1

Electric Charges and Fields

PREVIOUS YEARS' Questions

2007

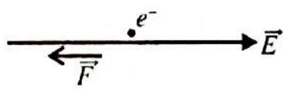
VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

1. Is the force acting between two point electric charges q_1 and q_2 kept at some distance apart in air, attractive or repulsive when
(i) $q_1q_2 > 0$? (ii) $q_1q_2 < 0$? *[Foreign]*

Ans. (i) When $q_1q_2 > 0$, force is repulsive,
(ii) When $q_1q_2 < 0$, force is attractive.

2. The force on an electron kept in an electric field in a particular direction is F. What will be the magnitude and direction of the force experienced by a proton kept at the same point in the field? Mass of the proton is about 1836 times the mass of the electron. *[Foreign]*

Ans. Same in magnitude and opposite in direction.



SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

3. The electric field E due to a point charge at any point near it is defined as $E = \lim_{q \to 0} \frac{F}{q}$ where q is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression? Draw the electric field lines of a point charge Q when (i) Q > 0 and (ii) Q < 0. *[Delhi]*

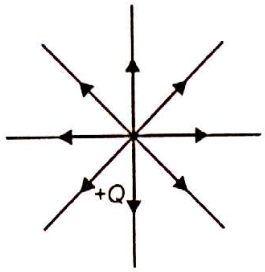
Ans. Electric field is defined as

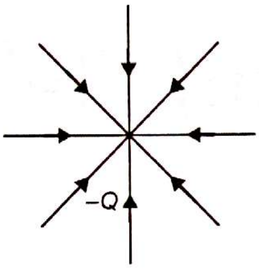
$$E = \lim_{q \to 0} \frac{F}{q}$$

We take limiting value of 'q'. It indicates that

(a) Charge is so small in magnitude that it does not change the position of source charge.

(b) It does not modify the electric field of the source charge.

(i) Q > 0 

(ii) Q < 0 

4. Define electric flux. Write its SI units. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason. *[Delhi]*

Ans. Total number of electric lines of force passing through a given area normally is called electric flux through that area.

$$\phi_E = \vec{E}.\vec{A}$$

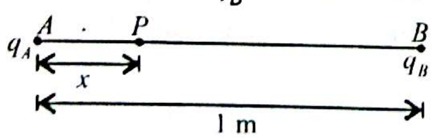
It's SI unit is N m^2 C^{-1}

As electric flux does not depend upon the shape and size of the closed surface.

The electric flux coming out of the surface will remain same as long as the charge enclosed by it remains same.

5. Two point charges 4 μC and –2 μC are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero. *[All India]*

Ans. $q_A = 4 \times 10^{-6}$ C; $q_B = -2 \times 10^{-6}$ C



$$V_{PA} = \frac{kq_A}{x}$$

$$V_{PB} = \frac{kq_B}{(1-x)}$$

$$V_{PA} + V_{PB} = 0$$

$$\frac{9\times10^9\times4\times10^{-6}}{x} = \frac{9\times10^9\times2\times10^{-6}}{1-x}$$

$$2 = \frac{x}{1-x}$$

$$2 - 2x = x$$

$$x = \frac{2}{3} \text{ m}$$

6. A cubical Gaussian surface encloses a charge of 8.85 × 10^{-10} C in vacuum at the centre. Calculate the electric flux passing through one of its faces. *[Foreign]*

Ans. $q = 8.85 \times 10^{-10}$ C

Electric flux through the whole cube is

$$\phi_E = \frac{q}{\varepsilon_0}$$

Through a single surface $\phi'_E = \frac{q}{6\varepsilon_0}$

$$\phi'_E = \frac{8.85\times10^{-10}C}{6(8.85\times10^{-12}C^2N^{-1}m^{-2})}$$

$$= 16.67 \text{ Nm}^2 \text{C}^{-1}$$

7. A spherical Gaussian surface encloses a charge of 8.85 × 10^{-10} C.

(i) Calculate the electric flux passing through the surface.

(ii) How would the flux change if the radius of the Gaussian surface is doubled and why? *[Foreign]*

Ans. Given: $q = 8.85 \times 10^{-10}$ C

(i) $$\phi_E = \frac{q}{\varepsilon_0} = \frac{8.85\times10^{-10}C}{8.85\times10^{-12}C^2N^{-1}m^{-2}}$$

$$= 10^2 \text{ Nm}^2\text{C}^{-1}$$

(ii) As flux depends only upon the charge enclosed by the surface, it will not change.

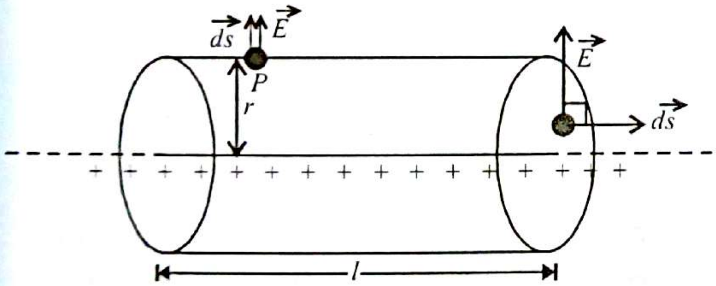
SHORT ANSWER TYPE QUESTIONS[II] **[3 Marks]**

8. State Gauss's theorem in electrostatics. Apply this theorem to derive an expression for electric field intensity at a point near an infinitely long straight charged wire. *[All India]*

Ans. Gauss's theorem: Total electric flux through a given closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\phi_E = \frac{q_0}{\varepsilon_0}$$

Consider an infinitely long straight charged wire having linear charged density 'λ'.



To find out electric field intensity at point '*P*' distant '*r*' from the wire we imagine a closed symmetrical Gaussian surface of radius '*r*' and length '*l*' around the wire.

Net electrical flux through the surface is

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = E(2\pi rl) \quad ...(i)$$

Flux through circular faces is zero because electric field vector is perpendicular to the area vector.

According to Gauss's theorem

$$\phi_E = \frac{\lambda l}{\varepsilon_0} \quad ...(ii)$$

where 'λl'is the charge enclosed by Gaussian surface.

From equations (*i*) and (*ii*), we get

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

2008

VERY SHORT ANSWER TYPE QUESTIONS **[1 Mark]**

9. A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square. *[Delhi]*

Ans. Zero, as the two points are at same potential.

10. Which orientation of an electric dipole in a uniform electric field would correspond to stable equilibrium? *[All India]*

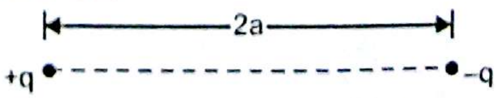
Ans. When dipole moment vector is parallel to electric field vector.

11. If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change? *[All India]*

Ans. Remains same.

12. Define the term electric dipole moment of a dipole. State its SI unit. *[All India]*

Ans. Strength of electric dipole is called dipole moment



$$|\vec{p}| = q|2\vec{a}|$$

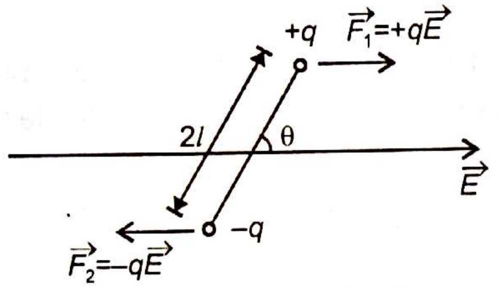
SI unit is Cm.

SHORT ANSWER TYPE QUESTIONS[I]

[2 Marks]

13. Derive an expression for the potential energy of an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$. *[Delhi]*

Ans. Expression for potential energy of an electric dipole:



Torque acting on the dipole, $\tau = pE \sin\theta$

It tends to rotate the dipole in clockwise direction. To rotate the dipole anticlockwise work has to be done on the dipole.

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

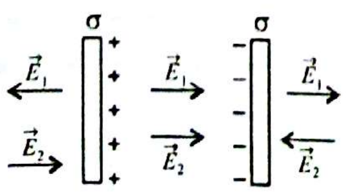
$$= \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$$= -pE(\cos\theta_2 - \cos\theta_1)$$

14. Two large parallel thin metallic plates are placed close to each other. The plates have surface charge densities of opposite signs and of magnitude 20 ×10^{-12} C/m^2. Calculate the electric field intensity (*i*) in the outer region of the plates and (*ii*) in the interior region between the plates. *[Foreign]*

Ans. Electric field due to (+ve) positively charged plate

$$E_1 = \frac{\sigma_1}{2\varepsilon_0}$$



Electric field due to (–ve) negatively charged plate is

$$E_2 = \frac{\sigma_1}{2\varepsilon_0}$$

For $\sigma_1 = \sigma_2 = \sigma$

(*i*) Net electric field in the outer regions is

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(*ii*) Net electric field in the interior region is

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{20\times10^{-12}\text{Cm}^{-2}}{8.85\times10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}}$$

$$= 2.26 \text{ NC}^{-1}$$

15. A point charge causes an electric flux of – 3 × 10^4 Nm^2/C to pass through a spherical Gaussian surface.

(*i*) Calculate the value of the point charge.

(*ii*) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface? *[Foreign]*

Ans. Given: $\phi = -3 \times 10^4 \text{ Nm}^2\text{C}^{-1}$

(*i*) From Gauss's Theorem, we know that

$$\phi = \frac{q}{\varepsilon_o}$$

$$q = \phi \times \varepsilon_o$$
$$= -3 \times 10^4 \times 8.854 \times 10^{-12}$$
$$= 2.65 \times 10^{-7} \text{ C}$$

(*ii*) Flux through the surface will remain the same *i.e.*, -3×10^4 Nm^2 C^{-1}.

16. **(a)** **Using Gauss's law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius R and charge density σ C/m^2. Draw the field lines when the charge density of the sphere is (*i*) positive, (*ii*) negative.**

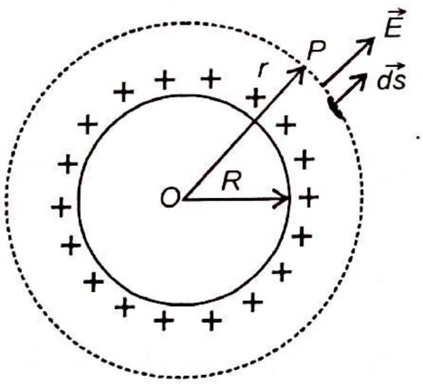
(b) **A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m^2. Calculate the**

(*i*) charge on the sphere

(*ii*) total electric flux passing through the sphere.

[*Delhi*]

Ans. (*a*) To find out electric field at a point outside a spherical charged shell we imagine a symmetrical Gaussian surface in such a way that the point lies on it.



We have $d\phi = \vec{E}.\vec{ds}$

$d\phi = Eds \quad (\because \theta = 0°)$

$\phi = E\oint ds$

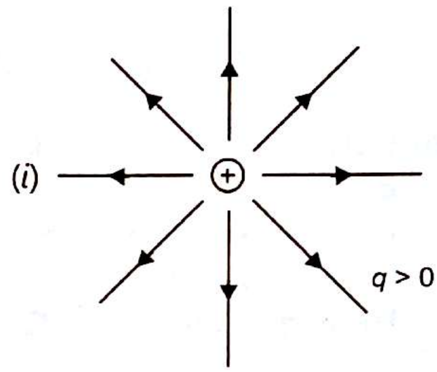
$\phi = E.4\pi r^2 \quad ...(i)$

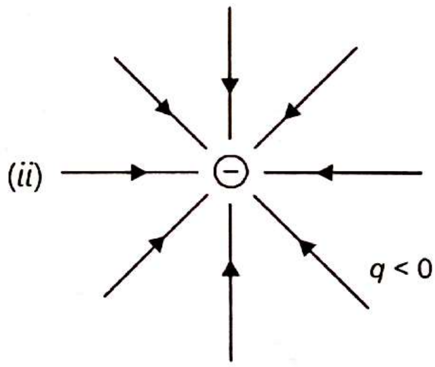
According to Gauss's theorem

$$\phi = \frac{q}{\varepsilon_0} = \frac{\sigma 4\pi R^2}{\varepsilon_0} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$E = \frac{\sigma}{\varepsilon_0}.\left(\frac{R}{r}\right)^2$$





(*b*) (*i*) Given : $r = \frac{2.5}{2}$ m;

$\sigma = 100\ \mu\ Cm^{-2}$

Charge on the sphere,

$Q = \sigma.\ 4\pi r^2$

$Q = 100 \times 10^{-6} \times 4 \times 3.14 \times \left(\frac{2.5}{2}\right)^2$

$= 19.6 \times 10^{-4}$ C

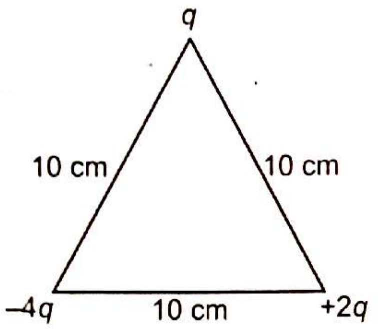
(*ii*) Flux passing through the sphere

$$\phi = \frac{Q}{\varepsilon_0}$$

$$\phi = \frac{19.6 \times 10^{-4}}{8.85 \times 10^{-12}}$$

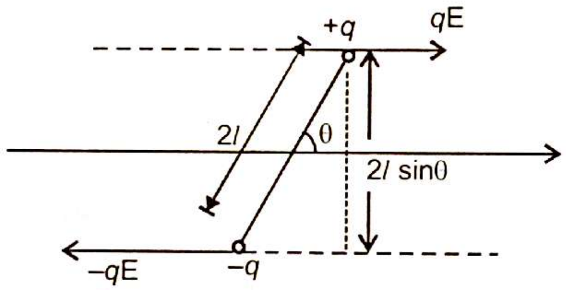
$= 2.2 \times 10^8\ Nm^2C^{-1}$

17. **(a)** **Derive an expression for the torque experienced by an electric dipole kept in a uniform electric field.**



(b) **Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown.**
Here, $q = 1.6 \times 10^{-10}$ C. [Delhi]

Ans. (a)



Electric force on – q charge,

$$\vec{F_1} = -q\vec{E}$$

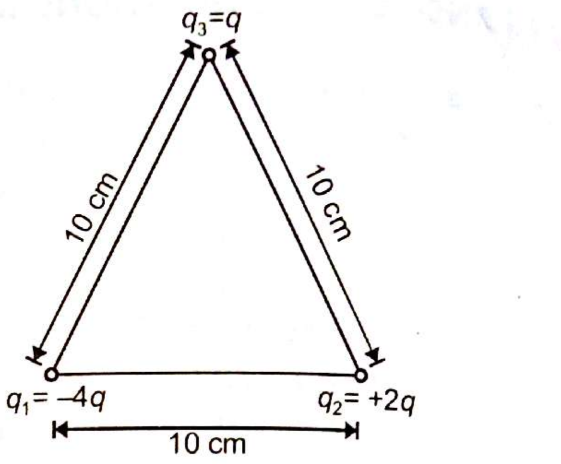
Electric force on + q charge,

$$\vec{F_2} = +q\vec{E}$$

$$\text{Torque } (\tau) = qE.\,2l \sin\theta$$
$$= (q.2l)\, E \sin\theta$$
$$\tau = PE \sin\theta$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

(b) Work done = – (Electric potential energy)



$$W = -\frac{1}{4\pi\varepsilon_0 l}(q_1q_2 + q_2q_3 + q_1q_3)$$

$$= -\frac{9\times10^9}{0.1}(-8q^2 + 2q^2 - 4q^2)$$

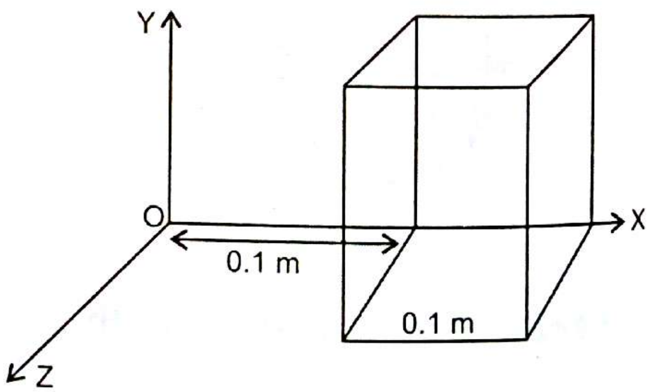
$$= \frac{9\times10^9\times10q^2}{0.1} = 9\times10^{11}q^2$$

$$= 9\times10^{11}\times(1.6\times10^{-10})^2 \text{ J}$$

$$W = 2.3\times10^{-8} \text{ J}$$

18. **(a)** **Define electric flux. Write its SI units.**

(b) **The electric field components due to a charge inside the cube of side 0.1 m are as shown:**



$E_x = \alpha x$, where α = 500 N/C-m

$E_y = 0$, $E_z = 0$

Calculate (i) the flux through the cube, and (ii) the charge inside the cube. [All India]

Ans. (*a*) Electric flux through a surface represents the total number of electric lines of force crossing the surface normally.

SI unit is Nm^2C^{-1}

(*b*) (*i*) Flux through R.H.S. of the cube is

$$\phi_1 = E_{x_1} \cdot A \qquad \because \theta = 0°$$
$$= (\alpha x).(l)^2$$
$$\because \quad \alpha = 500 \text{ N/C–m}$$
$$x = 0.2 \text{ m}$$
$$l = 0.1 \text{ m}$$
$$\phi_1 = (500\times0.2)(0.1)^2$$
$$= 1 \text{ Nm}^2\text{C}^{-1}$$

Flux through L.H.S. of the cube is

$$\phi_2 = E_{x_2} A \qquad \because \theta = 180°$$
$$= -(\alpha x)\cdot l^2$$
$$\because \quad \alpha = 500 \text{ N/C–m}$$
$$x = 0.1 \text{ m}$$
$$l = 0.1 \text{ m}$$
$$\phi_2 = -(500\times0.1)(0.1)^2$$
$$= -0.5 \text{ Nm}^2\text{C}^{-1}$$

Net flux,

$$\phi = \phi_1 + \phi_2 = 1 - 0.5$$
$$= 0.5 \text{ Nm}^2\text{C}^{-1}$$

(*ii*) As, $\phi = \dfrac{q}{\varepsilon_o}$

$$q = \varepsilon_o\phi$$
$$= 8.854 \times 10^{-12} \times 0.5$$
$$= 4.4 \times 10^{-12}\text{C}$$

19. An electric dipole of dipole moment $\vec{p}$ is held in a uniform electric field $\vec{E}$.

(*i*) Prove that no translatory force acts on the dipole.

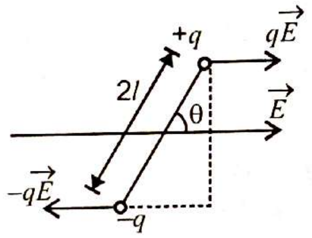
(*ii*) Hence prove that the torque acting on the dipole is given by pE sin θ, indicating the direction along which it acts.

(*iii*) How much work is required in turning the electric dipole, from the position of most stable equilibrium to the position of most unstable equilibrium?

[*Foreign*]

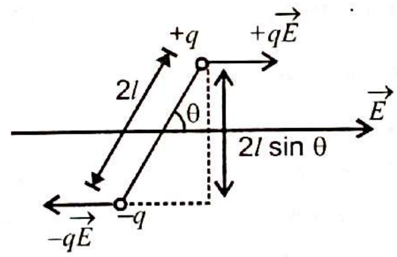
Ans. (*i*) Force on $+q$ charge $= +q\vec{E}$

Force on $-q$ charge $= -q\vec{E}$



As forces are equal in magnitude and opposite in direction net force = 0, *i.e.*, no translatory force acts on the dipole.

(*ii*) Magnitude of the torque,



$$\tau = qE(2l \sin \theta)$$
$$\vec{\tau} = \vec{P} \times \vec{E}$$

($\because P = \theta(2l)$)

(*iii*) $$W = -pE(\cos\theta_2 - \cos\theta_1)$$

Stable equilibrium corresponds to $\theta_1 = 0°$

Unstable equilibrium corresponds to

$$\theta_2 = 180°$$

Hence, $W = -pE(\cos 180° - \cos 0°)$

$$= -pE(-1-1)$$
$$W = 2\,pE$$

2009

SHORT ANSWER TYPE QUESTIONS

[2 Marks]

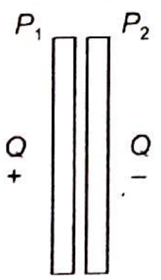
20. Define electric flux. Write its SI unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change? *[Foreign]*

Ans. Electric flux over an area in an electric field is the total number of lines of force passing through the area. It is represented by ϕ. It is a scalar quantity. Its SI unit is N $m^2 C^{-1}$ or V–m

i.e., $$\phi = \int_S \vec{E} \cdot \vec{dS}$$

Electric flux $\propto q_{en}$. So it is invariant with radius. [If the charge q is not at the centre, but at a distance x such that $\frac{R}{2} < x < R$, then ϕ will be zero.]

21. Figure shows two large metal plates, P_1 and P_2, tightly held against each other and placed between two equal and unlike point charges perpendicular to the line joining them.

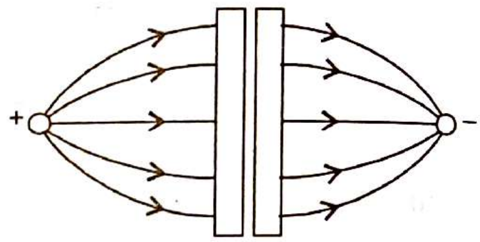


(i) What will happen to the plates when they are released?

(ii) Draw the pattern of the electric field lines for the system. *[Foreign]*

Ans. (*i*) They tend to move apart slightly due to polarisation of charges in them.

(*ii*)



22. The sum of two point charges is 7 μC. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge. *[Foreign]*

Ans. $q_1 + q_2 = 7\ \mu C \quad F = 1\ N \quad$ at $\quad r = 30$ cm $= 30 \times 10^{-2}$ m

$$\therefore \quad \frac{kq_1q_2}{r^2} = 1 \quad or$$

$$kq_1q_2 = 9 \times 10^{-2}$$

$$\because \quad k = 9 \times 10^9\ Nm^2C^{-2}$$

$$i.e., \quad q_1q_2 = 10^{-11}$$

$$= 10 \times 10^{-12}$$

Using $q_1 = (7 - q_2)$, we have

$$(7 - q_2)\, q_2 = 10 \text{ with charges in } \mu C.$$

$$7q_2 - q_2^2 = 10$$

$$q_2^2 - 7q_2 + 10 = 0$$

$$q_2 = \frac{7 \pm \sqrt{49 - 40}}{2} = \frac{7 \pm 3}{2}$$

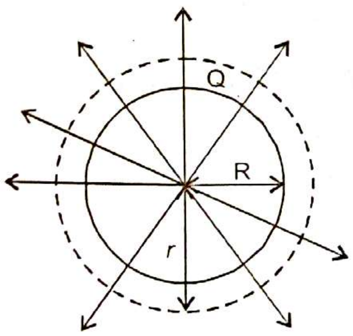
$$= 5\ \mu C \text{ or } 2 \mu C$$

$$\therefore \quad q_1 = 7 - q_2 = 2\ \mu C \text{ or } 5\ \mu C$$

23. **A thin conducting spherical shell of radius *R* has charge Q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell.**

Draw a graph of electric field *E*(*r*) with distance *r* from the centre of the shell for $0 \le r \le \infty$. *[Delhi]*

Ans. Consider the given spherical shell of radius *R* holding charge *Q*. Construct a gaussian surface of radius *r* (concentric and symmetrical). The field lines will pass perpendicularly through the gaussian surface in all the direction.

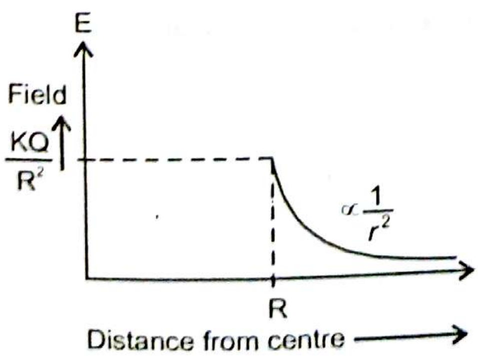


So, the effective Gaussian surface area having normal field will be $4\pi r^2$.

Using Gauss' theorem, $\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\varepsilon_0}$

we get

$$\int E\, ds \cos 0^\circ = \frac{Q}{\varepsilon_0}$$



$$\Rightarrow \quad E\int ds = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

For all points inside the shell, since $q_{en} = 0$, $E = 0$. The graph showing the variation of *E* with distance *r* from the centre is shown.

24. **State Gauss' law in electrostatics. Use this law to derive an expression for the electric field due to an infinitely long straight wire of linear charge density λ C m^{-1}.** *[Delhi]*

Ans. Gauss's Theorem states that the total normal electric flux over a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by it *i.e.*,

$$\phi = \int_s \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon_0}$$

The closed surface is called Gaussian surface, which has following characteristics:

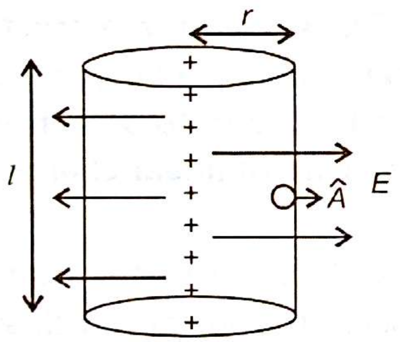
(*a*) It is an arbitrary surface chosen in such a way that the charge may lie either inside or outside the surface but never on the surface.

(*b*) It is generally taken symmetrical to the given charged structure.

Consider an infinitely long straight uniformly charged wire and with radius r and length l. Construct a symmetrical Gaussian surface. If the charge density is λ on the wire, the charge enclosed by the Gaussian surface = $q = \lambda l$. The electric field will go normal to the curved surface area of the cylinder while the cross-sectional area will carry a field along its surface. So the effective Gaussian surface area = $2\pi rl$.

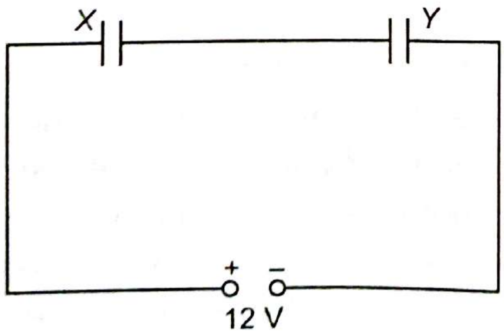
Using Gauss' theorem, we get

$$E \times 2\pi rl = \frac{\lambda l}{\varepsilon_o}$$



$$\therefore \quad E = \frac{\lambda}{2\pi r\varepsilon_o}$$

25. Two parallel plate capacitor, *X* and *Y*, have the same area of plates and same separation between them. *X* has air between the plates while *Y* contains a dielectric medium of $\varepsilon_r = 4$.



(*i*) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(*ii*) Calculate the potential difference between the plates of *X* and *Y*.

(*iii*) What is the ratio of electrostatic energy stored in *X* and *Y*? *[Delhi]*

Ans. Since area A and separation d are same, if $C_x = C$ then $C_y = 4\,C$.

(*i*) Since they are in series,

$$\frac{1}{4} = \frac{1}{C} + \frac{1}{4C}$$

$$\Rightarrow \quad 4 = \frac{4C}{5} \text{ or } C = 5\ \mu F$$

The two capacitors are therefore 5 μF and 20 μF.

(*ii*) Since the capacitance of capacitors are in the ratio 1 : 4, the potential drop across them should be in the ratio 4 : 1 making them $4 \times \frac{12}{5} : \frac{12}{5}$.

Therefore, V_x = 9.6 volt and V_y = 2.4 volt

(*iii*) Since they carry same charge, the ratio of the electrostatic energy is

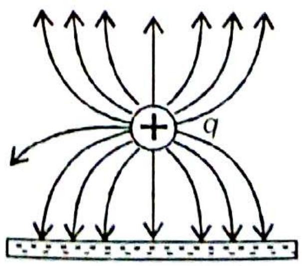
$$\frac{Q^2}{2C_x} : \frac{Q^2}{2C_y}$$

i.e., $C_y : C_x = 4 : 1$

26. A positive point charge (+ *q*) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate.

Derive the expression for the electric field at the surface of a charged conductor. *[All India]*

Ans.



Let σ be the surface charge density (charge per unit area) of the given sheet and let P be a point at distance r from the sheet where we have to find $\vec{E}$.

Choosing point P', symmetrical with P on the other side of the sheet, let us draw a Gaussian cylindrical surface cutting through the sheet as shown in the diagram. As at the cylindrical part of the Gaussian surface, $\vec{E}$ and $\vec{ds}$ are at right angle. The only surfaces having $\vec{E}$ and $\vec{ds}$ parallel are the plane ends.

$$\therefore \quad \phi_E = \oint \vec{E} \cdot \vec{ds} + \oint \vec{E} \cdot \vec{ds}$$

$$= \oint \vec{E}\,\vec{ds} + \oint \vec{E}\,\vec{ds}$$

$$= EA + EA = 2EA$$

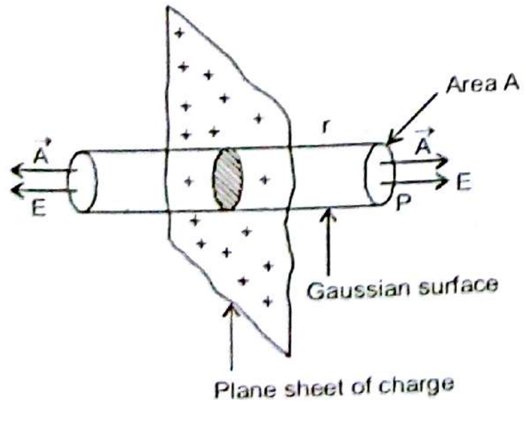
(As $\vec{E}$ is outgoing from both plane ends, the flux is positive.)

This is the total flux through the Gaussian surface.

Using the Gauss's law, $\phi_E = \dfrac{q}{\epsilon_0}$

$$\therefore \quad 2EA = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

(as $q = \sigma A$)



$\therefore E = \dfrac{\sigma}{2\epsilon_0}$. This value is independent of r. Hence, the electric field intensity is same for all points near the charged sheet.This is called uniform electric field intensity.

27. Use Gauss's law to derive the expression for the electric field between two uniformly charged large parallel sheets with surface charge densities σ and – σ respectively.

[All India]

Ans. Let σ be the surface charge density (charge per unit area) of the given sheet and let P be a point at distance r from the sheet where we have to find $\vec{E}$.

Choosing point P', symmetrical with P on the other side of the sheet, let us draw a Gaussian cylindrical surface cutting through the sheet as shown in the diagram. As at the cylindrical part of the Gaussian surface, $\vec{E}$ and $\vec{ds}$ are at right angle. The only surfaces having $\vec{E}$ and $\vec{ds}$ parallel are the plane ends.

$$\therefore \quad \phi_E = \oint \vec{E} \cdot \vec{ds} + \oint \vec{E} \cdot \vec{ds}$$

$$= \oint Eds + \oint Eds$$

$$= EA + EA = 2EA$$

(As $\vec{E}$ is outgoing from both plane ends, the flux is positive.)

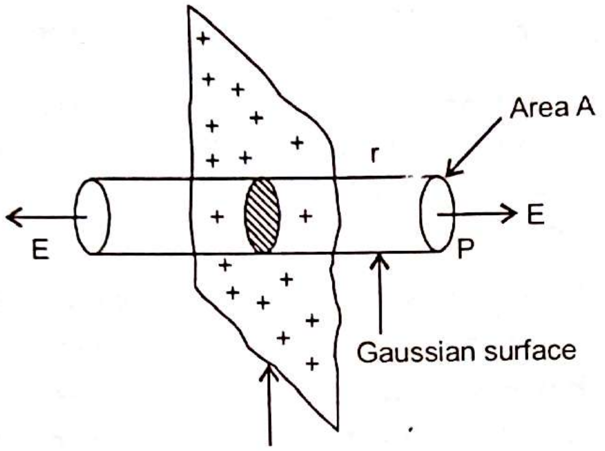
This is the total flux through the Gaussian surface.

Using the Gauss's law,

$$\phi_E = \frac{q}{\varepsilon_0}$$

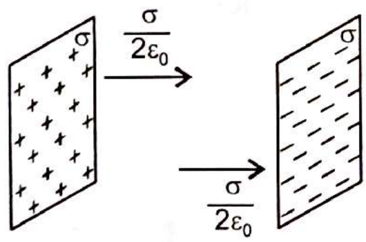
$$\therefore \quad 2EA = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

(as $q = \sigma A$)



$\therefore\ E = \frac{\sigma}{2\varepsilon_0}$. This value is independent of r. Hence, the electric field intensity is same for all points near the charged sheet.This is called uniform electric field intensity.

When two plates with charge densities $+\sigma$ and $-\sigma$ are separated by a distance then the region between them will experience a field E as shown



$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}.$$

28. Define the term 'electric dipole moment'. Is it a scalar or vector? Deduce an expression for the electric field at a point on the equatorial plane of an electric dipole of length 2*a*. *[Foreign]*

Ans. Electric Dipole moment: It is a measurement of the strength of electric dipole and is given by

$$\vec{p} = q(2\vec{a})\text{ cm}$$

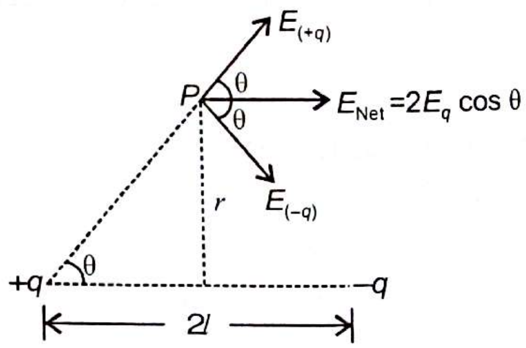
where $\vec{p}$ is electric dipole moment and $2a$ is the separation between the charges. It is a vector quantity directed from negative to positive on the line joining them.

Let the dipole be made of two equal and opposite charges $+q$ and $-q$, separated by $2l$. Consider a point P at a distance r from the mid-point. Field at P due to each charge will be of equal magnitude

$$|E_q| = \frac{kq}{(r^2 + a^2)}$$ pointing as shown.

Net field at $P = 2E_q \cos\theta$

$$= \frac{2kq}{(r^2 + a^2)} \cdot \frac{l}{(r^2 + a^2)^{1/2}}$$



$$E = \frac{2lqk}{(r^2 + a^2)^{3/2}} = \frac{kp}{(r^2 + a^2)^{3/2}}$$

pointing anti-parallel to dipole moment.

If $r >> a$, (*i.e.*) the point is far-off, $E = \frac{kp}{r^3}$ anti-parallel to $\vec{p}$.

2010

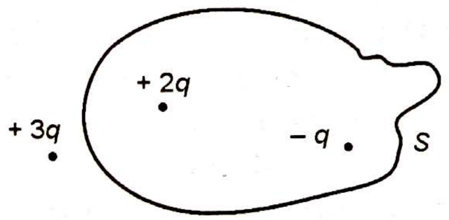
VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

29. In which orientation, a dipole placed in a uniform electric field is in (*i*) stable, (*ii*) unstable equilibrium? *[Delhi]*

Ans. (*i*) When dipole moment vector is parallel to electric field.

(*ii*) When dipole moment vector is anti-parallel to electric field.

30. Figure shows three point charges, + 2*q*, – *q* and + 3*q*. Two charges + 2*q* and – *q* are enclosed within a surface '*S*'. What is the electric flux due to this configuration through the surface '*S*'? *[Delhi]*



Ans. Electric flux, $\phi = \dfrac{q}{\varepsilon_0}$.

31. A charge Q μC is placed at the centre of a cube. What is the electric flux coming out from any one surface? *[Foreign]*

Ans. Electric flux $\phi = \dfrac{Q}{6\varepsilon_0}$

SHORT ANSWER TYPE QUESTIONS [I] [2 Marks]

32. Show that the electric field at the surface of a charged conductor is given by $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. *[All India]*

Ans. Electric field at a point on the surface of charged conductor,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

For simplicity we consider charged conductor as a sphere of radius '*R*'. If 'σ' is the surface charge density, then

$$Q = 4\pi R^2\sigma$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$

As the charge on the sphere is +ve electric field is normal to the surface directed outwards.

i.e., $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$.

2011

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

33. A proton is placed in a uniform electric field directed along the positive x-axis. In which direction will it tend to move? *[Delhi(C)]*

Ans. The proton has a tendency to move along the +ve x-axis.

34. Define electric dipole moment. Write its SI unit. *[All India]*

Ans. A vector having magnitude q times the separation between the pair of charges constituting the electric dipole and direction from $-q$ to $+q$ is called electric dipole moment.

$$\vec{p} = q \times (2a)\,\hat{p}$$

SI unit is Cm.

35. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? *[All India]*

Ans. 10 V

36. Two point charges having equal charges separated by 1 m distance experience a force of 8N. What will be the force experienced by them, if they are held in water, at the same distance? (Given : K_{water} = 80) *[All India(C)]*

Ans. The force F′ in water is given by

$$F' = \frac{F_{air}}{K} = \frac{8N}{80} = \frac{1}{10} = 0.1\text{ N}$$

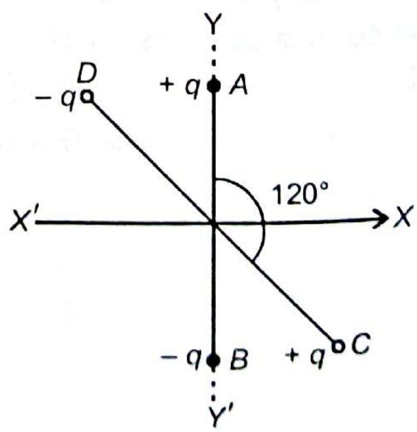
37. Two insulated charged copper spheres *A* and *B* of identical size have charges q_A and q_B respectively. A third sphere *C* of the same size but uncharged is brought in contact with the first and then in contact with the second and finally removed from both. What are the new charges on *A* and *B*? *[Foreign]*

Ans. Final charge on sphere *A*, $q'_A = \frac{q_A}{2}$

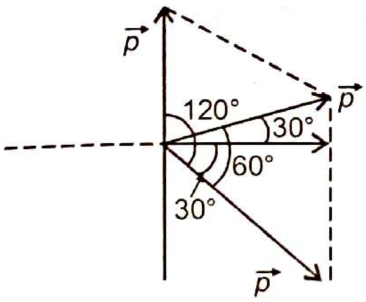
Final charge on sphere *B*, $q'_B = \frac{2q_B + q_A}{4}$

SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

38. Two small identical electrical dipoles *AB* and *CD*, each of dipole moment '*p*' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field ($\vec{E}$) directed along + *X* direction, what will be the magnitude and direction of the torque acting on this? *[Delhi]*



Ans.



(i) $p_R = \sqrt{p^2 + p^2 + 2p^2 \cos 120°}$

$p_R = p$

$$\tan \phi = \frac{p \sin \theta}{p + p \cos \theta}$$

$$\tan \phi = \frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}}$$

$$\tan \phi = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

$$\phi = \tan^{-1}\sqrt{3}$$
$$\phi = 60°$$

(ii) As the electric field and $\vec{P_R}$ are having an angle of 30°.

$$\tau = PE \sin 30° = \frac{PE}{2}$$

Direction of torque is clockwise when viewed from above *i.e.*, into the plane of the paper.

39. Calculate the amount of work done in turning an electric dipole of dipole moment 2×10^{-8} C m from its position of unstable equilibrium to the position of stable equilibrium, in a uniform electric field of intensity 10^3 NC^{-1}. [*Delhi(C)*]

Ans. The work done in rotating a dipole in an electric field from θ_1 to θ_2 is given by

$$W = pE [\cos \theta_1 - \cos \theta_2]$$

Here, $p = 2 \times 10^{-8}$ C m, $E = 10^3$ N C^{-1}

$$\theta_1 = 180°, \theta_2 = 0°$$

$\therefore$ $W = 2 \times 10^{-8} \times 10^3$ [cos 180° – cos 0°]

$= 2 \times 10^{-5}$ [– 1 – 1]

$= -4 \times 10^{-5}$ J

40. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. [*All India*]

Ans. According to Gauss's law electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface.

$$\phi = \frac{q}{\varepsilon_0}$$

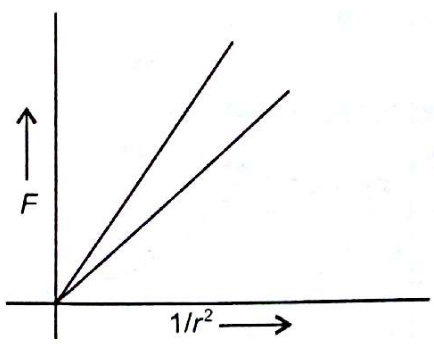
As the charge enclosed by the cylindrical surface is

$$q = \lambda l$$

$$\phi = \frac{\lambda l}{\varepsilon_0}$$

41. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance between the two charges of each pair of charges: (1μC, 2μC) and (2μC, –3μC). Interpret the graphs obtained. [*All India*]

Ans. Slope of the line is directly proportional to the force acting between the charges for a given separation.



42. Calculate the amount of work done in rotating a dipole, of dipole moment 3×10^{-8} cm, from its position of stable equilibrium to the position of unstable equilibrium, in a uniform electric field of intensity 10^4N/C. [*Foreign*]

Ans. $W = -pE(\cos\theta_2 - \cos\theta_1)$

$p = 3 \times 10^{-8}$ cm

$\theta_1 = 0°$

$\theta_2 = 180°$

$E = 10^4$ NC^{-1}

$W = -3 \times 10^{-8} \times 10^4(\cos 180° - \cos 0°)$

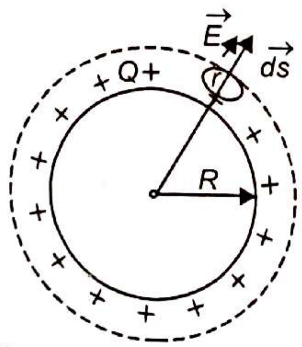
$= 6 \times 10^{-4}$ J

SHORT ANSWER TYPE QUESTIONS[II] [3 Marks]

43. Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius *R* at a point outside the shell. Draw a graph showing the variation of electric field with *r*, for *r* > *R* and *r* < *R*. [*Delhi*]

Ans.

$$\oint \vec{E}\cdot\vec{ds} = \oint \vec{E}\,\vec{ds} = E\oint \vec{ds} = E \times 4\pi r^2 \quad ...(i)$$

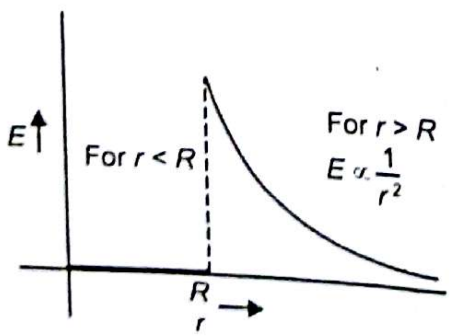


According to Gauss's law

$$\oint \vec{E}\cdot\vec{ds} = \frac{Q}{\varepsilon_0} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

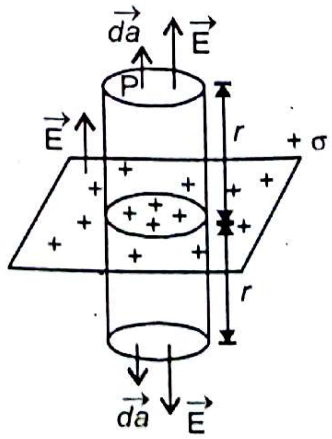


44. State Gauss's law in electrostatics. Use this law to derive the expression for the electric field due to a uniformly charged infinite thin plane sheet. [*Delhi*]

Ans. According to Gauss' theorem total electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\phi_E = \oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

Consider a thin infinite plane sheet of charge with uniform surface charge density σ. To calculate electric field at a point (P) at distant *r* from the sheet we imagine a symmetrical Gaussian surface in such a way that the point lies on it. Here we assume a cylinder of cross-sectional area *A* and length 2*r* with its axis perpendicular to the sheet.



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Flux through the curved surface of the cylinder (ϕ_1).

As electric lines are parallel to the curved surface flux is zero.

$$\phi_1 = \int \vec{E}.\vec{ds} = 0 \ (\because \theta = 90^o)$$

Flux through the plane and faces of the cylinder (ϕ_2).

$$\phi_2 = 2\int \vec{E}.\vec{ds} = 2EA$$
$$(\because \theta = 0^o)$$

Total flux through the cylindrical Gaussian surface is

$$\phi = 2EA.$$

Total charge enclosed by the surface

$$q = \sigma A$$

According to Gauss's theorem $\phi = \dfrac{q}{\varepsilon_0}$

i.e. $$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

LONG ANSWER TYPE QUESTIONS [5 Marks]

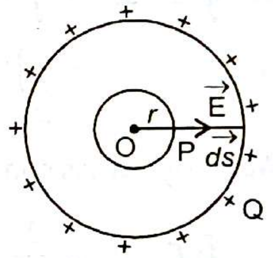
45. (a) State Gauss's law. Use it to deduce the expression for the electric field due to a uniformly charged thin spherical shell at points (*i*) inside and (*ii*) outside the shell.

(b) Two identical metallic spheres *A* and *B* having charges +4 *Q* and –10 *Q* are kept a certain distance apart. A third identical uncharged sphere *C* is first placed in contact with sphere *A* and then with sphere *B*. Spheres *A* and *B* are then brought in contact and then separated. Find the charges on the spheres *A* and *B*. [*Delhi*(*C*)]

Ans. (*a*) **Gauss's Law** states that the net outward flux through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the charge enclosed by the closed surface.

(*i*) **When the point *P* is inside the shell as shown.**

In this case, the Gaussian surface lies inside the spherical shell and hence no charge is enclosed by it.



$$\oint \vec{E}.\vec{ds} = \oint E.ds \cos 0$$
$$= E\oint ds$$
$$= E \times 4\pi r^2 \quad ...(i)$$

and by Gauss's law

$$\oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} \times 0$$

(since no charge is enclosed)

$$= 0 \quad ...(ii)$$

∴ From equations (*i*) and (*ii*), we have

$$E \times 4\pi r^2 = 0$$

or $E = 0$ *i.e.*, there is no electric field inside a charged spherical shell.

(*ii*) **When the point *P* lies outside the shell.**

Consider a spherical shell of radius r_1 charged to a potential *V* by a charge Q. To find the electric intensity at a point *P* at

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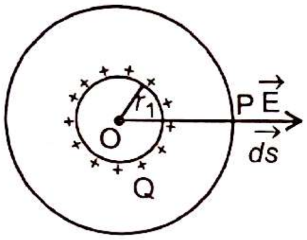
a distance r from the centre of the spherical shell imagine a spherical Gaussian surface to be drawn around the charged shell. At every point of this shell the $\vec{E}$ vector and $\vec{ds}$ vector are directed outwards in the same direction *i.e.*, $\theta = 0$



$$\therefore \quad \oint \vec{E} \cdot \vec{ds} = \oint E \cdot ds$$

$$= E \oint ds$$

$$= E \times 4\pi r^2 \quad \text{...}(i)$$

Also, by Gauss's law

$$\oint \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \cdot Q \quad \text{...}(ii)$$

From equations (*i*) and (*ii*)

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \cdot Q$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

(*b*) Initial charge on the sphere

$$A = +4\,Q$$

Initial charge on the sphere

$$B = -10\,Q$$

Since all the three spheres are identical, they have the same capacity. When uncharged sphere C is placed in contact with A, the total charge is equally shared between them.

$\therefore$ Charge on C after contact with

$$A = \frac{0+4}{2} = 2\,C$$

Charge on A after contact with C

$$= 2C.$$

When sphere C carrying a charge 2 C is placed in contact with B, again charges are equally shared between C and B.

Charge on C after it is in contact with

$$B = \frac{2-10}{2} = -4\,C$$

Now, when sphere A with a charge of 2 C is placed in contact with B, with charge – 4 C.

$$\text{Charge on } A = \frac{2-4}{2} = -1\,C$$

and charge on $B = -1$ C

2012

VERY SHORT ANSWER TYPE QUESTIONS **[1 Mark]**

46. Why should electrostatic field be zero inside a conductor? [Delhi]

Ans. In the static situation, there is no current inside, or on the surface of the conductor. Hence, electric field is zero everywhere inside the conductor.

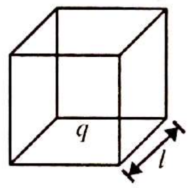
47. Why must electrostatic field be normal to the surface at every point of a charged conductor? [Delhi]

Ans. As charges on the surface of a conductor are static (*i.e.*, they do not experience any force), it is so only when there is no component of field along the surface of charged body.

Thus, we conclude that electric field is normal to the surface.

48. A charge 'q' is placed at the centre of a cube. What is the electric flux passing through a single face of the cube? *[All India]*

Ans. Total flux through the cube is



$$\phi_E = \frac{q}{\varepsilon_0}$$

As there are six faces of the cube, flux through a single face of the cube is

$$\phi'_E = \frac{\phi_E}{6} = \frac{q}{6\varepsilon_0}$$

49. Define dipole moment of an electric dipole. Is it a scalar or a vector? *[Foreign]*

Ans. The product of magnitude of either charge and distance between the charges constituting the dipole is called dipole moment.

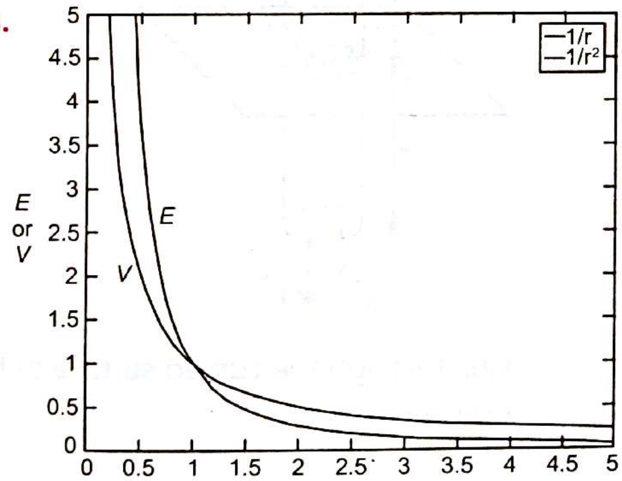
$$\vec{p} = q\vec{d}$$

It is a vector quantity. Its directon is from negative to positive charge.

SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

50. Draw a plot showing the variation of (*i*) electric field (*E*) and (*ii*) electric potential (*V*) with distance *r* due to a point charge *Q*. *[Delhi]*

Ans.

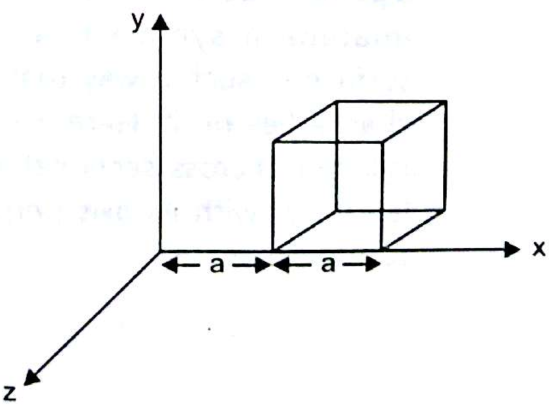


SHORT ANSWER TYPE QUESTIONS[II] [3 Marks]

51. State Gauss's law in electrostatics. *A* cube with each side '*a*' is kept in an electric field given by $\vec{E} = Cx\hat{i}$, (as is shown in the figure), where *C* is a positive dimensional constant. Find out

(*i*) the electric flux through the cube, and

(*ii*) the net charge inside the cube. *[Foreign]*



Ans. **Gauss's Law:** Surface integral of electric field (electric flux) over any closed surface is $\frac{1}{\varepsilon_0}$ times the charge enclosed in it

(i) Electric flux,

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

(ii) (a) $\phi = \oint \vec{E_1} \cdot \vec{ds_1} + \oint \vec{E_2} \cdot \vec{ds_2}$

$= -Ca^3 + 2Ca^3 = Ca^3$

(b) $\phi = \frac{q}{\varepsilon_0}$

$q = \varepsilon_0 Ca^3$

LONG ANSWER TYPE QUESTIONS [5 Marks]

52. (a) Define electric flux. Write its SI units.

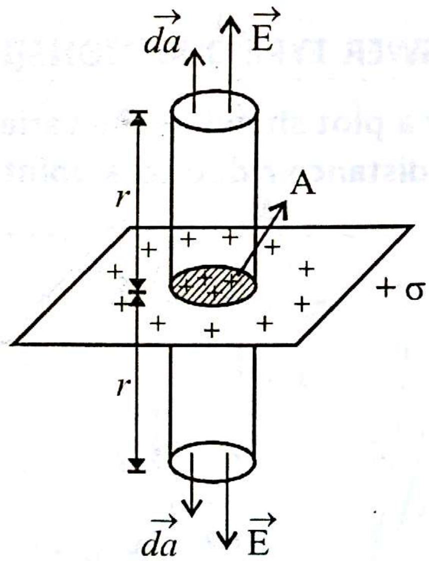
(b) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(c) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged? [Delhi]

Ans. (a) Number of electric lines of force passing through a given area normally is called the electric flux through that area.

SI unit is Nm^2C^{-1}.

(b) Consider a thin infinite sheet of charge with uniform surface charge density σ. To calculate electric field at a point '*P*' distant '*r*' from the sheet we imagine a symmetrical Gaussian surface in such a way that the point charge lies on it. Here we assume a cylinder of cross-sectional area *A* and length 2*r* with its axis perpendicular to the sheet.



Flux through the curved surface of the cylinder (ϕ_1).

$$\phi_1 = \oint \vec{E} \cdot \vec{ds} = 0 \quad (\because \theta = 90^\circ)$$

Total flux through plane faces of the cylinder (ϕ_2).

$$\phi_2 = 2\oint \vec{E} \cdot \vec{ds} = 2EA \quad (\because \theta = 0^\circ)$$

Net flux through the Gaussian surface is

$$\phi = \phi_1 + \phi_2$$

$$\phi = 2EA \quad ...(i)$$

Net charge enclosed by the Gaussian surface is

$$Q = \sigma A$$

According to Gaussian theorem,

$$\phi = \frac{Q}{\varepsilon_0}$$

i.e., $$\phi = \frac{\sigma A}{\varepsilon_0} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

(*c*) For positively charged sheet electric field is directed away from the sheet.

For negatively charged sheet electric field is directed towards the plane sheet.