Electric Charges and Fields

TEXTBOOK Questions

1.1. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Ans. Here, we are given

$$q_1 = 2 \times 10^{-7}\ \text{C}$$
$$q_2 = 3 \times 10^{-7}\ \text{C}$$
$$r = 30\ \text{cm} = 0.3\ \text{m}$$

Electrostatic force between the charges is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

where ε_0 = Permittivity of free space

and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$

So, $$F = \frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2}$$
$$= 6 \times 10^{-3}\ \text{N}$$

As both charges are having the same polarity, force will be repulsive in nature.

1.2. The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge –0.8 μC in air is 0.2 N.

(a) What is the distance between the two spheres?

(b) What is the force on the second sphere due to the first?

Ans. (*a*) Charge on first small sphere,

$$q_1 = 0.4\ \mu\text{C} = 0.4 \times 10^{-6}\ \text{C}$$

Charge on the second small sphere,

$$q_2 = -0.8\ \mu\text{C} = -0.8 \times 10^{-6}\ \text{C}$$

Electrostatic force on the first sphere due to second sphere $F = 0.2$ N

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$ where,

ε_0 = Permittivity of free space and

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$$

$$\therefore r^2 = \frac{q_1 q_2}{4\pi\varepsilon_0 F}$$

$$= \frac{0.4 \times 10^{-6} \times 0.8 \times 10^{-6} \times 9 \times 10^9}{0.2}$$
$$= 144 \times 10^{-4}$$

$$\Rightarrow r = \sqrt{144 \times 10^{-4}}$$
$$= 0.12\ \text{m} = 12\ \text{cm}$$

The distance between the two spheres is 12 cm.

(*b*) Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is also 0.2 N.

1.3. Check that the ratio $ke^2/G\, m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans. The given ratio is $\frac{ke^2}{Gm_e m_p}$.

where, G = Gravitational constant and its unit is N m^2 kg^{-2}.

m_e = mass of electron and

m_p = mass of proton

Their unit is kg.

e = charge on an electron and its unit is C.

k = constant = $\frac{1}{4\pi\varepsilon_0}$ and its unit is N m^2 C^{-2}.

Therefore, unit of the given ratio

$$\frac{ke^2}{Gm_e m_p} = \frac{[\text{Nm}^2\ \text{C}^{-2}][\text{C}^2]}{[\text{Nm}^2\ \text{kg}^{-2}]\,[\text{kg}]\,[\text{kg}]}$$

$$= M^0L^0T^0$$

Hence, the given ratio is dimensionless.

$e = 1.6 \times 10^{-19}$ C

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$m_e = 9.1 \times 10^{-31}$ kg;

$m_p = 1.67 \times 10^{-27}$ kg

Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_e m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$\approx 2.4 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

1.4. (a) Explain the meaning of the statement 'electric charge of a body is quantised.'

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic *i.e.*, large scale charges?

Ans. (a) Any charge (Q) in nature is an integral multiple of the electronic charge $|e|$ or $Q = n|e|$, where n is an integer.

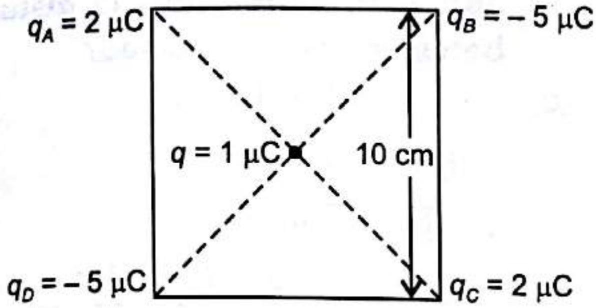
(b) Because $|e|$ is very small and n is generally very large, charges on objects behave as if they are continuous.

1.5. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Ans. When a pair of bodies are rubbed against each other, the transfer of charge takes place. In the process, one body receives charge and other loses it and they become negatively and positively charged respectively. There is no new charge created or destroyed in the process. This implies that in an isolated system, the total charge is always conserved and hence, the observation is consistent with the law of conservation of charge.

1.6. Four point charges q_A = 2 μC, q_B = –5 μC, q_C = 2 μC and q_D = – 5 μC are located at the corners of a square *ABCD* of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?

Ans. In the given figure, it can be easily seen that the charges of equal magnitude and same sign are at the corners of same diagonal. So, they will exert equal and opposite forces at the charge placed at center, cancelling out each other. So, the net force on the charge at centre of the square is zero Newton.



1.7. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two field lines never cross each other at any point.

Ans. (a) An electrostatic field line represents the actual path of a unit positive charge which is continuous. This field line cannot have sudden breaks because moving charges cannot jump from one position to another.

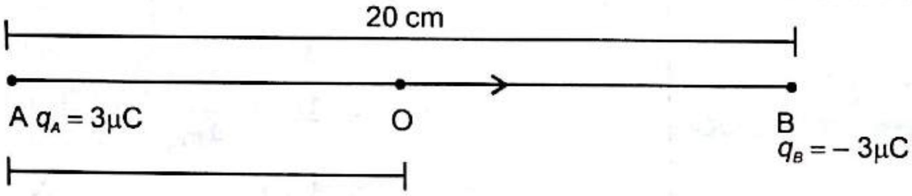
(b) They do not cross each other because at the point of intersection two tangents can be drawn but two directions of force is impossible at any point.

1.8. Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Ans.



(a) $q_A = 3\ \mu C = 3 \times 10^{-6} C$; $AB = 20\ cm = 0.2\ m$

$q_B = -3\ \mu C = -3 \times 10^{-6} C$; $AO = OB = 10\ cm = 0.1\ m$

$$\vec{E}_O = \vec{E}_A + \vec{E}_B = 9 \times 10^9 \left[\frac{3\times10^{-6}}{(0.1)^2} + \frac{3\times10^{-6}}{(0.1)^2}\right] \text{ along } OB$$

$$= 5.4 \times 10^6\ NC^{-1} \text{ along } OB$$

(b) When $q_o = -1.5 \times 10^{-9} C$ is placed at O.

$$F_o = q_o \cdot \vec{E}_o = (-1.5 \times 10^{-9}) \times 5.4 \times 10^6$$

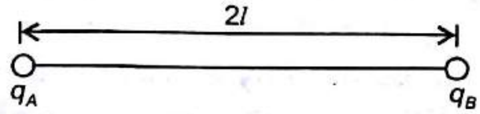
$$= 8.1 \times 10^{-3}\ N \text{ along } OA.$$

1.9. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, –15 cm) and B: (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?

Ans. $q_A = 2.5 \times 10^{-7}$ C

$q_B = -2.5 \times 10^{-7}$ C

$2l = 30\ cm = 30 \times 10^{-2}\ m$



(i) Total charge is zero because both have equal and opposite charge.

(ii) $p = q(2l) = 2.5 \times 10^{-7} \times 30 \times 10^{-2}$

$= 75.0 \times 10^{-9} = 7.5 \times 10^{-8}$ Cm

1.10. An electric dipole with dipole moment 4×10^{-9} Cm is aligned at 30° with direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.

Ans. Use formula

$$\tau = pE \sin\theta$$
$$= 4 \times 10^{-9} \times 5 \times 10^4 \times 0.5$$
$$= 10^{-4} \text{ Nm} \quad (\because \sin 30° = \frac{1}{2} = 0.5)$$

1.11. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?).

(b) Is there a transfer of mass from wool to polythene?

Ans. $q = -3 \times 10^{-7}$C; $e = 1.6 \times 10^{-19}$ C

(a) Electrons are transferred from wool to polythene.

$$n = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}}$$

(–ve sign can be neglected)

$$= 1.8 \times 10^{12}$$
$$\approx 2 \times 10^{12} \text{ electrons.}$$

(b) Yes, there is a transfer of mass. But of negligible amount.

Mass transferred is (m)

$$= 2 \times 10^{12} \times 9.1 \times 10^{-31} \text{ kg}$$
$$= 1.82 \times 10^{-18} \text{ kg}$$
$$\approx 2 \times 10^{-18} \text{ kg.}$$

1.12. (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Ans. (a)
$$q_A = 6.5 \times 10^{-7} \text{ C}$$
$$q_B = 6.5 \times 10^{-7} \text{ C}$$
$$r = 0.5 \text{ m}$$
$$F = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2}$$
$$= 1.5 \times 10^{-2} \text{ N}$$

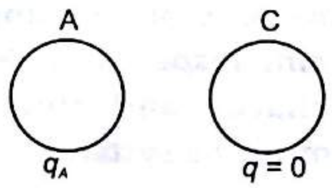
(b)
$$q'_A = 2q_A,\ q'_b = 2q_B,\ r' = \frac{r}{2}$$
$$F^1 = \frac{1}{4\pi\varepsilon_0}\frac{q'_A q'_B}{(r')^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2q_A 2q_B}{\left(\frac{r}{2}\right)^2}$$
$$= 16 \times \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A q_B}{r^2}\right)$$
$$= 16 \times 1.52 \times 10^{-2} \text{ N}$$
$$= 0.2432 \text{ N or } 0.24 \text{ N}$$

1.13. Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans. Initially the charge on sphere A is,

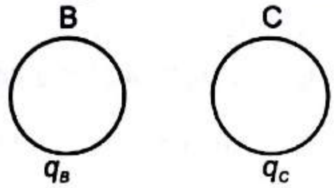
$q_A = 6.5 \times 10^{-7}$ C

When A and C are brought in contact, charges are shared equally.



i.e.
$$q_A = 3.25 \times 10^{-7} \text{ C}$$
$$q_C = 3.25 \times 10^{-7} \text{ C}$$
$$q_B = 6.5 \times 10^{-7} \text{ C}$$

When B and C are brought in contact, again charges are shared equally.



i.e. $q_B = \dfrac{3.25 \times 10^{-7} + 6.5 \times 10^{-7}}{2}$

$= 4.875 \times 10^{-7}$ C

$q_C = 4.875 \times 10^{-7}$ C

If $r_{AB} = 0.5$ m

So, the new force of repulsion between A and B is

$$F_{AB} = \frac{9 \times 10^9 \times 3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(0.5)^2}$$

$F_{AB} = 5.7 \times 10^{-3}$ N

1.14. Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio? Explain.

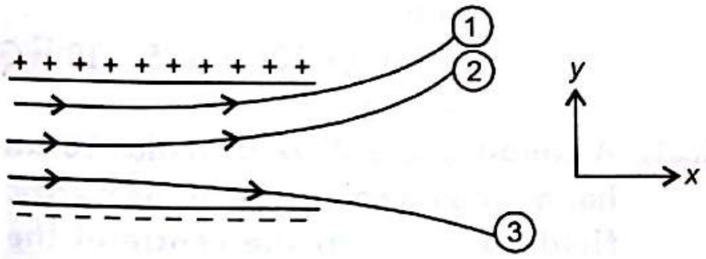


Figure 1.33

Ans. We know $y = ut + \frac{1}{2}at^2$

Assuming $u = 0$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2$$

If 't' is same for all the particles,

Then $\frac{q}{m} \propto y$

Thus, particle having maximum deflection along vertical direction will be having the highest charge to mass ratio. Thus particles 1 and 2 are negatively charged while particle 3 is positively charged. Particle 3 has the highest charge to mass ratio.

1.15. Consider a uniform electric field $E = 3 \times 10^3 \hat{i}$ N/C.

(*a*) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the $y - z$ plane?

(*b*) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

Ans. Given, $\vec{E} = 3 \times 10^3 \hat{i}$ NC^{-1}

$\vec{S} = (0.10)^2 \hat{i}$ m^2

(*a*) $\phi_E = \vec{E} \cdot \vec{S}$

$= 3 \times 10^3 \times 0.10 \times 0.10 (\hat{i}.\hat{i})$

$\phi_E = 30$ Nm^2 C^{-1}

(*b*) $\phi_E = E\,S \cos 60°$

$= 3 \times 10^3 \times 10^{-2} \times \frac{1}{2}$

$= 15$ Nm^{2}C^{-1}

1.16. What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans. As all the faces of the cube are parallel to the coordinate axes, therefore, the number of field lines entering the cube is equal to the number of field lines leaving the cube. As a result, net flux through the cube is zero.

1.17. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 Nm^2/C.

(*a*) What is the net charge inside the box?

(b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Ans. (a) Flux (ϕ) = $\frac{q}{\varepsilon_0}$

i.e., $q = \varepsilon_0 \phi$

$= 8.85 \times 10^{-12} \times 8.0 \times 10^3$ C

$q = 0.07\ \mu C$

(b) We cannot conclude this as according to Guass's theorem there are two possibilities.

(i) No charge inside.

(ii) Net charge is zero inside.

1.18. A point charge + 10 μC is at a distance 5 cm directly above the centre of square of side 10 cm as shown in Figure 1.34. What is the magnitude of the electric flux through the square? [Hint: Think of the square as one face of a cube with edge 10 cm.]

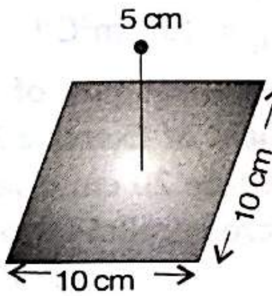


Figure 1.34

Ans. As we can imagine that given square is one of the faces of a cube of side 10 cm.

$$\phi = \frac{1}{6}\frac{q}{\varepsilon_0} = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}}$$

[∵ number of faces in a cube is 6]

$\phi = 1.88 \times 10^5$ Nm^2 C^{-1}

$\approx 2 \times 10^5$ Nm^2 C^{-1}

1.19. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans. Charge inside the cubic Gaussian surface

$q = 2.0\ \mu C$

$= 2.0 \times 10^{-6}$C.

Total flux through surface of cube

$$\phi = \frac{q}{\varepsilon_0} = \frac{2.0 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$= 2.2 \times 10^5$ Nm^{2}C^{-1}

1.20. A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(b) What is the value of the point charge?

Ans. (a) As same charge is enclosed by the Gaussian surface, electric flux remains unchanged, i.e. -10^3 Nm^2 C^{-1}.

(b) $\phi = \frac{q}{\varepsilon_0}$

$q = \phi\, \varepsilon_0$

$= (-1.0 \times 10^3) \times 8.85 \times 10^{-12}$ C

$q = -8.8 \times 10^{-9}$ C

1.21. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Ans. R = 10 cm, r = 20 cm,

$E = 1.5 \times 10^3$ N/C

$q = 4\pi\varepsilon_0 r^2 E = -6.67$ nC

1.22. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Ans. (a) Diameter of the sphere, d = 2.4 m

∴ Radius of the sphere, r = 1.2 m

Surface charge density,

$\sigma = 80.0\ \mu C/m^2 = 80 \times 10^{-6}\ C/m^2$

We know that, total charge on the surface of the sphere,

Q = Charge density × Surface area

$= \sigma \times 4\pi r^2$

$= 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$

$= 1.447 \times 10^{-3}\ C \approx 1.45 \times 10^{-3}\ C$

Therefore, the charge on the sphere is 1.45×10^{-3} C.

(*b*) Total electric flux (ϕ_{Total}) leaving the surface of the sphere containing net charge Q is,

$$\phi_{Total} = \frac{Q}{\varepsilon_0}$$

where, ε_0 = Permittivity of free space

$= 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$Q = 1.45 \times 10^{-3}\ C$

$$\phi_{Total} = \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$= 1.6 \times 10^8\ N\ m^2\ C^{-1}$

Therefore, the total electric flux leaving the surface of the sphere is $1.6 \times 10^8\ N\ m^2\ C^{-1}$.

1.23. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Ans. $E = 9 \times 10^4$ N/C; $r = 2$ cm

$$\lambda = \frac{E\,(4\pi\varepsilon_0 r)}{2} = 10^{-7}\ C/m$$

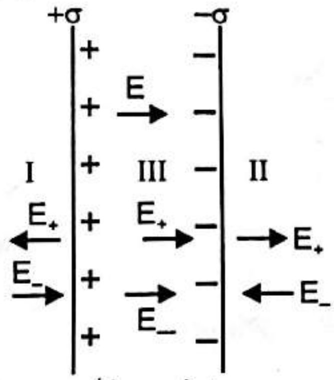
i.e., $\lambda = 0.1\ \mu C/m$

1.24. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m². What is E : (*a*) in the outer region of the first plate, (*b*) in the outer region of the second plate and (*c*) between the plates?

Ans. (*a*) and (*b*) In the I and II regions of the plates, electric field is

$E_I = E_{II} = 0$

as electric fields are equal in magnitude and opposite in directions in regions I and II



(*c*) Electric field between the plates

$$E_{III} = \frac{\sigma}{\varepsilon_0} = \frac{17 \times 10^{-22}}{8.85 \times 10^{-12}}\ NC^{-1}$$

$E_{III} = 1.9 \times 10^{-6}\ NC^{-1}$

1.25. An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4\ NC^{-1}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g cm⁻³. Estimate the radius of the drop. ($g = 9.81\ ms^{-2}$; $e = 1.60 \times 10^{-19}$ C).

Ans. Here, $n = 12$

$e = 1.6 \times 10^{-19}$ C

$E = 2.55 \times 10^4\ NC^{-1}$

$\rho = 1.26 \times 10^{+3}\ kg\ m^{-3}$

$g = 9.81\ ms^{-2}$

$r = ?$

As $mg = neE$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g = neE$$

$$r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$$

$= 9.81 \times 10^{-7}$ m

$= 9.81 \times 10^{-4}$ mm

1.26. Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic field lines?

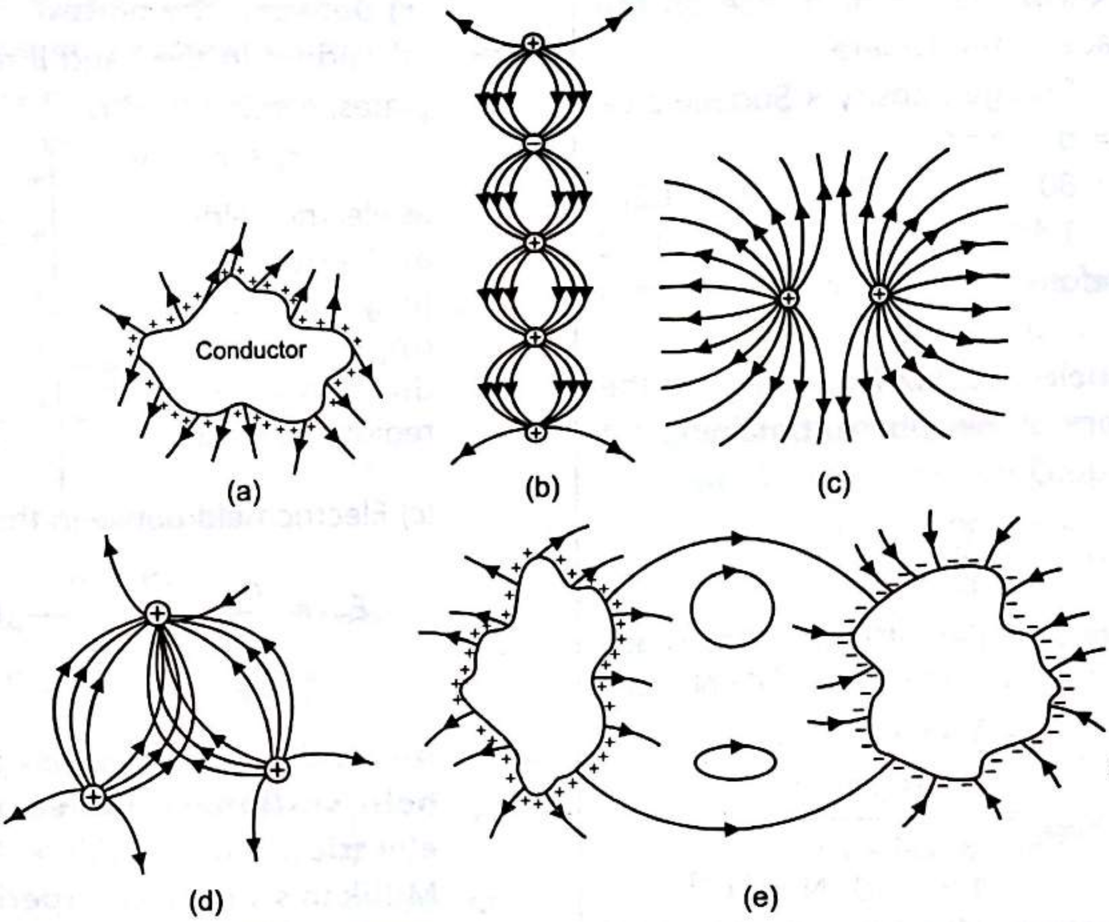


Figure 1.35

Ans. (*a*) The field lines showed in (*a*) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.

(*b*) The field lines showed in (*b*) do not represent electrostatic field lines because the field lines cannot start from a negative charge and end at a positive charge.

(*c*) The field lines showed in (*c*) represent electrostatic field lines. This is because the field lines are emerging from the positive charges and are repelling each other.

(*d*) The field lines showed in (*d*) do not represent electrostatic field lines because the field lines cannot intersect each other.

(*e*) The field lines showed in (*e*) do not represent electrostatic field lines because field lines cannot form closed loops.

1.27. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction?

Ans. In non uniform electric field

$$F = P_x \frac{\partial E}{\partial x} + P_y \frac{\partial E}{\partial y} + P_z \frac{\partial E}{\partial z}$$

$$\frac{\partial E}{\partial z} = 10^5 \text{ NC}^{-1} \text{ m}^{-1}$$

$$P_z = -10^{-7} \text{ Cm}, \; P_x = P_y = 0$$

$$\frac{\partial E}{\partial x} = 0, \; \frac{\partial E}{\partial y} = 0$$

$$\vec{F} = -10^{-7} \times 10^5 \text{ N} = -10^{-2} \text{ N}$$

Here –ve sign indicates that force is directed along –ve z-axis.

As both $\vec{P}$ and $\vec{E}$ are along z-axis, $\sin\theta = 0$

$\therefore \quad \tau = PE\sin\theta = 0 \qquad [\because \theta = 0°]$

1.28. **(a) A conductor *A* with a cavity as shown in Figure 1.36(a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor.**

(b) Another conductor *B* with charge *q* is inserted into the cavity keeping *B* insulated from *A*. Show that the total charge on the outside surface of *A* is Q + *q* [Figure 1.36(b)].

(c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

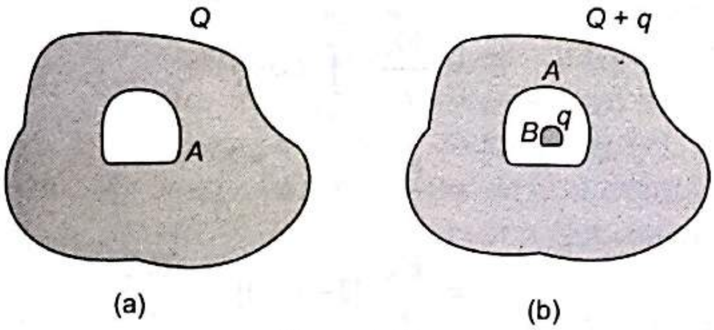


Figure 1.36

Ans. (a) Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity, *E* inside the charged conductor is zero.

Let q is the charge inside the conductor and ε_0 is the permittivity of free space.

According to Gauss's law,

$$\text{Electric flux, } \phi = \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

Here, $E = 0$

$$\Rightarrow \frac{q}{\varepsilon_0} = 0$$

$\because \varepsilon_0 \neq 0 \quad \therefore \quad q = 0$

Therefore, charge inside the conductor is zero.

Hence, the entire charge Q must appear on the outer surface of the conductor.

(b) If another conductor B having charge $+q$ is kept inside the conductor A and insulated from A, a charge of amount $-q$ will be induced in the inner surface of conductor A and $+q$ is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is $Q + q$.

(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully by a metallic surface. A closed metallic body acts as an electrostatic shield.

1.29. **A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $(\sigma/2\varepsilon_0)\hat{n}$ where $\hat{n}$ is the unit vector in the outward normal direction and σ is the surface charge density near the hole.**

Ans. In case hole is filled up, total electric field outside is

$$\vec{E}_1 + \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \text{...}(i)$$

Total field inside the hollow conductor

$$\vec{E}_1 + \vec{E}_2 = 0 \qquad \text{...}(ii)$$

Here $\vec{E}_1$ is the electric field due to the portion filling up the hole and $\vec{E}_2$ is the electric field due to rest of the charged conductor.

From relation (*ii*) we conclude that

$$|\vec{E}_1| = |\vec{E}_2|$$

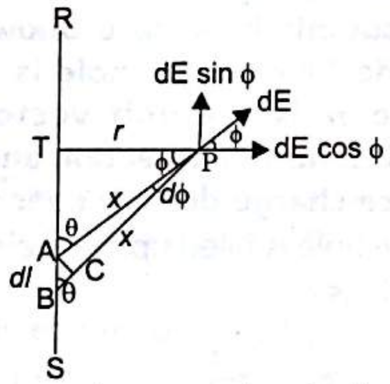
Hence, from (*i*), we get

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

1.30. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Hint: Use Coulomb's law directly and evaluate the necessary integral.]

Ans. Let RS be a thin wire having a linear charge density λ. *p* is any point at normal distance *r* from the mid point of wire RS, *i.e.*, T. Let AB be a small elementary length *dl*. The charge on AB is $dq = \lambda\, dl$. Let AP = BP = *x* (as *dl* is very small). Let AC be the perpendicular from A on BP meeting it at C. $d\phi$ be the angle subtended by elementary length *dl* at P. The elementary electric field *dE* at P due to charge on AB will be

$$dE = k\frac{dq}{x^2} = k\frac{\lambda dl}{x^2} \quad \text{...(A)}$$



It can be resolved in mutually perpendicular components $dE \cos\phi$ and $dE \sin\phi$ as shown. As there will be symmetrically opposite elementary charge in the upper half of the wire (TR), the component $dE \sin\phi$ will be cancelled, while $dE \cos\phi$ will be added up for each pair of symmetrically opposite elementary lengths.

Hence *E* at *P* will be $E = \int dE \cos\phi$ ...(B)

In ΔABC $\quad \sin\theta = \dfrac{AC}{dl}$

∴ $\quad dl = \dfrac{AC}{\sin\theta}$ or $AC = dl \sin\theta$

In ΔACP $\quad d\phi = \dfrac{AC}{x} = \dfrac{dl \sin\theta}{x}$

In ΔAPT $\quad \sin\theta = \dfrac{r}{x}$

∴ $\quad d\phi = \dfrac{dl\, r}{x^2}$

or $\quad dl = \dfrac{x^2}{r}\, d\phi$

∴ $\quad dE = \dfrac{k\lambda}{x^2} \cdot \dfrac{x^2}{r}\, d\phi = \dfrac{k\lambda}{r}\, d\phi$

Substituting this value in equation (B)

$$E = \int \frac{k\lambda}{r} \cos\phi\, d\phi$$

If the wire is infinitely large, then limits of integral on R.H.S. will be from $-\pi/2$ to $\pi/2$

∴ For infinitely large, charged, thin wire the intensity of electric field at *P* will be

$$E = \frac{k\lambda}{r}\int_{-\pi/2}^{\pi/2} \cos\phi\, d\phi$$

$$= \frac{k\lambda}{r}\left[\sin\phi\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{k\lambda}{r}\,[1-(-1)] = \frac{2k\lambda}{r}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \left(\because\ k = \frac{1}{4\pi\varepsilon_0}\right)$$

1.31. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) *e*, and the 'down' quark (denoted by *d*) of charge (–1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give

rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Ans. Given, $q_u = +2/3$

$q_d = -1/3$

Possible quark composition of proton is $2u + 1d$ *i.e.* u, u, d so total charge on it is

$$\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$$

Possible quark composition of neutron is $1u + 2d$. *i.e.* $u, d, d,$ so total charge is

$$\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

1.32. (*a*) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (*i.e.* where *E* = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(*b*) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Ans. (*a*) Suppose equilibrium at null point is stable. This means that a test charge placed at null point if displaced in any direction experiences a restoring force directed towards the null point. From this we conclude that all electrostatic field lines near the null point should be directed towards the null point *i.e.* there should be a net inward flux around the null point. As the surface encloses no charge, there should not be any flux. Thus our assumption is wrong. Test charge at null point is unstable.

(*b*) Point 'O' is a null point in the diagram. Test charge placed at 'O' will not experience any net force. As soon as we displace it in any direction it experiences a net force.

1.33. A particle of mass m and charge (-q) enters the region between the two charged plates initially moving along *x*-axis with speed v_x (like particle 1 in Figure 1.33). The length of plate is L and a uniform electric field *E* is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\, v_x^2)$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Ans. Charge on a particle of mass $m = -q$

Velocity of the particle = v_x

Length of the plates = L

Magnitude of the uniform electric field between the plates = E

We know that,

Mechanical force,

$$F = \text{Mass } (m) \times \text{Acceleration } (a)$$

$$\Rightarrow \quad a = \frac{F}{m}$$

Acceleration in the uniform electric field, E

$$a = \frac{qE}{m} \qquad \text{...}(i)$$

Time taken by the particle to cross the field of length L is given by,

$$t = \frac{\text{Length of the plate}}{\text{Velocity of the particle}}$$

$$= \frac{L}{v_x} \qquad \text{...}(ii)$$

In the vertical direction, initial velocity, $u = 0$

The vertical deflection 's' of the particle can be obtained according to the third equation of motion which is:

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

[Using (*i*) and (*ii*)]

$$s = \frac{qEL^2}{2mv_x^2}$$

Hence, vertical deflection of the particle at the far edge of the plate is $qEL^2/(2mv_x^2)$.

1.34. Suppose that the particle in Exercise in 1.33 is an electron projected with velocity v_x = 2.0 × 10^6 m s^{-1}. If E between the plates separated by 0.5 cm is 9.1 × 10^2 N/C, where will the electron strike the upper plate? (|*e*|=1.6 × 10^{-19} C, m_e = 9.1 × 10^{-31} kg.)

Ans. Velocity of the particle,

$$v_x = 2.0 \times 10^6 \text{ m/s}$$

Separation of the two plates,

$$d = 0.5 \text{ cm} = 0.005 \text{ m}$$

Electric field between the two plates,

$$E = 9.1 \times 10^2 \text{ N/C}$$

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Let the electron strike the upper plate at the end of plate *L*, when deflection is *s*. Therefore,

$$s = \frac{qEL^2}{2mv_x^2}$$

$$L = \sqrt{\frac{2smv_x^2}{qE}}$$

$$= \sqrt{\frac{2 \times 0.005 \times 9.1 \times 10^{-31} \times (2.0 \times 10^6)^2}{1.6 \times 10^{-19} \times 9.1 \times 10^2}}$$

$$= \sqrt{0.025 \times 10^{-2}} = \sqrt{2.5 \times 10^{-4}}$$

$$\simeq 1.6 \times 10^{-2} \text{ m} \simeq 1.6 \text{ cm}$$

Therefore, the electron will strike the upper plate after travelling 1.6 cm.