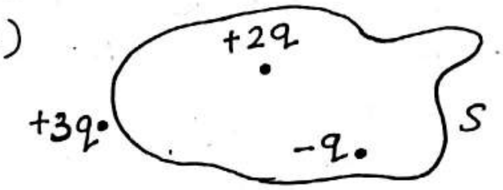


figure shows three point charges +2q, -q and +3q. Two charges +2q and -q are enclosed within the surface 'S'. what is the electric flux due to this configuration through the surface 'S'? [D-2010]

Electric flux $= \frac{1}{\varepsilon_0}$ [Net charge enclosed]

$$= \frac{1}{\varepsilon_0}\left[2q - q\right].$$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

(2) The distance of the field point, on the equatorial plane of a small electric dipole is halved. By what factor does the electric field due to the dipole change? [D-2004, 08]

Formulas

simil..

1. $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

2. $\varepsilon_0 = 8.85 \times 10^{-12} N^{-1} m^{-2} c^2$

3. $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

4. $\varepsilon = \varepsilon_0 \varepsilon_r$, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{F_0}{F_m}$

6. $\lambda = \frac{q}{l}$, $\sigma = \frac{q}{A}$, $\rho = \frac{q}{V}$

7. $q = \pm ne$

10. $\vec{F}_{12} = -\vec{F}_{21}$, $\hat{r}_{12} = -\hat{r}_{21}$

11. $E = \frac{F}{q}$, $q = \frac{F}{E}$, $F = qE$

12. $E = \frac{V}{d}$

13. $p = q \times 2l$

14. $E_{(\text{Point charge})} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$

15. $E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2Pr}{(r^2 - l^2)^2}$

$$\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2P}{r^3}}$$

16. $E_{eq} = \frac{1}{4\pi\varepsilon_0} \frac{P}{(r^2 + l^2)^{3/2}}$

$$\boxed{E_{eq} = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^3}}$$

17. $\tau = PE \sin\theta$

18. $W = PE[\cos\theta_1 - \cos\theta_2]$

19. $F = Eq$

20. $U = -PE\cos\theta$

21. gauss's Theorem

$$\oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \Sigma q$$

22. Electric field strength

(i) Inside shell $r < R$, $E_{in} = 0$ (sphere)

(ii) surface $r = R$, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$

(iii) outside $r > R$, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$

23. Infinite line charge

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r}$$

24. Infinite thin sheet of charge.

$E = \frac{\sigma}{2\varepsilon_0}$, $E_2 =$

Imp 5 Mark Questions

1. Find the expression for force and torque on an electric dipole kept in a uniform electric field? [CBSE AI 2008]

2. An electric dipole is held in a uniform electric field (i) using suitable diagram show that it does not undergo any translatory motion, and (ii) derive an expression for torque acting on it and specify its direction? [D-2005]

3. Write the expression for the torque $\vec{\tau}$ experienced by the dipole (i) Maximum (ii) Half the maximum value (iii) zero.