2

Electrostatic Potential and Capacitance

PREVIOUS YEARS' Questions

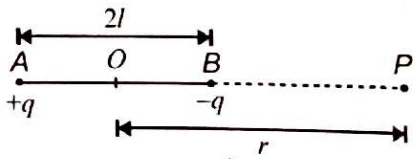
2007

SHORT ANSWER TYPE QUESTIONS[II] **[3 Marks]**

1. Deduce an expression for the electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electric potential of a dipole at a point as compared to that due to a single charge. [*Delhi*]

Ans. Potential at '*P*' due to charge at *A* is

$$V_{PA} = \frac{kq}{r+l}$$



Potential at '*P*' due to charge at *B* is

$$V_{PB} = -\frac{kq}{r-l}$$

Net potential at '*P*' is

$$V_P = V_{PA} + V_{PB}$$

$$= kq\left[\frac{1}{r+l} - \frac{1}{r-l}\right]$$

$$= kq\left[\frac{r-l-r-l}{r^2-l^2}\right]$$

$$= -\frac{kq(2l)}{r^2-l^2}$$

$$= -\frac{kp}{r^2-l^2} \quad [\therefore p = q(2l)]$$

In case $r >> l$

$$V_P = -\frac{kp}{r^2}, \text{ i.e., } V_P \propto \frac{1}{r^2}$$

Whereas, due to a single charge potential at a point is $V \propto \frac{1}{r}$.

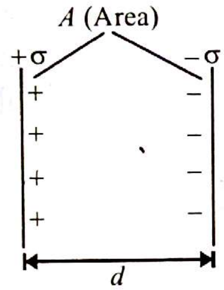
2. Explain the underlying principle of working of a parallel plate capacitor. If two similar plates, each of area *A* having surface charge densities +σ and −σ are separated by a distance *d* in air, write expressions for

(*i*) the electric field at points between the two plates.

(ii) the potential difference between the plates.

(iii) the capacitance of the capacitor so formed. [All India]

Ans. Capacitor is based on the principle of electrostatic induction.



(i) $E_0 = \frac{\sigma}{\varepsilon_0}$

(ii) $V = E_0 d = \frac{\sigma d}{\varepsilon_0}$

(iii) $C_0 = \frac{\varepsilon_0 A}{d}$

LONG ANSWER TYPE QUESTIONS [5 Marks]

3. **Derive the expression for the energy stored in a parallel plate capacitor of capacitance *C* with air as medium between its plates having charges Q and –Q. Show that this energy can be expressed in terms of electric field as $\frac{1}{2}\varepsilon_0 E^2 Ad$, where *A* is the area of each plate and *d* is the separation between the plates.**
How will the energy stored in a fully charged capacitor change when the separation between the plates is doubled and a dielectric medium of dielectric constant 4 is introduced between the plates? [Foreign]

Ans. (i) Suppose at any instant of time potential difference between the capacitor plates be *V*. Then amount of work required to supply a charge *dq* to the capacitor is

$$dW = Vdq$$

To supply a charge *Q* work done is

$$W = \int_0^Q Vdq$$

$$W = \int_0^Q \frac{q}{C}dq \quad \left(\because V = \frac{q}{C}\right)$$

$$W = \frac{1}{2}\frac{Q^2}{C} \quad (\because Q = CV)$$

or $$W = \frac{1}{2}CV^2$$

Here $C = KC_0$. $W = \frac{1}{2}KC_0V^2$

As $C_0 = \frac{\varepsilon_0 A}{d}$, $V = Ed$

$$W = \frac{1}{2}K\frac{\varepsilon_0}{d}E^2d^2A,$$

$$W = \frac{1}{2}K\varepsilon_0E^2Ad.$$

(ii) $$C_0 = \frac{\varepsilon_0 A}{d}$$

$$C'_0 = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2}$$

On introducing dielectric medium of dielectric constant *K* = 4 between the plates.

$$C = KC'_0$$

$$E'_V = \frac{1}{2}\frac{Q^2}{C}$$

$$E'_v = \frac{1}{2}\frac{Q^2}{KC'_0} \qquad \left[\because C'_0 = \frac{C_0}{2}\right]$$

$$= \frac{1}{2}\left(\frac{Q^2}{4\frac{C_0}{2}}\right) \qquad [\because K = 4]$$

$$= \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C_0}\right)$$

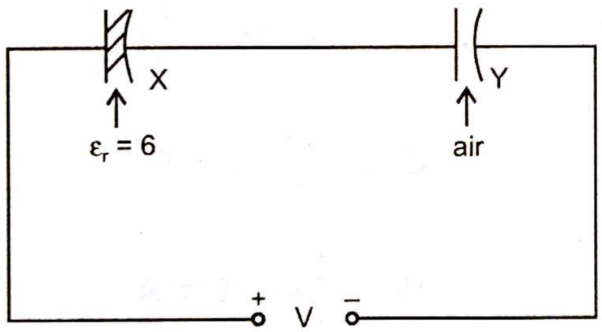
$$E'_v = \frac{1}{2}E_n$$

Energy stored in the capacitor reduces to one half of its original value.

2008

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

4. In the figure given below, *X*, *Y* represent parallel plate capacitors having the same area of plates and the same distance of separation between them. What is the relation between the energies stored in the two capacitors? *[Foreign]*



Ans. Capacitance of capacitor *Y*, $C_Y = \frac{\varepsilon_0 A}{d}$

Capacitance of capacitor $X, C_X = \varepsilon_r\left(\frac{\varepsilon_0 A}{d}\right)$

i.e., $C_X = 6\, C_Y$

As capacitors are joined in series, *Q* is same for both the capacitors.

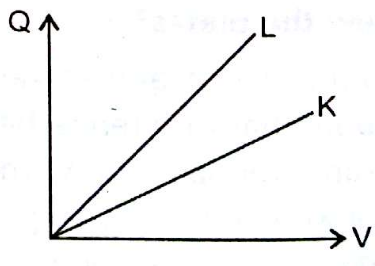
Now $E_Y = \frac{1}{2}\frac{Q^2}{C_X}$

$$E_X = \frac{1}{2}\frac{Q^2}{C_X}$$

$$= \frac{1}{6}\left(\frac{1}{2}\frac{Q^2}{C_Y}\right) = \frac{1}{6}E_Y$$

Thus, $\frac{E_X}{E_Y} = \frac{1}{6}$

5. The following graph shows the variation of charge Q, with voltage *V*, for two capacitors *K* and *L*. In which capacitor is more electrostatic energy stored? *[Foreign]*

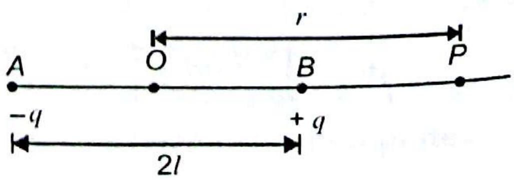


Ans. In capacitor *L*.

SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

6. Derive the expression for the electric potential at any point along the axial line of an electric dipole? *[Delhi]*

Ans.



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Potential at '*P*' due to dipole is

$$V_p = V_{PA} + V_{PB}$$

$$V_p = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-l} - \frac{1}{r+l}\right]$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{q.2l}{r^2 - l^2}$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - l^2}$$

7. Two point charges, $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air.

(i) Find at what distance from the 1st charge, q_1, would the electric potential be zero.

(ii) Also, calculate the electrostatic potential energy of the system.

[All India]

Ans. Given :

$$q_1 = 10 \times 10^{-8}\text{C}$$

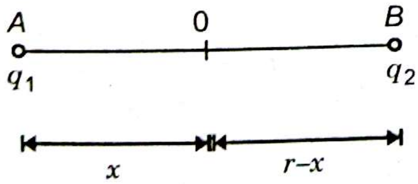
$$q_2 = -2 \times 10^{-8}\text{C}$$

$$r = 60 \times 10^{-2}\text{m}$$

(*i*) $V_{OA} = V_{OB}$.

$$\frac{kq_1}{x} + \frac{kq_2}{r-x} = 0$$

$$\frac{10^{-7}}{x} - \frac{2 \times 10^{-8}}{r-x} = 0$$



$$\frac{10}{x} = \frac{2}{r-x}$$

$$10(r-x) = 2x$$

$$10r = 12x$$

$$x = \frac{10}{12}r$$

$(r = 60 \times 10^{-2}\text{ m})$

$$x = 0.5\text{ m}$$

$$x = 50\text{ cm}$$

(*ii*) Electrostatic energy of the system is

$$E_n = \frac{kq_1q_2}{r}$$

$$= \frac{-9 \times 10^9 \times 10^{-7} \times 2 \times 10^{-8}}{60 \times 10^{-2}}$$

$$E_n = -3 \times 10^{-5}\text{ J}$$

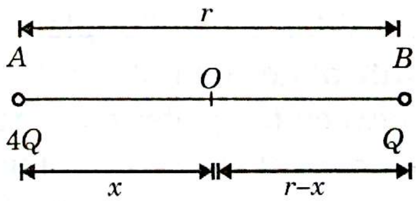
8. Two point charges 4Q, Q are separated by 1 m in air. At what point on the line joining the charges is the electric field intensity zero?

Also, calculate the electrostatic potential energy of the system of charges, taking the value of charge, Q = 2 × 10⁻⁷ C.

[All India]

Ans. Given: $r = 1$m, $q_1 = 4Q$, $q_2 = Q$.

$$E_{OA} = E_{OB}$$



$$\frac{k(4Q)}{x^2} = \frac{kQ}{(r-x)^2}$$

$$\frac{4}{x^2} = \frac{1}{(r-x)^2}$$

$$\frac{2}{x} = \frac{1}{r-x}$$

$$2r - 2x = x$$

$$3x = 2r$$

$$x = \frac{2}{3}r \quad (r = 1\text{ m})$$

$$x = \frac{2}{3}\text{ m}$$

Electrostatic potential energy of the system is

$$E_n = \frac{kq_1q_2}{r}$$

Given $r = 1\text{m}, Q = 2 \times 10^{-7}\text{C}$

$$= \frac{9 \times 10^9 \times 4Q \times Q}{r}$$

$$= \frac{9 \times 10^9 \times 4(2 \times 10^{-7})^2}{1}$$

$$= 144 \times 10^{-5}$$

$$= 1.44 \times 10^{-3}\text{J}$$

LONG ANSWER TYPE QUESTIONS [5 Marks]

9. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential *V*, the spacing between the plates is halved, and a dielectric medium of $\varepsilon_r = 10$ is introduced between the plates, without disconnecting the d.c. source. Explain, using suitable expressions, how the (*i*) capacitance, (*ii*) electric field and (*iii*) energy density of the capacitor change. *[All India]*

Ans. (*a*) Consider a parallel plate capacitor with plate area '*A*' and separation between the plates equal to '*d*'.

Suppose at any instant of time charge on the capacitor plate is '*q*' and potential difference due to this charge is '*V*'.

To supply a charge '*dq*' further to the capacitor, amount of work required is

$$dW = Vdq$$

$$dW = \frac{q}{C}dq \quad (\because q = CV)$$

In order to supply a charge '*Q*', work required is,

$$W = \int_0^Q \frac{q}{c}dq = \frac{Q^2}{2C}$$

Thus energy stored by capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C}$$

(*b*) (*i*)

$$C_f = \frac{K\varepsilon_o A}{d'} = \frac{K\varepsilon_o A}{d/2} \quad (\because d' = d/2)$$

$$C_f = \frac{20\varepsilon_o A}{d} = 20C_i \quad \left(\because C_i = \frac{\varepsilon_o A}{d}\right)$$

Capacitance becomes twenty times.

(*ii*)

$$E_f = \frac{V}{d'} = \frac{V}{d/2} \quad (\because d' = d/2)$$

$$E_f = 2\frac{V}{d} = 2E_i \quad (\because E_i = V/d)$$

Electric field is doubled.

(*iii*) Energy density,

$$U_i = \frac{1}{2}\frac{\sigma^2}{\varepsilon_o} = \frac{1}{2}\varepsilon_o\left(\frac{\sigma}{\varepsilon_o}\right)^2$$

$$= \frac{1}{2}\varepsilon_o E_i^2 \quad \left(\because E_i = \frac{\sigma}{\varepsilon_o}\right)$$

$$U_f = \frac{1}{2}\varepsilon_r\varepsilon_o E_f^2 = \frac{10}{2}\varepsilon_o(2E_i)^2$$

$$= 40\left(\frac{1}{2}\varepsilon_o E_i^2\right)$$

$$U_f = 40U_i$$

Energy density is fourty times.

2009

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

10. What is the electrostatic potential due to an electric dipole at an equatorial point? [*All India*]

Ans. Zero.

11. Why is it necessary that the field lines from a point charge placed in the vicinity of a conductor must be normal to the surface of the conductor at every point? [*Foreign*]

Ans. Surface of a conductor is equipotential and the field lines cannot be at any angle but perpendicular to the surface.

12. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor? [*Foreign*]

Ans. Capacitance increases.

SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

13. (*i*) Can two equipotential surfaces intersect each other? Give reasons.

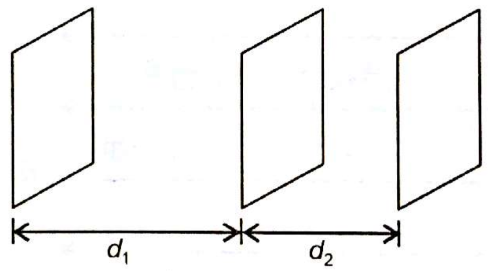
(*ii*) Two charges $-q$ and $+q$ are located at points $A(0, 0, -a)$ and $B(0, 0, +a)$ respectively. How much work is done in moving a test charge from point $P(7, 0, 0)$ to $Q(-3, 0, 0)$? [*Delhi*]

Ans. (*i*) No, if they intersect, there will be two different directions of electric field at that point which is not correct.

(*ii*) Since both the points are in the equatorial line of the dipole and $V = 0$ at every point on it, work done will be zero. Also the force on any charge is perpendicular to the equatorial line. So, work done is zero.

14. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along z-direction. How are these surfaces different from that of a constant electric field along z-direction? [*All India*]

Ans.

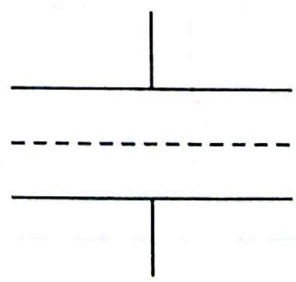


$d_2 < d_1$ for increasing field and $d_2 = d_1$ for uniform field.

15. Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected if

(*i*) the foil is electrically insulated?

(*ii*) the foil is connected to the upper plate with a conducting wire? [*Foreign*]



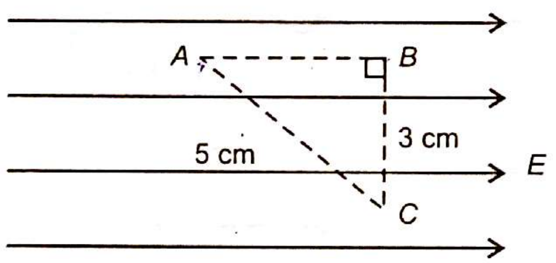
Ans. (*i*) Capacitance increases with increase in the thickness t of conductor electrically insulated foil in the space between the plates of capacitor as

$C_c = \frac{\varepsilon_0 A}{d-t}$, where d is the separation between the plates.

If t is negligible, the increase also is very small.

(*ii*) When the foil is connected to the upper plate with a conducting wire, the space between the foil and the connected upper plate becomes ineffective and so the effective separation becomes less than d and so capacitance increases.

16. Three points *A*, *B* and *C* lie in a uniform electric field (*E*) of 5×10^3 NC^{-1} as shown in the figure. Find the potential difference between *A* and *C*. [*Foreign*]



Ans. Points B and C will be at the same potential and the separation between A and B equalling 4 cm will contribute to the change in the potential. So the potential difference between A and C is equal to the potential difference between A and B and is given by

$$V = E.dr = 5 \times 10^3 \times 4 \times 10^{-2}$$
$$= 20 \times 10 = 200 \text{ J/C.}$$

17. A 800pF capacitor is charged by a 100V battery. After sometime the battery is disconnected. The capacitor is then connected to another 800pF capacitor. What is the electrostatic energy stored? [*Foreign*]

Ans. On charging, 800 pF will acquire a charge of $Q = CV = 800 \times 100 = 8 \times 10^4$ pC.

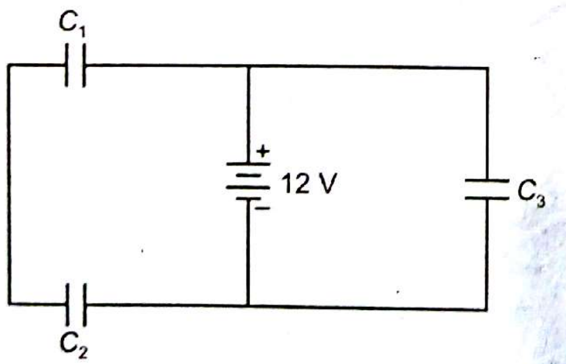
On connecting another 800 pF in parallel, $C_{eq} = 400$ pF.

Charge in the system will be the same as the source is disconnected.

$$\text{Therefore, energy} = \frac{Q^2}{2C_{eq}}$$
$$= \frac{(8\times10^4 \text{pC})^2}{2\times400 \text{ pF}}$$
$$= 100 \times 10^{-12}$$
$$= 10^{-10} \text{ joule}$$

SHORT ANSWER TYPE QUESTIONS[II] [3 Marks]

18. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown.



Find

(*i*) charge on each capacitor.

(*ii*) equivalent capacitance of the network.

(*iii*) energy stored in the network of capacitors. [*Delhi*]

Ans. C_1 and C_2 in series, make $C_4 = 3$ μF using

$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2} = \left(\frac{1}{6}+\frac{1}{6}\right) = \frac{2}{6}.$$

$C_4 = 3$ μF

(*i*) 12 V of potential is available in C_4 and C_3.

Charge in $C_3 = Q_3 = C_3V$

$= 6 \times 10^{-6} \times 12 = 72$ μC.

Charge in $C_4 = Q_4 = C_4 V$
$= 3 \times 10^{-6} \times 12 = 36\ \mu C.$

$\therefore$ Charge on C_1 and C_2 will also be $36\ \mu C$.

(ii) C_4 and C_3 are in parallel to the source

So, $C_{eq} = 3 + 6 = 9\ \mu F$.

(iii) Energy stored $= \frac{1}{2} C_{eq}. V^2$

$$= \frac{1}{2} \times 9 \times 10^{-6} \times 12^2$$
$$= 648\ \mu \text{ joule.}$$

19. The equivalent capacitance of the combination between *A* and *B* in the given figure is 4 μF.

(i) Calculate capacitance of the capacitor *C*.

(ii) Calculate charge on each capacitor if a 12 V battery is connected across terminals *A* and *B*.

(iii) What will be the potential drop across each capacitor? [Delhi]

Ans. $C_{eq} = 4\ \mu F$

(i) Since 20 μF and *C* are in series, we have

$$\frac{1}{4} = \frac{1}{20} + \frac{1}{C}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{4} - \frac{1}{20} = \frac{5-1}{20}$$

$$\Rightarrow \quad C = \frac{20}{4} = 5\ \mu F$$

(ii) Charge drawn from 12 V battery is

$$Q = C_{eq} . V = 4 \times 12$$
$$= 48\ \mu C.$$

So charge on each capacitor

$$= 48\ \mu C.$$

(iii) Potential drop across

$$20\ \mu F \Rightarrow V_{20} = \frac{Q}{C} = \frac{48\ \mu C}{20\ \mu F}$$
$$= 2.4 \text{ volt}$$

$$5\ \mu F \Rightarrow V_5 = \frac{48\ \mu C}{5\ \mu F}$$
$$= 9.6 \text{ volt.}$$

20. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab of dielectric constant *K* is inserted between the plates. How would (*i*) the capacitance, (*ii*) the electric field between the plates and (*iii*) the energy stored in the capacitor, be affected? Justify your answer. [All India]

Ans. Let *C* be the capacitance and *V* be the potential difference. The charge on the capacitor plates will then be, $Q = CV$.

The electric field between the plates =

$$E = \frac{V}{d} \text{ and the energy stored } =$$

$$E_n = \frac{Q^2}{2C} \text{ or } \frac{1}{2} CV^2.$$

As the dielectric (*K*) is introduced after disconnecting the battery, we have, the new values of

Charge $\quad Q' = Q$

Capacitance $\quad C' = KC$

Potential $\quad V' = \frac{Q}{KC} = \frac{V}{K}.$

(i) New capacitance is *K* times its original.

(ii) New electric field

$$E = \frac{V'}{d} = \frac{V}{Kd} = \frac{E}{K}$$

i.e., $\frac{1}{K}$ times the original field.

(iii) New energy

$$= \frac{Q^2}{2C'} = \frac{Q^2}{2KC} = \frac{1}{K}(E_n) \text{ i.e., } \frac{1}{K} \text{ times}$$

the original energy.

2010

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

21. Name the physical quantity whose SI unit is JC^{-1}. Is it a scalar or a vector quantity? *[All India]*

Ans. (*i*) Electric potential
(*ii*) Scalar quantity

SHORT ANSWER TYPE QUESTIONS [3 Marks]

22. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will (*i*) the capacitance of the capacitor, (*ii*) potential difference between the plates and (*iii*) the energy stored in the capacitor be affected? Justify your answer in each case. *[Delhi]*

Ans. (*i*) Capacitance of the capacitor,

$$C = \frac{K\varepsilon_0 A}{d}$$

Hence, capacitance increases K times.

(*ii*) Potential difference between the plates,

$$V = \frac{V_0}{K}$$

Hence, potential difference decreases by a factor K.

(*iii*) Energy stored in the capacitor,

$$E = \frac{1}{2}CV^2$$

$$E = \frac{1}{2}(KC_0) \times \frac{V_0^2}{K^2}$$

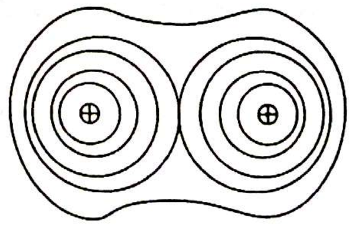
$$E = \frac{1}{2K}C_0V_0^2 = \frac{E_0}{K}$$

Energy stored becomes $\frac{1}{K}$ times.

23. (*a*) Depict the equipotential surfaces for a system of two identical positive point charges placed at a distance '*d*' apart.

(*b*) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the presence of external electric field $\vec{E}$. *[Delhi]*

Ans. (*a*) Equipotential surfaces for a system of two identical positive point charges placed a distance '*d*' apart.



(*b*) Work done in bringing the charge q_1 from infinity to $\vec{r}_1$. Against the external electric field

$$W_1 = q_1V(r_1)$$

Work done in bringing the charge q_2 from infinity to $\vec{r}_2$.

$$W_2 = q_2V(r_2)$$

Work done on q_2 against the field due to q_1

$$W_3 = \frac{q_1q_2}{4\pi\epsilon_0 r_{12}}$$

Potential energy of the system

$$= W_1 + W_2 + W_3$$

$$= q_1V(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}$$

24. A parallel plate capacitor, each with plate area *A* and separation *d*, is charged to a potential difference *V*. The battery used to charge it remains connected. A dielectric slab of thickness *d* and dielectric constant *K* is now placed between the plates. What change, if any, will take place in:

(*i*) charge on plates?

(*ii*) electric field intensity between the plates?

(*iii*) capacitance of the capacitor?

Justify your answer in each case. [*Delhi*]

Ans. (*i*) Charge on plates $Q = CV$

$$C = KC_0$$

So, for increase in the value of capacitance charge also increases.

(*ii*) Electric field intensity between the plates

$$E = \frac{E_0}{K}$$

Due to polarisation of dielectric slab, net electric field get reduced.

(*iii*) Capacitance of the capacitor

$$C = \frac{K\varepsilon_0 A}{d}$$

Capacitance increases.

25. A parallel plate capacitor is charged to a potential difference *V* by a *dc* source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change;

(*i*) electric field between the plates,

(*ii*) capacitance, and

(*iii*) energy stored in the capacitor. [*Delhi*]

Ans. (*i*) Electric field between the plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

As surface charge density remains same E is constant.

(*ii*) Capacitance $C = \frac{\varepsilon_0 A}{d}$

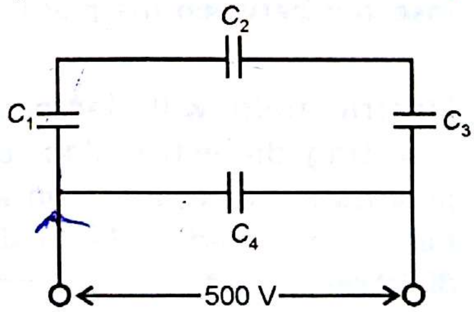
If distance between the plates is doubled, then, capacitance reduces to half.

(*iii*) Energy stored in the capacitor,

$$E_n = \frac{Q^2}{2C}$$

As, capacitance reduces to half energy get doubled.

26. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (*a*) equivalent capacitance of the network and (*b*) charge on each capacitor. [*All India*]

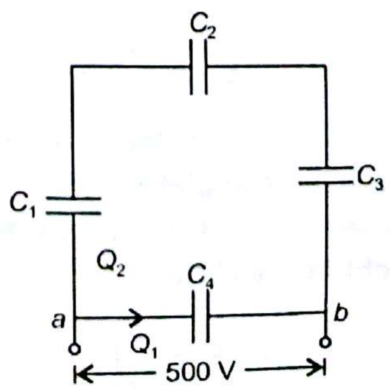


Ans. (*a*) Equivalent capacitance of the network,

$$C_{123} = \frac{12\mu F}{3} = 4\mu F$$

$$C_{eq} = C_{123} + C_4 = (4 + 12)\,\mu F$$

$$C_{eq} = 16\,\mu F$$



(*b*) (*i*) $Q = CV$

$Q_1 = C_4V = 500 \times 12 \times 10^{-6}$

$= 6000 \times 10^{-6} = 6 \times 10^{-3}$ C

Charge on capacitor C_4 is 6×10^{-3} C

(*ii*) $Q_2 = C_{123}V = 4 \times 500 \times 10^{-6}$

$= 2 \times 10^{-3}$ C

Charge on each of the capacitors C_1, C_2 and C_3 is 2×10^{-3} C.

27. **(*a*) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?**

(*b*) A slab of material of dielectric constant *K* has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where *d* is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.

[Foreign]

Ans. (*a*) Electric field will decrease. On inserting dielectric slab, due to polarisation of charges, an electric field is developed inside the slab in a direction opposite to the electric field due to capacitor plates. Thus, a reduced value of electric field is obtained.

(*b*) In the absence of dielectric slab, let the electric field between the capacitor plates be

$$E_0 = \frac{V_0}{d}$$

where V_0 is potential difference between the plates.

When the dielectric is inserted electric field in it will be

$$E = \frac{E_0}{K}$$

Now potential difference between capacitor plates will be

$$V = E_0\left(\frac{d}{2}\right) + \frac{E_0}{K}\left(\frac{d}{2}\right)$$

$$= E_0 d\left[\frac{1}{2} + \frac{1}{2K}\right]$$

$$= \frac{E_0 d}{2}\left[\frac{K+1}{K}\right]$$

$$V = \frac{V_0(K+1)}{2K} \quad \left[\because E_0 = \frac{V_0}{d}\right]$$

As the charge on capacitor plates remains same.

$$C = \frac{Q_0}{V} = \frac{2KQ_0}{V_0(K+1)}$$

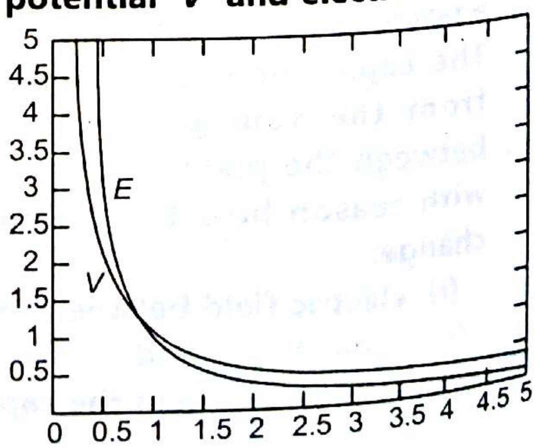
$$= \frac{2K}{K+1}C_0$$

28. **(*a*) Plot a graph comparing the variation of potential '*V*' and electric field '*E*' due to a point charge '*Q*' as a function of distance '*R*' from the point charge.**

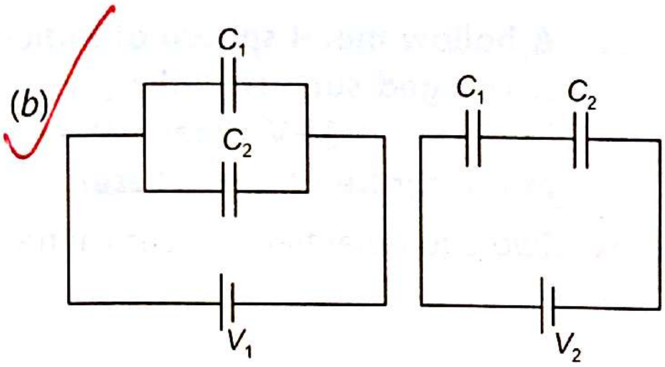
(*b*) Find the ratio of the potential differences that must be applied across the parallel and the series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2, so that the energy stored in the two cases, becomes the same.

[Foreign]

Ans. (*a*) **Graph comparing the variation of potential '*V*' and electric field '*E*'**



(b)



Given $\frac{C_1}{C_2} = \frac{1}{2}$.

We know that,

$$C_p = C_1 + C_2$$

$$C_S = \frac{C_1C_2}{C_1 + C_2}$$

$$E_P = \frac{1}{2}C_PV_1^2$$

$$E_S = \frac{1}{2}C_SV_2^2$$

$$\frac{1}{2}C_PV_1^2 = \frac{1}{2}C_SV_2^2$$

$$\frac{V_1^2}{V_2^2} = \frac{C_S}{C_P} = \frac{C_1C_2}{(C_1 + C_2)^2}$$

$$= \frac{\frac{C_1C_2}{C_2^2}}{\left(\frac{C_1 + C_2}{C_2}\right)^2}$$

$$= \frac{C_1/C_2}{\left(\frac{C_1}{C_2}+1\right)^2}$$

$$= \frac{1/2}{\left(\frac{1}{2}+1\right)^2} = \frac{1/2}{(3/2)^2}$$

$$= \frac{1}{2}\times\frac{4}{9} = \frac{2}{9}$$

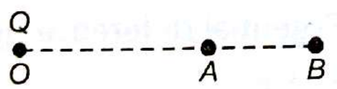
Required potential differences

$$\frac{V_1}{V_2} = \frac{\sqrt{2}}{3}$$

2011

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

29. A point charge Q is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero, if Q is (*i*) positive (*ii*) negative? [*Delhi*]



Ans. As $V \propto \frac{1}{r}$

For Q having +ve value.

(*i*) $V_A > V_B$ *i.e.* $V_A - V_B$ is positive.

For Q having –ve value.

(*ii*) $V_A < V_B$ *i.e.* $V_A - V_B$ is negative.

30. Can two equipotential surfaces intersect each other? Justify your answer. [*Delhi*(*C*)]

Ans. Two equipotential surfaces cannot intersect. The direction of electric field is always perpendicular to the equipotential surface. If they intersect, there will be two directions of the electric field at the point of intersection which is not possible.

31. Define the dielectric constant of a medium. What is its unit? [*Delhi* (*C*)]

Ans. Dielectric constant of a medium is defined as the ratio of the force between two charges placed a certain distance apart in vacuum to the force between the same two charges placed the same distance apart in the medium. It has no units.

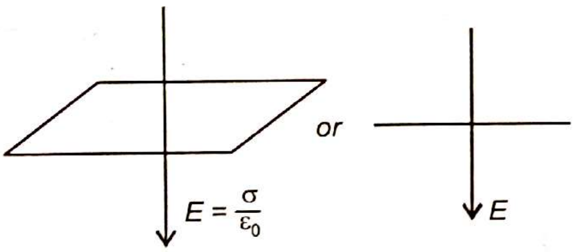
32. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? [Delhi(C)]

Ans. Two equipotential surfaces canno

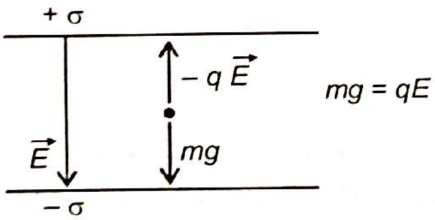
SHORT ANSWER TYPE QUESTIONS [2 Marks]

33. Two uniformly large parallel thin plates having charge densities +σ and –σ are kept in the *X-Z* plane at a distance '*d*' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass *m* and charge '–*q*' remains stationary between the plates, what is the magnitude and direction of this field? [Delhi]

Ans. (*i*)



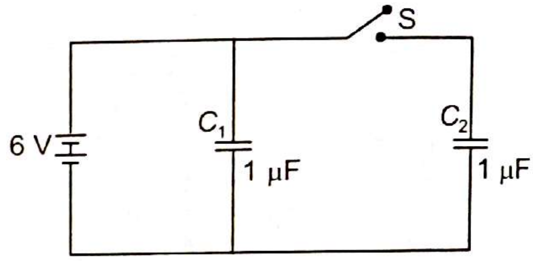
(*ii*)



(*a*) $E = \frac{mg}{q}$

(*b*) Directed upside down.

34. Figure shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch 'S' is closed. After sometimes 'S' is left open and dielectric slabs of dielectric constant *K* = 3 are inserted to fill completely the space between the plates of the two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted? [Delhi]



Ans. (*i*) Charge on the capacitor C_1

New capacity of capacitor

$C_1' = KC_1 = 3 \times 1\ \mu F = 3\mu F$

As the supply remains connected with C_1, new charge on the capacitor is

$$Q_1' = C_1' \times V = 3 \times 10^{-6} \times 6 = 18 \times 10^{-6}\ C$$

Charge on the capacitor, C_2

As the supply has been disconnected.

$$Q_2 = C_2 \times V = 1 \times 10^{-6} \times 6 = 6\ \mu C.$$

(*ii*) Potential difference between plates of C_1.

As the supply remains connected it is same.

For capacitor C_2

$$C_2' V' = C_2 V$$

$$V' = \frac{C_2 V}{KC_2} = \frac{V}{K}$$

$$V' = \frac{6}{3} = 2 \text{ volt}$$

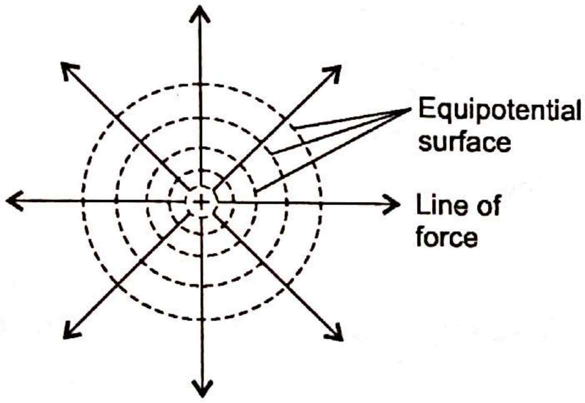
35. **(a) Draw equipotential surfaces due to a point Q > 0.**

(b) Are these surfaces equidistant from each other? If not, explain why.

[*Delhi*(*C*)]

Ans. The equipotential surfaces due to a charge Q are as shown below:

(a) The equipotential surfaces are spherical concentric spheres.



(b) The equipotential surfaces are not equidistant.

$$\text{Since} \qquad E = -\frac{dV}{dr}$$

$$dr = -\frac{dV}{E}$$

Clearly, as E decreases, the distance between the equipotential surfaces goes on increasing as shown in the fig.

36. **Deduce the expression for the energy stored in a parallel plate capacitor *C* having charges +*Q* and –*Q* on its plates.** [*Delhi*(*C*)]

Ans. Let a capacitor of capacity *C* be charged to a potential *V* by a charge *Q*. Work has to be done in charging a capacitor. This work is stored in the capacitor in the form of potential energy. It exists in the electric field between the plates of the capacitor. Let *q* be the charge on the capacitor at any stage and *V* the potential, then

$$V = \frac{q}{C}$$

Then small amount of work done *dw* in giving a further small charge *dq* to the capacitor is

$$dW = Vdq = \frac{q}{C}dq$$

∴ Work done in charging the capacitor to a charge *Q* is

$$W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$= \frac{1}{2C}[Q^2 - 0] = \frac{1}{2}\frac{Q^2}{C}$$

Also, $\qquad Q = CV$

$$\therefore \qquad W = \frac{1}{2}\frac{C^2V^2}{C} = \frac{1}{2}CV^2$$

37. **A parallel plate capacitor is being charged by a time varying current. Explain briefly how Ampere's circuital law is generalized to incorporate the effect due to the displacement current.**

[*All India*]

Ans. Generalized Ampere's circuital law included displacement current in such a way that condition of continuity is obeyed, *i.e.*

$$\oint \vec{B} \cdot \vec{dl} = \mu_0(i_c + i_d)$$

$$= \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

38. **Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?**

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

[*All India*]

Ans. $C_s = 1\ \mu F$

If *C* is the capacitance of a single capacitor.

$$\frac{C}{3} = C_s$$

i.e. $C = 3C_s$

In parallel,

$$C_P = 3C$$
$$= 3\,(3C_S) = 9 \times C_S$$
$$C_P = 9 \times 1\ \mu F = 9\ \mu F$$

Energy stored by the series combination

$$U_S = \frac{1}{2} C_S V^2$$

Energy stored by parallel combination

$$U_P = \frac{1}{2} C_P V^2$$

$$\frac{U_S}{U_P} = \frac{C_S}{C_P} \quad \left(\because \frac{C_S}{C_P} = \frac{1}{9}\right)$$

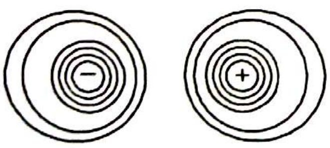
$$\frac{U_S}{U_P} = \frac{1}{9}$$

39. Two point charges 2μC and –2μC are placed at points *A* and *B*, 6 cm apart.

(*i*) Draw the equipotential surfaces of the system.

(*ii*) Why do the equipotential surfaces get closer to each other near the point charges? *[All India]*

Ans. (*i*) The equipotential surfaces of a system of two equal and opposite charges *i.e.*, a dipole are as shown below :

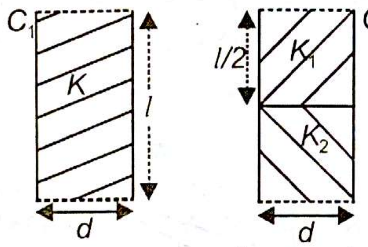


(*ii*) We know that $E = \frac{-dV}{dr}$

Therefore, $dr = \frac{-dV}{E}$

Since, near the charge, electric field *E* is large, *dr* will be less *i.e.*, the equipotential surface will get closer.

40. Two identical parallel plates (air) capacitors C_1 and C_2 have capacitances *C* each. The space between their plates is now filled with dielectrics as shown. If the two capacitors still have equal capacitance, obtain the relation between dielectric constants K, K_1 and K_2. *[Foreign]*



Ans. New capacitance of capacitor, C_1

$$C'_1 = K\,C_1 = K\,C \quad (\because C_1 = C)$$

Resultant capacitance of capacitor, C_2.

$$C'_2 = (K_1 + K_2)\frac{C}{2}$$

As the capacitances are still equal

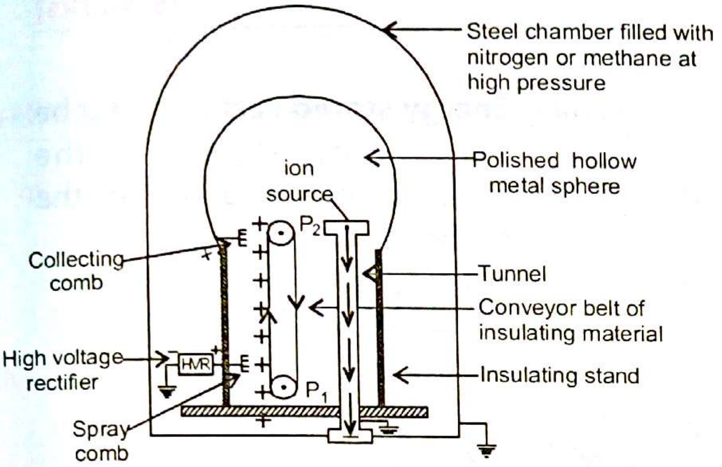
$$C'_1 = C'_2$$
$$2K = K_1 + K_2$$

41. State the principle of the device that can build up high voltages of the order of a few million volts. Draw its labelled diagram. A stage reaches in this device when the potential at the outer sphere cannot be increased further by piling up more charge on it. Explain why. *[Foreign]*

Ans. (*a*) **Principle:**

(*i*) Electric discharge taken place readily at pointed ends (corona discharge).

(*ii*) Charge given to a hollow conductor gets shifted to outer surface and is distributed uniformly.



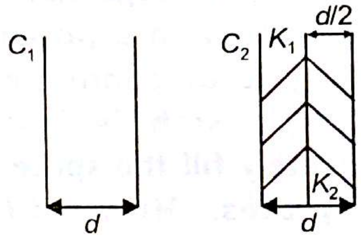
Van de Graaff Generator

Working: Spray comb leaks charge by the phenomenon of corona discharge to the insulating belt. Belt moves up towards the collecting comb. At the collecting comb due to induction charge is induced. Charge of the same nature gets transferred to the hollow metallic sphere. This charge increases its potential. Again corona discharge takes place at collecting comb and charge on the belt gets neutralised. This process continues. A charge in the form of ion is produced inside the sphere. This ion moves with some velocity and acquires energy qV.

To minimise the leakage of charge from the generator, it is housed inside a steel chamber filled with nitrogen or methane at high pressure.

(*b*) At a particular value of electric potential medium surrounding the generator begin to ionize *i.e.*, corona discharge begins. So we can not further increase the charge on the generator.

42. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in C_2. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d. [*Foreign*]



Ans. $C'_2 = \dfrac{2K_1\varepsilon_0 A}{d}$

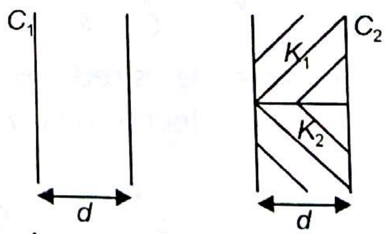
$$C''_2 = \frac{2K_2\varepsilon_0 A}{d}$$

$$C_2 = \frac{C'_2C''_2}{C'_2 + C''_2} = \frac{\dfrac{4K_1K_2\varepsilon_0^2A^2}{d^2}}{\dfrac{2\varepsilon_0 A}{d}(K_1 + K_2)}$$

$$C_2 = \frac{4K_1K_2\varepsilon_0^2A^2}{d^2} \times \frac{d}{2\varepsilon_0 A(K_1 + K_2)}$$

$$= \frac{2K_1K_2\varepsilon_0 A}{d(K_1 + K_2)}$$

43. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in C_2. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d. [*Foreign*]



Ans. Capacitance of the capacitor

$$C'_2 = \frac{K_1\varepsilon_0 A}{2d}$$

$$C''_2 = \frac{K_2\varepsilon_0 A}{2d}$$

$$C_2 = C'_2 + C''_2 = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

44. (a) A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates. How will (*i*) its capacitance, (*ii*) electric field between the plates and (*iii*) energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expressions for each case.

(b) Sketch the pattern of electric field lines due to (*i*) a conducting sphere having negative charge in it, (*ii*) an electric dipole. [*All India*(*C*)]

Ans. (*a*) Let *C* be capacity of the parallel plate capacitor charged to a potential *V* of the battery. When the battery is disconnected the charge on the capacitor remains the same.

(*i*) **Capacitance:** The capacitance of the capacitor becomes *K* times the original value *i.e.*, $C = KC_0$.

(*ii*) **Electric field:** The new potential *V* is given by

$$V = \frac{Q}{C} = \frac{Q}{KC_0} = \frac{V_0}{K}$$

i.e., potential is reduced *K* times. The new electric field *E* is given by

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}$$

$$\left(\because E_0 = \frac{V_0}{d}\right)$$

i.e., the electric field is reduced *K* time.

(*iii*) **Energy stored :** Let U_0 and U be the energy stored in the capacitor before and after the dielectric is introduced.

Then $U_0 = \frac{1}{2}C_0V_0^2$

and $U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0\left(\frac{V_0}{K}\right)^2$

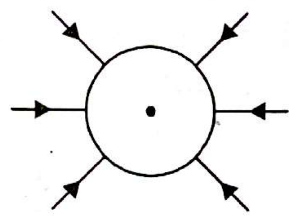
$$U = \frac{1}{2}\frac{C_0V_0^2}{K} = \frac{U_0}{K}$$

$\therefore \quad U = \frac{U_0}{K}$

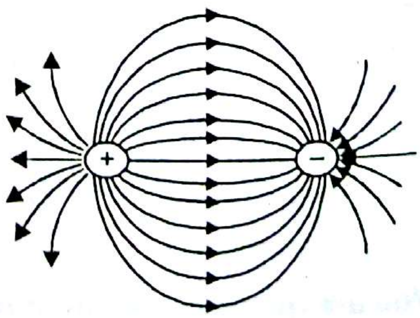
i.e., the energy stored in the capacitor is reduced *K* times.

(*b*) Pattern of electric lines of force of a conducting sphere having a negative charge.

(*i*) There is no electric line of force inside the conducting sphere.



(*ii*) Pattern of the electric lines of force of a magnetic dipole.



2012

VERY SHORT ANSWER TYPE QUESTIONS [1 Mark]

45. Why is electrostatic potential constant throughout the volume of the conductor and has the same value (as inside) on its surface? *[Delhi]*

Ans. As we know that the electrostatic field inside the conductor is zero and on the surface, the field is normal to the surface at every point (Gauss's theorem).

No work is done in moving a small test charge, within the conductor and on its surface. We find there is no potential difference between the two points inside or on the surface, which implies the potential being constant throughout.

46. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface? *[Foreign]*

Ans. Electric field inside a hollow charged conductor is zero and there is no tangential component on the surface of the conductor. So, work done is zero to moving a charge inside and on the surface of the conductor.

$$E = -\frac{dV}{dr},$$

From this we conclude that potential inside the spherical charged conductor is constant and it is same on the surface. In inside a hollow charge conductor,

$$E = 0$$

$$V = \text{constant}$$

47. Why is there no work done in moving a charge from one point to another on an equipotential surface? *[Foreign]*

Ans. Potential difference between any two points of equipotential surface is zero. So no work is done in moving a charge from one point to another.

48. Why do the equipotential surfaces due to a uniform electric field not intersect each other? *[Foreign]*

Ans. In case equipotential surfaces intersect each other at the point of intersection there would be two values of potential which is not possible.

SHORT ANSWER TYPE QUESTIONS[I] [2 Marks]

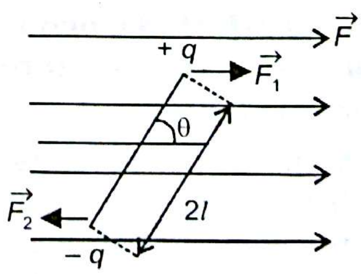
49. An electric dipole is held in a uniform electric field.

(i) Show that the net force acting on it is zero.

(ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180°. *[All India]*

Ans. (*i*) Force experienced by charge (+*q*) is

$$\vec{F_1} = q\vec{E} \quad ...(i)$$



Force experienced by charge (–*q*) is

$$\vec{F_2} = -q\vec{E} \quad ...(ii)$$

As $\vec{F_1} = -\vec{F_2}$

$$\vec{F_1} + \vec{F_2} = 0$$

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Thus, net force experienced by the electric dipole held in uniform electric field is zero.

(*ii*) For initial orientation of the dipole,

$$\theta_1 = 0°$$

For find orientation of the dipole,

$$\theta_2 = 180°$$

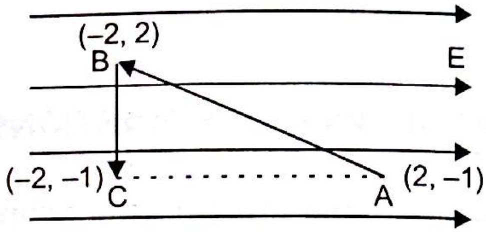
We know

$$W = -pE(\cos\theta_2 - \cos\theta_1)$$

$$W = -pE(\cos 180° - \cos 0°) = -pE(-1-1)$$

$$W = 2pE$$

50. A test charge '*q*' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure. (*i*) Calculate the potential difference between A and C. (*ii*) At which point (of the two) is the electric potential more and why? *[All India]*



Ans. (*i*) Electric field is conservative. So amount of work done will depend upon initial and final position only. Amount of work done in moving charge '*q*' from *A* to *C* is

$$W = \vec{F}\cdot\vec{d} = q\vec{E}\cdot\vec{d}$$

$$W = -qEd$$

$[\because \theta = 180°]$

$$W = -qE(-4) = 4qE$$

$[d = 2 - 6 = -4]$

As $$V_C - V_A = \frac{W}{q}$$

$$V_C - V_A = 4E$$

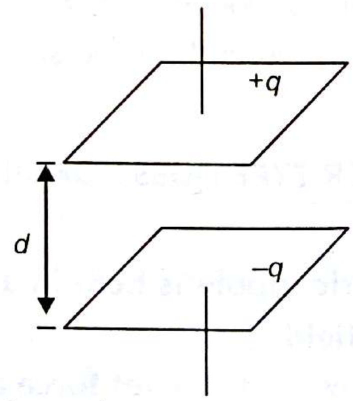
(*ii*) As the work is being done on the charge in moving it from *A* to *C*, the potential at '*C*' would be more.

SHORT ANSWER TYPE QUESTIONS[II] [3 Marks]

51. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C' and having charge 'Q'.

How will the (*i*) energy stored and (*ii*) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'K'? *[All India]*

Ans. (*i*) Consider a parallel plate capacitor with separation '*d*' between the plates. Suppose at any instant of time charge on each plate of capacitor is '*q*' and potential difference is '*V*'.



In order to supply a charge '*dq*' further to the capacitor amount of work done is

$$dW = Vdq$$

$$dW = \frac{q}{C}dq \quad (\because q = CV)$$

To charge a capacitor from 0 to Q charge amount of work done is

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

or, $$U = \frac{1}{2}\frac{Q^2}{C}$$

So, this work done get stored in the form of electrostatic energy between the capacitor plates.

(*ii*) When capacitor is filled with a dielectric material of dielectric constant '*K*'

(*a*) $$E_n = \frac{1}{2}\frac{Q^2}{C}$$

On introducing dielectric slab.

$$C' = KC$$

$$Q' = Q$$

So, $$E_n = \frac{1}{2}\frac{Q^2}{KC} = \frac{1}{K}(E_n)$$

Energy get reduced by *K* times

(*b*) $$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

On introducing dielectric slab

$$E' = \frac{Q}{A(K\varepsilon_0)} \quad \left(\because K = \frac{\varepsilon_m}{\varepsilon_0}\right)$$

$$E' = \frac{E}{K}$$

Electric field get reduced by a factor of *K*.

52. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. [*Foreign*]

Ans. $C_1 = 200 \text{ pF} = 200 \times 10^{-12}$ F

$V_1 = 300$ or $V_1 = 3 \times 10^2$ V

Initial energy,

$$E_{ni} = \frac{1}{2}C_1V_1^2$$

$$E_{ni} = \frac{1}{2}(200 \times 10^{-12}) \times 9 \times 10^4$$

$$= 9 \times 10^{-6} \text{ J}$$

Common potential attained by the combination

$$V_2 = \frac{C_1V_1}{C_1 + C_2} = 200 \text{ V}$$

$$= 2\times10^2 \text{ V}$$

$$C_2 = 100 \text{ pE} = 100 \times 10^{-12} \text{ F}$$

Final energy of the combination

$$E_{nf} = \frac{1}{2}(C_1 + C_2) V_2^2$$

$$= \frac{1}{2} \times 300 \times 10^{-12} \times 4 \times 10^4$$

$$= 6 \times 10^{-6} \text{ J}$$

Difference in energy

$$\Delta E = E_{ni} - E_{nf}$$

$$= 9 \times 10^{-6} - 6 \times 10^{-6}$$

$$= 3 \times 10^{-6} \text{ J}$$