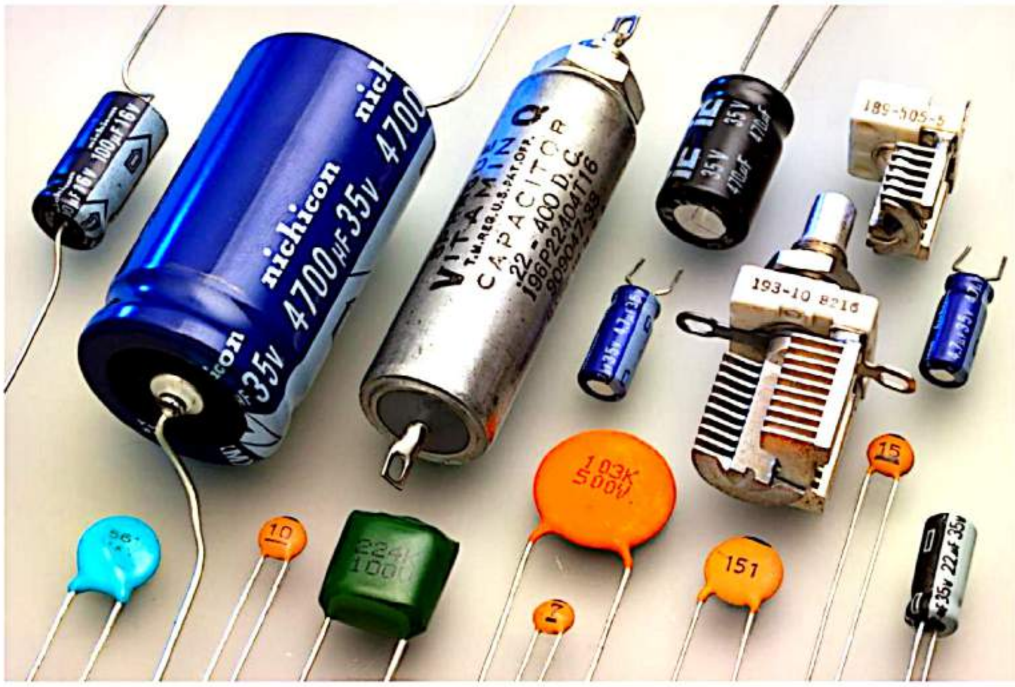
Chapter–2

Electrostatic Potential and Capacitance



CBSE CLASS XII NOTES

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Dr. SIMIL RAHMAN

Electric Potential

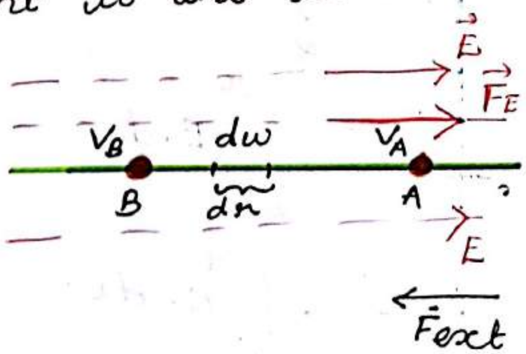
The work done is bringing unit positive charge from ∞ to that point (given point) [* without acceleration]

$$V = \frac{W}{q}$$

S.I unit is $\frac{J}{C}$ or V (volt)

Electric potential difference.

Electric potential difference between two points is the work done in moving a unit positive charge from one point to the other



$$dw = \vec{F}ext \cdot \vec{dr}$$

$$dw = -\vec{F_E} \cdot dr$$

$$\int_A^B dw = -\int_A^B \vec{E}q \cdot \vec{dr}$$

$$W_{AB} = -q\int_A^B \vec{E} \cdot \vec{dr}$$

$$\cancel{V_B - V_A} = \frac{W_{AB}}{q} = -\int_A^B \vec{E} \cdot \vec{dr}$$

$$V_{AB} = V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dr}$$

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dr}$$

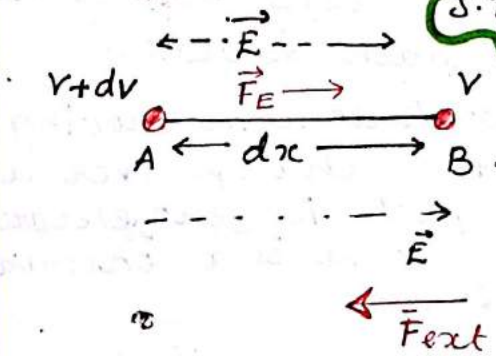
* If 'A' is at infinity $V_A = 0$

$$\therefore \boxed{V = -\int \vec{E} \cdot \vec{dr}}$$

* Potential difference between 2 points in a electrostatic field is the negative integral of electric Intensity between 2 points.

Relation b/w (E) and (V)

S.T $E = -\frac{dV}{dx}$



$$dw = F_{ext} \cdot dx$$

$$dw = -F_E \cdot dx$$

$$dw = -Eq \cdot dx$$

$$\frac{dw}{q} = -E \cdot dx \quad \text{when } q = 1$$

$\left\{ \text{since } \frac{dw}{q} = dV \right\}$

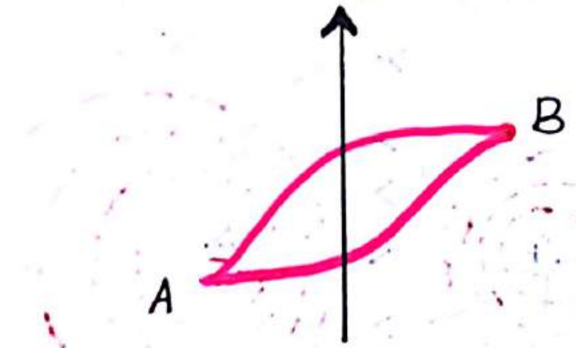
$$dV = -E \cdot dx$$

$\left\{ \cancel{\frac{dw}{q}} \right\}$

$$\therefore \boxed{E = -\frac{dV}{dx}}$$

S.T the line Integral of electric field along any closed path is zero.

S.T electrostatic force is a conservative force.



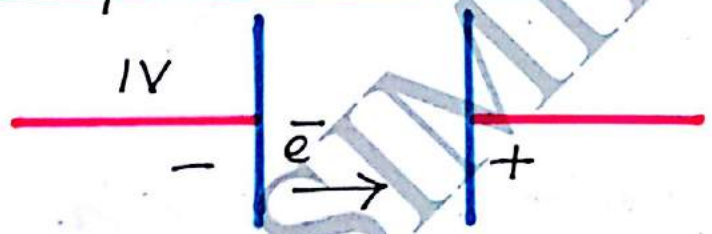
$$\oint_{ABA} \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l} + \int_B^A E\cdot d\vec{l}$$

$$= \int_A^B \vec{E}\cdot d\vec{l} - \int_A^B \vec{E}\cdot d\vec{l}$$

$\boxed{W=0}$.

As work done is moving unit positive charge over a closed path is zero, electrostatic force is conservative force.

* Define electron volt.



eV is the KE acquired by an $\bar{e}$ under a potential difference of 1V.

$$V = \frac{W}{q}$$

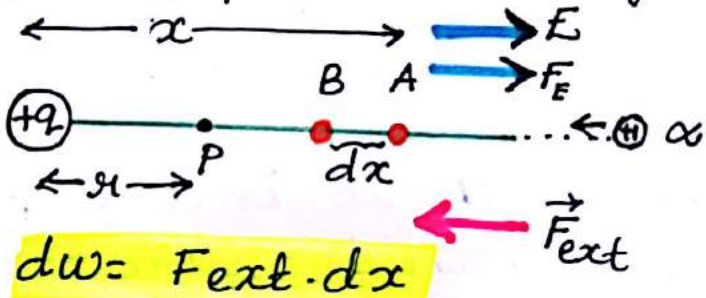
$$K.E = W = Vq = eV$$

$$= 1.6 \times 10^{-19} \times 1$$

$$\boxed{1eV = 1.6 \times 10^{-19} J}$$

$$\boxed{1J = \frac{1}{1.6 \times 10^{-19}} eV}$$

* Electric potential due to a point charge.



$$dW = F_{ext}\cdot dx$$

$$dW = -F_E \cdot dx$$

$$dW = -Eq\cdot dx$$

$$q = +1$$

$$dW = -E\cdot dx$$

$$\int dW = -\int_{\infty}^{r} E\cdot dx$$

$$W = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$W = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}dx$$

$$W = -\frac{q}{4\pi\varepsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r}$$

$$W = \frac{-q}{4\pi\varepsilon_0}\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

→

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$V = \frac{-q}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\therefore \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

medium change → * $V = \frac{1}{4\pi\varepsilon_r\varepsilon_0}\frac{q}{r}$

extra

* V is positive for +ve charge.
* V is '−'ve for '−ve charge.

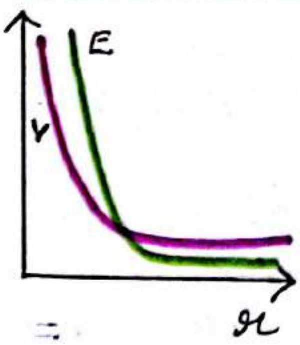
$$a \rightarrow \begin{cases} dw = F_{ext} \cdot dx \\ dw = -F_E \cdot dx \end{cases}$$

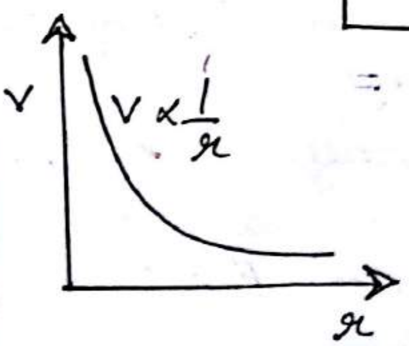
$$dw = -Eq \cdot dx$$

$$dw = -E \cdot dx \quad (q = +1)$$

$$W = \int dw = -\int_{\infty}^{r} E \cdot dx$$

graphs





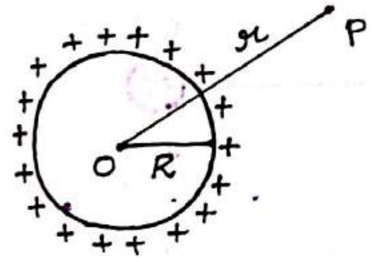
→ potential due to point charge versus distance

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\boxed{V \propto \frac{1}{r}}$$

* Electric potential is a scalar quantity

Electric potential due to a uniformly charged spherical shell.
[Vout, Von, Vin]



(i) potential outside the shell (any point i.e. $r > R$)

$$\boxed{V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

(ii) on the surface of shell ($r = R$).

$$\boxed{V_{on} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

(iii) V_{in} [any point inside the shell]

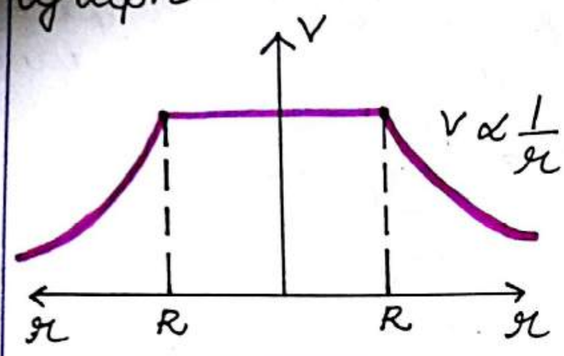
as $E = 0$ inside
no extra work is done to move a charge from surface to inside

$$\therefore V_{in} = V_{on}$$

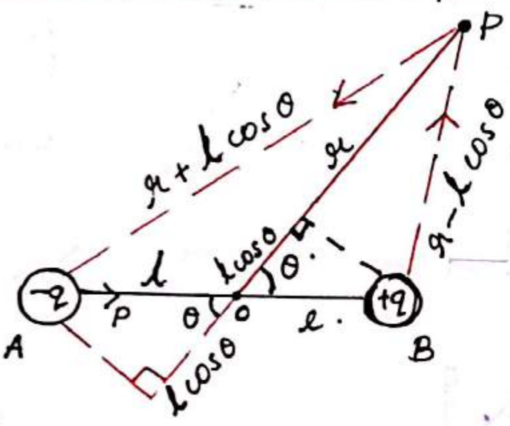
$$\boxed{V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

→ ∴ potential inside a shell ≠ 0

graph



★★ Electric potential at any point due to an electric dipole.



$$V_{+q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r - l\cos\theta}$$

$$V_{-q} = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r + l\cos\theta}$$

Hence potential due to dipole AB at C

$$V = V_{+q} + V_{-q}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\left[\frac{p\cos\theta}{r^2 - l^2\cos\theta}\right]}$$

For short dipole $(l << r)$

$$\therefore \boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}}$$

In vector form

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\hat{r}}{r^2}$$

special cases

case (i) V_{axial} (at a distance 'r' from the centre of dipole) near +q

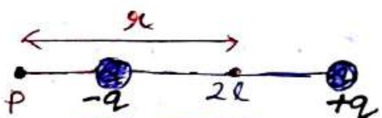
If point P is near +q $\theta = 0°$, $\cos\theta = 1$

$$V_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}$$

case (ii) V_{axial} near −q

$\theta = 180°$

$\cos 180° = -1$



$$V_{axial} = -\frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}$$

case (iii) $V_{equatorial}$.

If point P on equatorial line

$\theta = 90°$, $\cos 90° = 0$.

$$V_{eqt} = 0$$

Give an example at which (i) $E \neq 0, V = 0$

(ii) $E = 0, V \neq 0$

(i) At a point on equatorial line $E_{eq} \neq 0$ but $V = 0$

(ii) At any point Inside a charged shell $V \neq 0$, but $E = 0$.

Equipotential surface

Any surface that has same electric potential at every point on it is called equipotential surface.

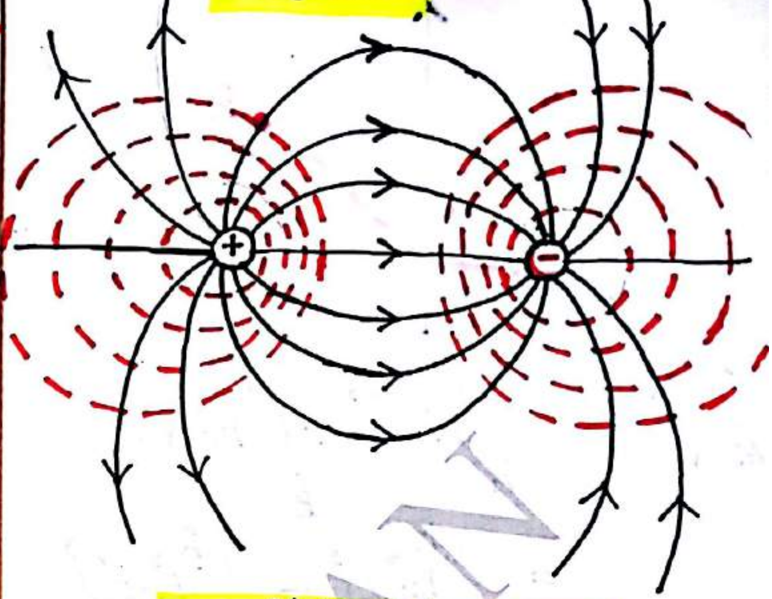
properties

1. Electric lines of force are perpendicular to the equipotential surface.
2. No need to do any work to move a charge from one point to another point on equipontential surface.
3. Never Intersect
4. closer together in the regions of strong field and farther apart in the regions of weak field.

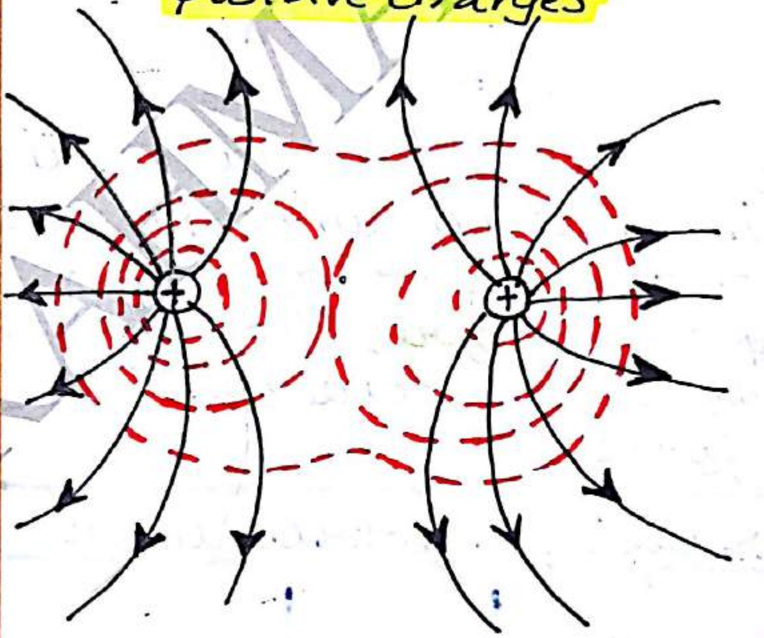
Equi potential surfaces

(a) point charge.

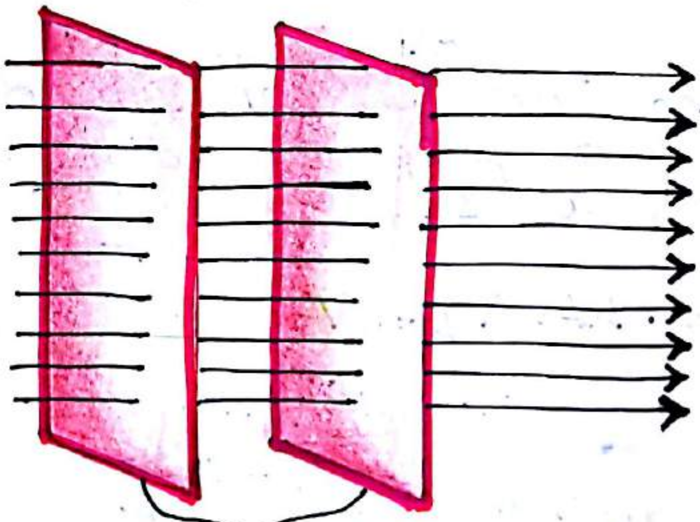
dipole.



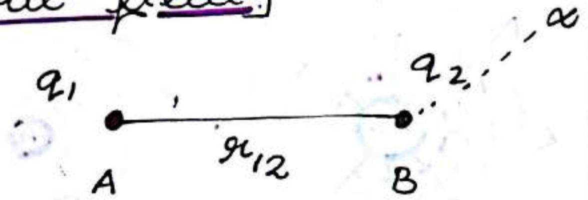
positive charges



uniform Electric field



potential energy of a system of 2 charges [in the absence of electric field]



(i) Bring charge q_1 from infinity to a point A (no work is done).

(ii) calculate the potential at B due to q_1

$$V = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1}{r_{12}}$$

(iii) Bring q_2 from α to B against repulsive force by doing work.

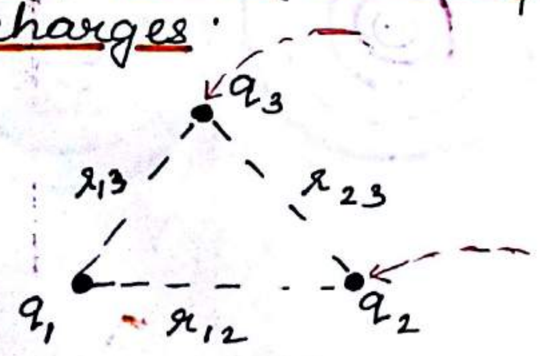
$$W = U_{12} = V \times q_2$$

$$W = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}}$$

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r_{12}}$$

This W is stored as P.E

★ P.E of a system of 3 charges.



$q_1, q_2, q_3 \rightarrow$ charges

$r_{12}, r_{13}, r_{23} \rightarrow$ distance.

∴ Algebraically.

$$W = W_{12} + W_{13} + W_{23}$$

$$U = U_{12} + U_{13} + U_{23}.$$

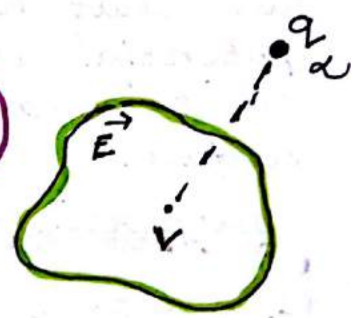
$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

P.E of a Single charge in a electric field.

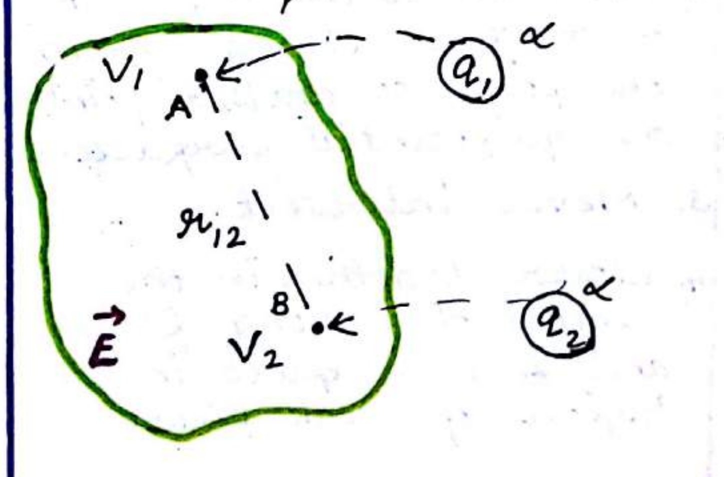
It is the work done in bringing a charge q (unit positive) from infinity to a point at which electric potential is V

$$\boxed{W = Vq}$$

$$\therefore \boxed{U = Vq}$$



P.E of a system of two point charges in an external field.



(i) Work done in bringing q_1 from ∞ to A by doing work against electric field.

$$\boxed{W_A = U_A = V_1 q_1}$$

(ii) work done in bringing q_2 from ∞ to point B by doing work against electric field.

$$\boxed{W_B = U_B = V_2 q_2}$$

(iii) Assemble q_1 q_2 at a distance r_{12} by doing work on q_2.

$$U_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

$$\therefore U = U_A + U_B + U_{AB}.$$

$$\boxed{U = V_1 q_1 + V_2 q_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

CONDUCTORS & INSULATORS

★ why are Insulators called dielectrics? and conductors non-electrics?

The rubbed insulators were able to retain charges placed on them, so they are called dielectrics.

Rubbed conductors could not retain charges placed on them. So they are called non-electrics.

Behaviour of conductors in electrostatic field.

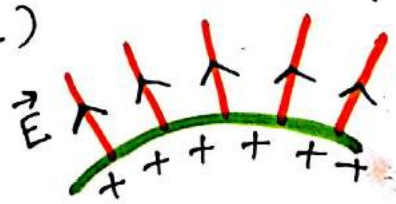
(i) Inside a conductor, electrostatic field is zero.

(ii) Net charges in the interior of a conductor is zero.

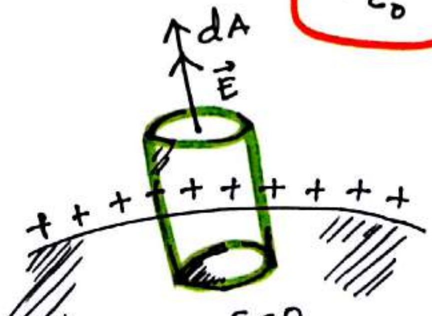
(iii) Inside the conductor electric potential is constant.

(iv) charges always reside on the outer surface of the conductor.

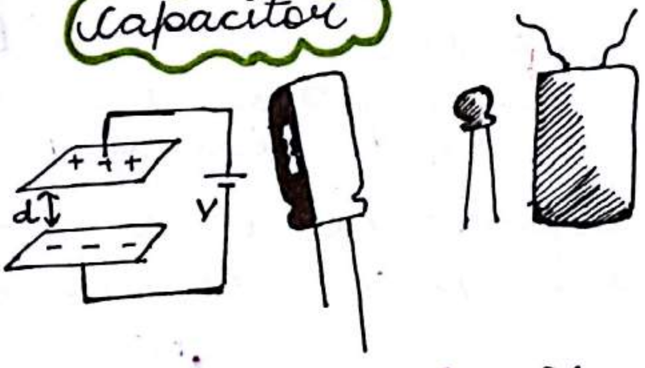
(v) electrostatic field ~~at~~ on the surface of the charged conductor must be normal to the surface. ($\perp r$)



(vi) Electric field at any point on the surface of a charged conductor is σ/ε_0



Capacitor



A capacitor consists of two conductors of any shape seperated by an insulating material.

Definition

capacitance of a conductor may be defined as the charge required to increase the potential by a unit amount

$\boxed{C = \frac{Q}{V}}$ $\boxed{\begin{matrix} Q \propto V \\ Q = CV \end{matrix}}$

$C \rightarrow$ capacitance

Define units of capacitance.

capacitance of a capacitor is said to be 1 F when 1 C charge increases the potential by 1 volt.

$C = Q/V$

$1 \text{ Farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$

S.I unit Farad [F]

$1\,mF = 10^{-3} F$ $1\,pF = 10^{-12} F$

$1\,\mu F = 10^{-6} F$

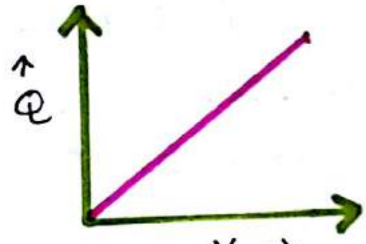
$1\,nF = 10^{-9} F$.

capacitor

variable capacitor

graphs

(i) Q V

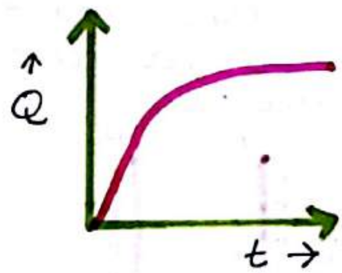


$Q = CV$

$Q \propto V$

$C \rightarrow$ capacitance, constant

(ii) Q Vs t



uses

used to store electric charge and electric energy

used in electronic circuits

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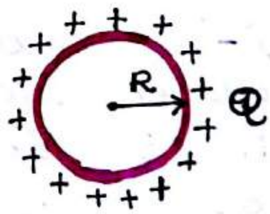
capacitance of a spherical Body.



$C = \frac{Q}{V}$ $\left\{ \text{since } V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \right.$

$C = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$

$\therefore C = 4\pi\varepsilon_0 R$

For a medium

$C = 4\pi\varepsilon_0\varepsilon_r R$

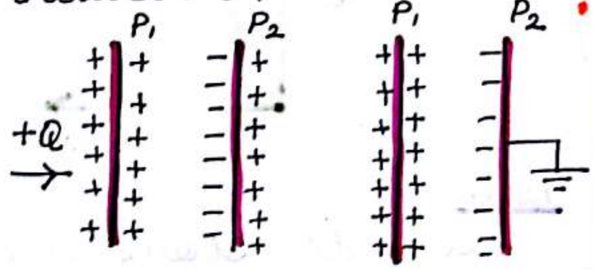
principle of capacitor, parallel plate capacitor

It consists of two large plane parallel conducting plates, seperated by a small distance.

principle

(i) Electrostatic Induction

(ii) capacitance can be increased by bringing an earthed uncharge conductor near a charged conductor.



(a) charge $+Q$ given to plade P_1 which increases the potential to V $\quad C = \frac{Q}{V}$

(b) Bring uncharged plate P_2 near by P_1.

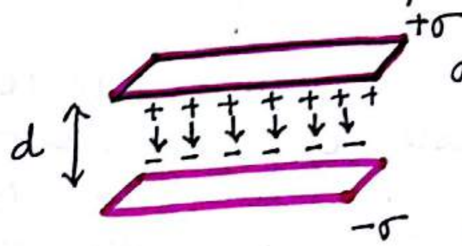
(c) '-ve' charges induced on sides facing P_1. It decreases 'V' as $C \propto \frac{1}{V}$

→ It results in increase of 'C'

(d) '+'ve charges induced on the other side of P_2 it increases V and decreases C as $C \propto \frac{1}{V}$

(e) To Increase capacitance, earth positively charged side of P_2

capacitance of a parallel plate capacitor



$\sigma = \frac{Q}{A}$

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed}$$

$\therefore$ $V = Ed$, $\sigma = \frac{Q}{A}$, $Q = \sigma A$, $E = \frac{1}{\varepsilon_0}\sigma$, $E = \frac{\sigma}{\varepsilon_0}$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} \cdot d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

If it is placed in a medium of dielectric constant

$$\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$$

$\therefore$ $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$, $\varepsilon = \varepsilon_r \varepsilon_0$

** Factors on which capacitance of a parallel plate capacitor depends upon.

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

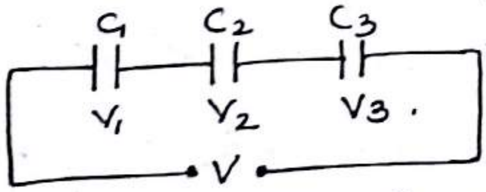
1, size and shape of conductor [plates -area]

2, distance between plates

3, Nature of dielectric kept in between the plates.

capacitance - series combination

V-ADDS



$Q \rightarrow$ constr

charge 'Q' is constant in series combination

$V = V_1 + V_2 + V_3$

$C = \frac{Q}{V}$

$V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$

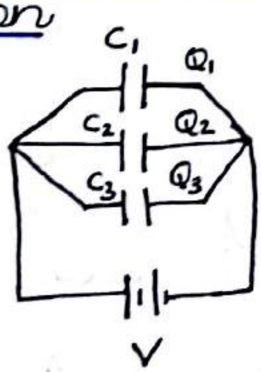
$V = \frac{Q}{C}$.

$V_3 = Q/C_3$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

capacitance - Parallel combination



Q - ADDS

$V \rightarrow$ const

$Q = Q_1 + Q_2 + Q_3$

$\therefore CV = C_1V + C_2V + C_3V$

$Q = CV$

$Q_1 = C_1V$

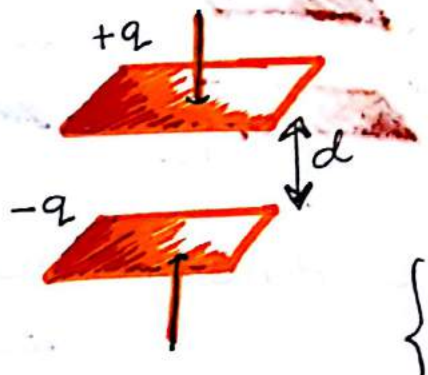
$Q_2 = C_2V$

$Q_3 C_3 V$

$$\therefore \boxed{C = C_1 + C_2 + C_3}$$

Energy stored in a capacitor

The amount of work done in raising the potential of a capacitor is stored in the form of energy of the capacitor.



$C = \frac{Q}{V}$

$V = \frac{Q}{C}$

$Q = CV$

let 'q' and V be the charge and potential difference at any instant of time.

'dw' - work must be done to impart charge 'dq'

$$dW = Vdq$$

$\boxed{W = Vq}$

$$dw = \frac{q}{C} dq$$

$\boxed{V = \frac{W}{q}}$

$$W = \int dw = \int_0^Q \frac{q}{C} dq$$

$$W = \frac{1}{C} \int_0^Q q\, dq$$

$\int x^n = \frac{x^{n+1}}{n+1}$

$$W = \frac{1}{C} \left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{1}{C} \left[\frac{Q^2}{2} - \frac{0^2}{2}\right]$$

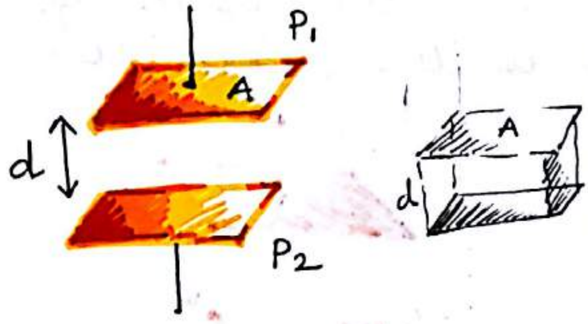
$$W = \frac{1}{C} \left[\frac{Q^2}{2}\right] = \frac{Q^2}{2C}$$

$$W = \boxed{U = \frac{Q^2}{2C}}$$

$Q = CV \Rightarrow \therefore U = \frac{C^2V^2}{2C} = \boxed{\frac{1}{2} CV^2}$

$C = \frac{Q}{V} \Rightarrow U = \frac{Q^2}{2\frac{Q}{V}} = \boxed{\frac{1}{2} QV}$

Energy density.



Energy density = $\frac{\text{Energy}}{\text{Volume}}$

$$= \frac{\frac{1}{2}CV^2}{A \cdot d} = \frac{\frac{1}{2}\frac{\varepsilon_0 A V^2}{d}}{Ad}$$

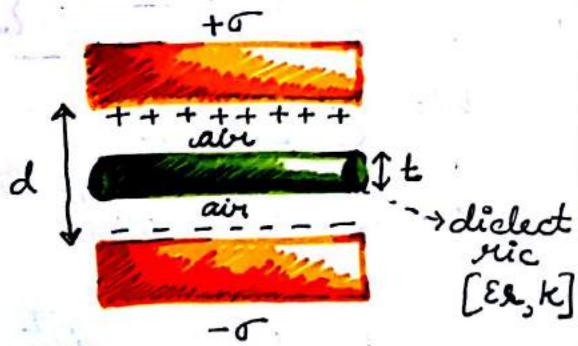
$$= \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$$

$$= \frac{1}{2}\varepsilon_0 \frac{E^2 d^2}{d^2}$$

$\because$ $C = \frac{\varepsilon_0 A}{d}$, $E = \frac{V}{d}$, $V = Ed$

$$\boxed{\text{Energy density} = \frac{1}{2}\varepsilon_0 E^2}$$

capacitance when dielectric is placed between the plates.



consider a parallel plate capacitor of plate area 'A' and seperation 'd' with air as dielectric

Hence the potential difference b/w the plates

$$V = V_0 + V_m$$

$$V = E_0(d-t) + E_m t$$

$$V = E_0(d-t) + \frac{E_0}{\varepsilon_r} t$$

$V = Ed$, $\frac{V}{d} = E$

$$V = E_0\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

$\varepsilon_r = \frac{E_0}{E_m}$, $\therefore E_m = \frac{E_0}{\varepsilon_r}$

$$V = \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

$\therefore \varepsilon_r = k$

$$V = \frac{Q}{A\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right] \quad \text{--- (1)}$$

$$C = \frac{Q}{V} \Rightarrow \frac{Q}{\frac{Q}{A\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]}$$

$$\boxed{C = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{\varepsilon_r}}}$$

If the space is completely filled by dielectric slab ($t = d$).

$$C = \frac{\varepsilon_0 A}{d/\varepsilon_r} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0 k A}{d}$$

$$\boxed{C = C_0 \varepsilon_r}$$

Dielectrics

A dielectric is an Insulator which does not conduct electric current.

A dielectric material may be defined as an insulating material which transmits electric effect without conducting.

dielectric constant

$$\varepsilon_r = K = \frac{\varepsilon}{\varepsilon_0} = \frac{F_0}{F_m} = \frac{E_0}{E_m} = \frac{C_m}{C_0}$$

$$\boxed{\varepsilon_r = \frac{C_m}{C_0}}$$

$$\boxed{\varepsilon_r = \frac{E_0}{E_m} = \frac{F_0}{F_m}}$$

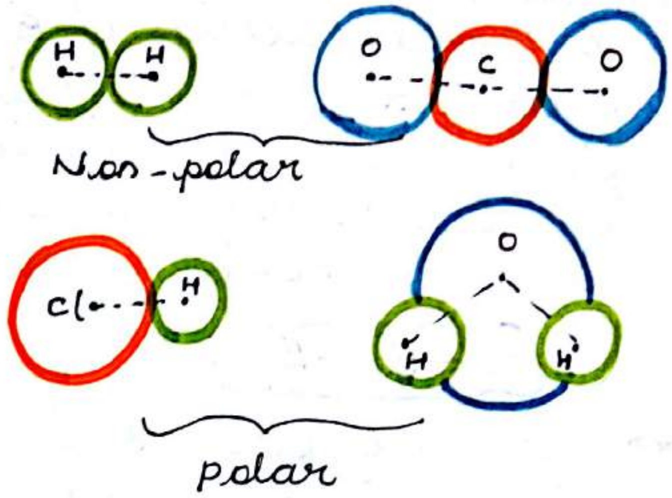
Dielectric strength

Maximum electric field 'E' that a dielectric medium can withstand without loosing its Insulating property is called dielectric strength.

Break down voltage of air is 3×10^6 V/m

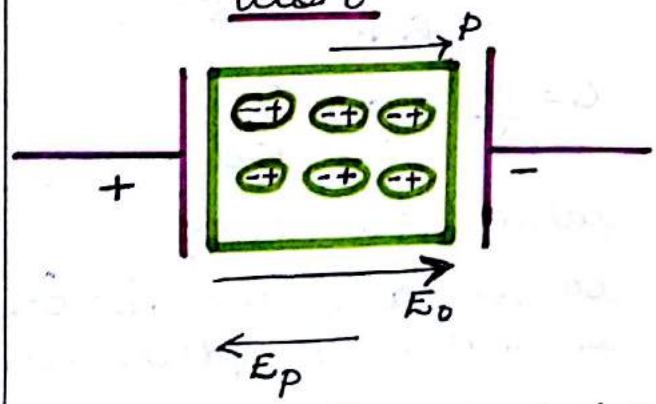
Types of dielectric

1. Polar dielectric
2. Non-polar dielectric.



Non Polar dielectrics	Polar Dielectrics
1. centres of +ve and -ve charges coincide.	1. centres of +ve and -ve charge do not coincide even in the absence of electric field.
2. Dipole moment is zero. Eg; CO_2, H_2	2. They have a permanent dipole moment. Eg; HCl, NH_3

Dielectric Polarisation



In an external electric field $\bar{E_0}$, dielectrics aquire a dipole moment $\vec{p}$ in the direction of $\vec{E}_0$ produces an electric field inside dielectric opposite to that of external electric field.

$$\boxed{E = E_0 - E_p}$$

** Electric field Inside dielectric is reduced due to dielectric polarisation.

** <u>Electric field change when dielectric is Introduced.</u>

$$\varepsilon_r = \frac{E_0}{E_m}$$

$$E_m = \frac{E_0}{\varepsilon_r} = \frac{E_0}{K}$$

'E' decreases by 'K' times.

** <u>capacitance change when dielectric is Introduced.</u>

$$E = E_0 - E_p .$$

$$E_m = \frac{E_0}{K}.$$

$E \downarrow$

$$C = \frac{Q}{V} = \frac{Q}{E \cdot d}$$

when $E \downarrow$ $C \uparrow$

so capacitance increase when (E) decreases

** <u>Relation b/w surface charge density and radius of curvature</u>

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi r^2}.$$

$$\sigma \propto \frac{1}{r^2}$$

If 'r' increases $\sigma \downarrow$

If $r \downarrow$ $\sigma \uparrow$

** <u>action of points</u>

leakage of charges near sharp points due to Induction is called action of points

uses: lightning arrestor
Vande graff generator.

Van de Graff generator

<u>principle</u>

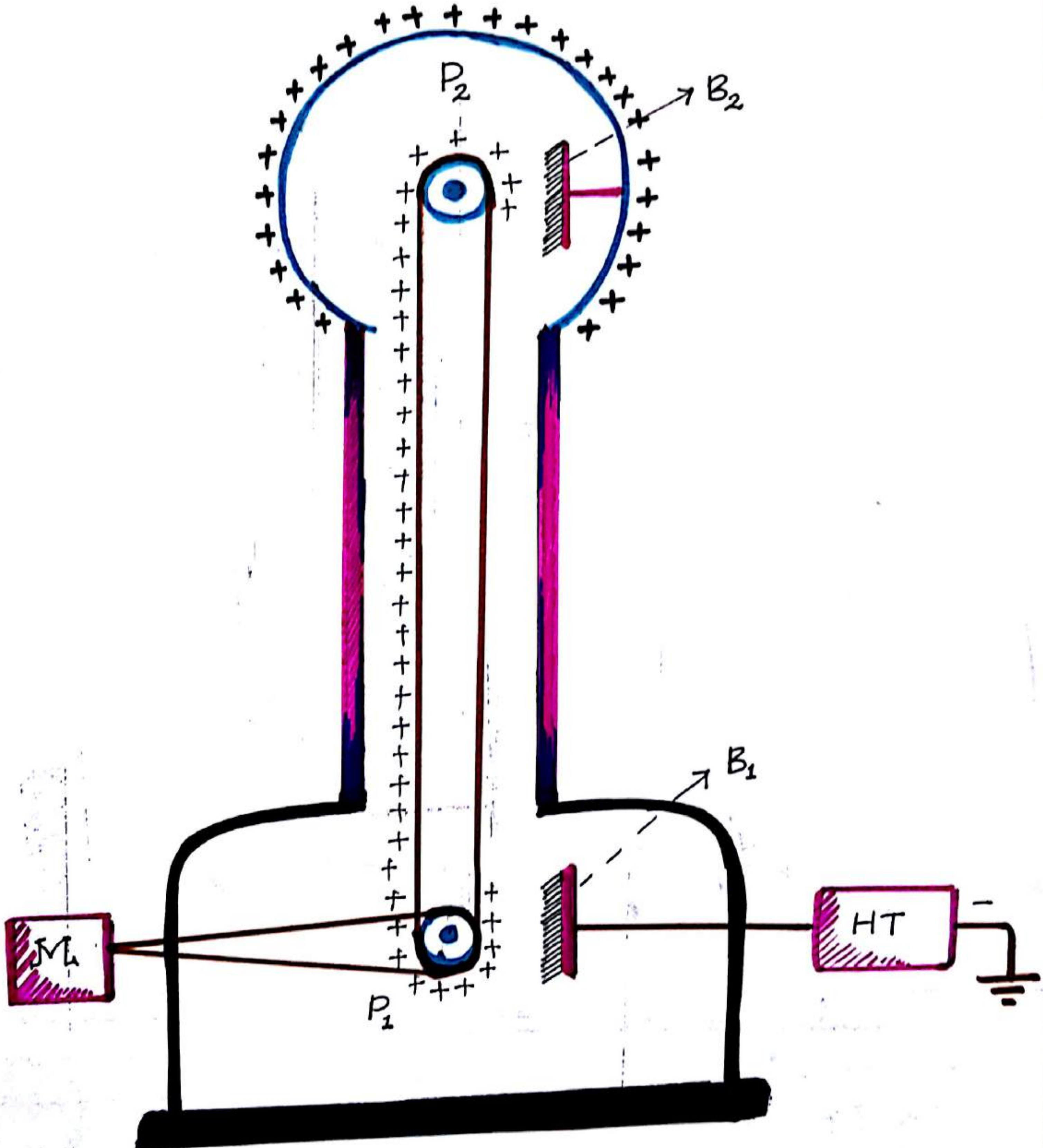
(i) Action of points:- Electric discharge takes place at the sharp ends.

(ii) Electrostatic Induction

(iii) Uniform distribution of charges at on the surface of a spherical conducting sphere.

<u>working</u>

(i) A sprayer comb is given a '+ve' potential [10^4 V]



Vande Graaff Generator

simil

(ii) By the phenomenon of action of points, the spray comb sprays positive charges on the belt.

(iii) The belt moves up towards the collecting comb.

(iv) By electrostatic Induction, tips of collecting comb acquires -ve charges and metal spheres aquires +ve charges. +ve charges immediatly shift to outer surface of hollow metal sphere, and negative charges are neutrilised by the +ve charges in the belt.

(V) The uncharged portions of the belt goes down and collects positive charges from spray comb which is in turn collected by the collecting comb. This is repeated.

(Vi) Thus positive charges on metal sphere goes on accumulating.

** capacity of metal sphere $C = 4\pi\varepsilon_0 R$

** Potential of spherical shell $V = \frac{Q}{C}$

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$R \rightarrow$ Radius of metal sphere.

** Vande graff generator is capable of building up potential difference of a few million volt, and electric field close to the break down field of air about 3×10^6 V/m.

** To minimise the leakage of charges the entire arrangement is enclosed inside a steel chamber filled with N_2 or CH_4 (methane) at high pressure.

uses

(1) To increase the potential

(2) It is used to bring out nuclear disintegration using accelerated particles.