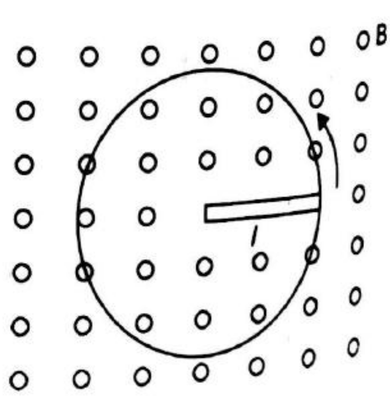
NUMERICALS

NCERT Numericals

Induced emf

1. **A 1.0 m metallic rod is rotated with an angular velocity of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.**



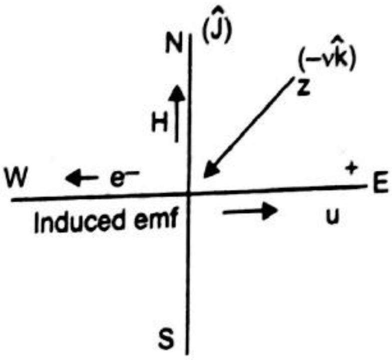
Sol. EMF developed between the centre of ring and the point on the ring.

$$\varepsilon = \frac{1}{2} B \omega l^2$$

Given $B = 0.5$ T, $\omega = 400$ rad/s, $l = 1.0$ m.

$$\therefore \quad \varepsilon = \frac{1}{2} \times 0.5 \times 400 \times (1.0)^2 = \mathbf{100\ volt}$$

2. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of earth's magnetic field equal to 0.30×10^{-4} Wbm^{-2}.



(a) What is the instanteous value of the emf induced in the wire?

(b) What is the direction of emf?

(c) Which emf of the wire is at the higher electrical potential?

Sol. (a) Instantaneous emf, $\varepsilon = B_n vl = Hvl$

Given $H = 0.30 \times 10^{-4}$ T, $v = 5.0\ ms^{-1}$, $l = 10$ m

$$\therefore \quad \varepsilon = 0.30 \times 10^{-4} \times 5.0 \times 10 = 1.5 \times 10^{-3}\ \text{V} = \mathbf{1.5\ mV}$$

(b) By Fleming's right hand rule, the direction of induced current in wire is from west to east, therefore direction of emf is from west to east.

(c) The direction of electron flow according to relation

$$\vec{F}_m = q\,\vec{v} \times \vec{B} = -e(-v\,\hat{k}) \times B\,\hat{j}) = -evB\,\hat{i}$$

i.e., along negative *x*-axis *i.e.*, from east to west.

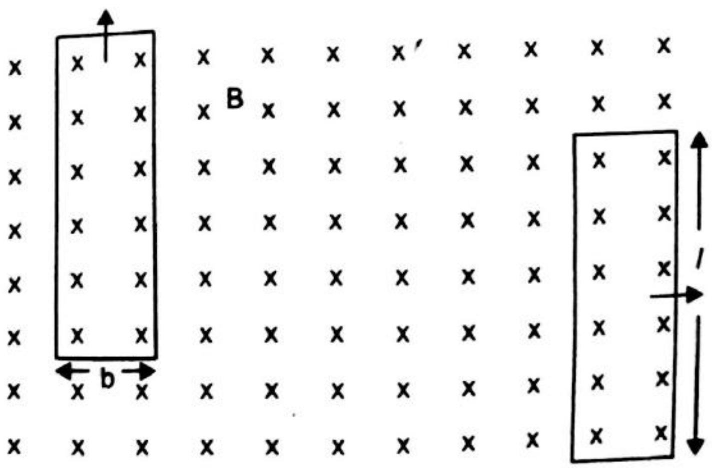
The induced emf will oppose the flow of electrons from east to west, so the flow will be eastern and at a higher potential.

3. A rectangular wire loop of sides 8 cm × 2 cm (fig.) with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed if the velocity of the loop is 1 cms^{-1} in a direction normal to the (i) longer side (ii) shorter side of the loop? For how long does the induced voltage last in each case?

Sol. Given $l = 8$ cm $= 8 \times 10^{-2}$ m,

$b = 2$ cm $= 2 \times 10^{-2}$ m,

$v = 1$ cm $s^{-1} = 1 \times 10^{-2}$ m / s, $B = 0{\cdot}3$ T



(i) When velocity is normal to the longer side

Induced emf $\quad \varepsilon = Bvl$

$= 0.3 \times 1 \times 10^{-2} \times 8 \times 10^{-2} = \mathbf{24 \times 10^{-5}\ V}$

EMF will last only so long as the loop is in the magnetic field.

$$\text{Time taken} = \frac{\text{distance}}{\text{velocity}} = \frac{b}{v} = \frac{2 \times 10^{-2}}{1 \times 10^{-2}} = \mathbf{2\ s}$$

(ii) When velocity is normal to the shorter side

$$\varepsilon_2 = Bvb$$

$$= 0.3 \times 1 \times 10^{-2} \times 2 \times 10^{-2} = 6 \times 10^{-5}\ \text{V}$$

$$\text{Time taken} = \frac{l}{v} = \frac{8 \times 10^{-2}}{1 \times 10^{-2}} = 8\ \text{s}$$

4. A jet plane is travelling westward at a speed of 1800 km/h. What is the potential difference developed between the ends of a wing 25 m long, its earth's magnetic field at the location has a magnitude of 5.0×10^{-4} T and the dip angle is 30°. *[CBSE (AI) 2009]*

Sol. **Key idea:** The wing of horizontal travelling plane will cut the vertical component of earth's magnetic field, so emf is induced across the wing. The vertical component of earth's field is given by

$$V = B_e \sin \theta;$$

where B_e is earth's magnetic field and θ is angle of dip

Induced emf of wing $\quad \varepsilon = V v l = (B_e \sin \theta) v l$

Given $\quad B_e = 5.0 \times 10^{-4}$ T, $l = 25$ m, $\theta = 30°$,

$$v = 1800 \text{ km/h} = 1800 \times \frac{5}{18} \text{ m/s} = 500 \text{ m/s}$$

$$\therefore \quad \varepsilon = (5.0 \times 10^{-4} \times \sin 30°) \times 500 \times 25$$

$$= (5.0 \times 10^{-4} \times 0.5) \times 500 \times 25 = 3.1\ \text{V}$$

Induced emf and Power

5. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to joule heating. Where does the power come from?

Sol. Magnetic flux linked with the coil, $\phi = \vec{B} \bullet \vec{A}$

$$= NBA \cos \theta = NBA \cos \omega t \text{ (where } \theta = \omega t)$$

EMF induced in the coil $\quad \varepsilon = -N \dfrac{d\phi}{dt}$

$$= -N \frac{d}{dt}(BA \cos \omega t) = NBA\omega \sin \omega t$$

Maximum emf induced

$$\varepsilon_{max} = NBA\omega = NB(\pi r^2)\,\omega$$

Given $N = 20$, $r = 8.0$ cm $= 8.0 \times 10^{-2}$ m, $B = 3.0 \times 10^{-2}$ T, $w = 50$ rad/s

$$\therefore \quad \varepsilon_{max} = 20 \times 3.0 \times 10^{-2} \times 3.14 \times (8.0 \times 10^{-2})^2 \times 50$$

$$= 0.603 \text{ volt}$$

Average emf $= NBA\omega(\sin \omega t)_{av} = 0$

(Since average value of sin ωt over a complete cycle is zero.)

Maximum current induced,

$$I_{max} = \frac{\varepsilon_{max}}{R} = \frac{0.603}{10} = 0.0603\ \text{A}$$

Average power loss due to joule heating

$$P_{max} = (I^2)_{av}\ R = \frac{(\varepsilon^2)_{av}}{R}$$

Since average value of $\sin^2 \omega t$ for a complete cycle is $\frac{1}{2}$ *i.e.*, $(\sin^2 \omega t)_{av} = \frac{1}{2}$

$$\therefore \quad P_{max} = \frac{1}{2}\frac{N^2B^2A^2\omega^2}{R}$$

$$= \frac{1}{2}(NBA\omega)\left(\frac{NBA\omega}{R}\right) = \frac{1}{2}\varepsilon_{max}\ I_{max}$$

$$= \frac{1}{2} \times 0.603 \times 0.0603 = 0.018\ \text{W}$$

The current induced causes a torque which opposes the rotation of the coil. An external agency (rotor) must supply torque to counter this torque in order to keep the coil rotating uniformly. The source of power dissipated as heat is the rotor.

6. A rectangular loop of sides 8 cm × 2 cm with a small cut is stationary in a uniform magnetic field produced by an electromagnet. If the current feeding the electromagnet is gradually reduced so that the magnetic field decreases from its initial value of 0.3 T at the rate of 0.02 Ts^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?

Sol. Area of loop,

$$A = 8\ \text{cm} \times 2\ \text{cm} = 16\ \text{cm}^2 = 16 \times 10^{-4}\ \text{m}^2$$

Induced emf, $\varepsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{\Delta}{\Delta t}(BA) = -A\frac{\Delta B}{\Delta t}$

Here $\frac{\Delta B}{\Delta t} = -0.02\ \text{Ts}^{-1}$

$\therefore$ Induced emf $\varepsilon = -(16 \times 10^{-4}) \times (-0{\cdot}02)$

$$= 3.2 \times 10^{-5}\ \text{V}$$

Induced current,

$$I = \frac{\varepsilon}{R} = \frac{3.2 \times 10^{-5}}{1.6} = 2 \times 10^{-5}\ \text{A}$$

Power dissipated, $P = I^2R = (2 \times 10^{-5})^2 \times 1.6$

$$= 6.4 \times 10^{-10}\ \text{W}$$

The source of the power is the external source feeding the electromagnet.

Self Inductance and Mutual Inductance

7. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced. Calculate the self-induction of the circuit.

Sol. Induced emf $E = -L\frac{\Delta I}{\Delta t}$ ...(*i*)

Here, $E = 200$ V,

$$\frac{\Delta I}{\Delta t} = \frac{I_2 - I_1}{\Delta t} = \frac{0.0 - 5.0}{0.1} = -50\ \text{A/s}$$

∴ Substituting these values in (*i*)

$$L = \frac{E}{(-\Delta I / \Delta t)} = \frac{200}{50} = 4 \text{ H}$$

8. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of magnetic flux linkage with the other coil?

Sol. We have $\phi_2 = MI_1$

Change in magnetic flux, $\Delta\phi_2 = M\Delta I_1$

Given $M = 1.5 \text{ H}, \Delta I_1 = 20 - 0 = 20 \text{ A}$

∴ $\Delta\phi_2 = 1.5 \times 20 = \mathbf{30 \text{ weber}}$

9. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside normal to the axis of the solenoid. The current carried by the solenoid changes steadily from 2A to 4A in 0.1 s, what is the induced emf in the loop while the current is changing?

Sol. Mutual inductance of solenoid coil system

$$M = \frac{\mu_0 N_1 N_2 A_2}{l}$$

Here $N_1 = 15, N_2 = 1, l = 1 \text{ cm} = 10^{-2}$ m,

$$A_2 = 2.0 \text{ cm}^2 = 2.0 \times 10^{-4} \text{ m}^2$$

∴ $$M = \frac{4\pi \times 10^{-7} \times 15 \times 1 \times 2.0 \times 10^{-4}}{10^{-2}} = 120\,\pi \times 10^{-9} \text{ H}$$

Induced emf. in the loop

$$\varepsilon_2 = M \frac{\Delta I_1}{\Delta t} \quad \text{(numerically)}$$

$$= 120\,\pi \times 10^{-9} \frac{(4-2)}{0.1}$$

$$= 120 \times 3.14 \times 10^{-9} \times \frac{2}{0.1} = 7.5 \times 10^{-6} \text{ V}$$

$$= \mathbf{7.5\,\mu V}$$

10. An air cored solenoid with length 30 cm, area of cross-section 25 cm² and number of turns 500 carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Sol. Induced emf in a solenoid,

$$\varepsilon = -L \frac{\Delta I}{\Delta t} \quad \ldots(i)$$

Inductance of solenoid $$L = \frac{\mu_0 N^2 A}{l} \quad \ldots(ii)$$

∴ Induced emf $$\varepsilon = -\left(\frac{\mu_0 N^2 A}{l}\right)\frac{\Delta I}{\Delta t}$$

Here $N = 500$, $A = 2.5 \text{ cm}^2 = 2.5 \times 10^{-4} \text{ m}^2$,

$l = 30 \text{ cm} = 0{\cdot}30 \text{ m}$

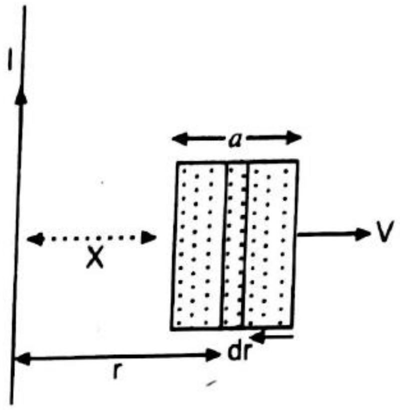
$$\frac{\Delta I}{\Delta t} = \frac{I_2 - I_1}{t} = \frac{0 - 2.5}{10^{-3}} = -2.5 \times 10^3 \text{ A/s}$$

$$\therefore \quad \varepsilon = -\frac{4\pi \times 10^{-7} \times (500)^2 \times 25 \times 10^{-4}}{0{\cdot}30} \times (-2.5 \times 10^3)$$

$$= \frac{3.14 \times 25 \times 2.5}{3} \times 10^{-1} = 6.5 \text{ V}$$

11. (*a*) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side 'a' as shown in fig.

(*b*) Evaluate the induced emf in the loop if the wire carries a current of 50 A and the loop has an instantaneous velocity $v = 10\ ms^{-1}$ at the location $x = 0.2$ m as shown, Take $a = 0.1$ m and assume that the loop has a large resistance.



Sol. (*a*) Suppose the loop is formed of a number of small elements parallel to the length of wire. Consider an element of width dr at a distance r from the wire. The magnetic field at the vicinity of wire, $B = \frac{\mu_0 I}{2\pi r}$, downward perpendicular to the plane of paper.

The magnetic flux linked with this element $\phi_2 = |\vec{B} \cdot d\vec{A_2}|$

$$= |B\, dA_2 \cos \pi| = \frac{\mu_0 I}{2\pi r}(a\, dr) = \frac{\mu_0 I a}{2\pi} \frac{dr}{r}$$

Total magnetic flux linked with the loop

$$\phi_2 = \frac{\mu_0 I a}{2\pi} \int_x^{x+a} \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} [\log_e r]_x^{x+a}$$

$$= \frac{\mu_0 I a}{2\pi} \log_e \left(\frac{x+a}{x}\right)$$

$\therefore$ Mutual inductance

$$M = \frac{\phi_2}{I} = \frac{\mu_0 a}{2\pi} \log_e \left(1 + \frac{a}{x}\right)$$

(*b*) Induced emf in the element $d\varepsilon = Bv\, dr$

$\therefore$ Net emf induced in the loop

$$\varepsilon = \int_x^{x+a} Bv\, dr = \int_x^{x+a} \frac{\mu_0 I}{2\pi r} v\, dr$$

$$= \frac{\mu_0 I v}{2\pi} \int_x^{x+a} \frac{dr}{r} = \frac{\mu_0 I v}{2\pi} [\log_e r]_x^{x+a}$$

$$\therefore \quad \varepsilon = \frac{\mu_0 I v}{2\pi} \log_e \left(\frac{x+a}{x}\right)$$

Given $I = 50$ A, $v = 10 \text{ ms}^{-1}$,

$x = 0.2$ m, $a = 0.1$ m

$$\therefore \quad \varepsilon = \frac{4\pi \times 10^{-7} \times 50 \times 10}{2\pi} \log_e \left(\frac{0.2 + 0.1}{0.2}\right)$$

$$= 10^{-4} \log_e 1.5 = 10^{-4} \times 2.3 \log_{10} 1.5 = 2.3 \times 10^{-4} \times 176$$

$$= 4.05 \times 10^{-5} \text{ V}$$

AC Circuits

12. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply:

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Sol. Given $V_{rms} = 220$ V, $f = 50$ Hz, $R = 100\,\Omega$

(a) RMS value of current $I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2$ A

(b) Net power consumed $P = I_{rms}^2 R$

$$= (2.20)^2 \times 100 = 484 \text{ W}$$

13. A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate

(a) the resistance of the bulb;

(b) the rms current through the bulb **[CBSE (AI) 2013, 2012]**

Sol. (a) $R = \frac{V_{rms}^2}{P} = \frac{220 \times 220}{100} = 484\Omega$

(b) $I_{rms} = \frac{P}{V_{rms}} = \frac{100}{220} = 0.45A$

14. (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Sol. (a) Given $V_0 = 300$ V

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \approx 212 \text{ V}$$

(b) Given $I_{rms} = 10$ A

$$I_0 = I_{rms}\sqrt{2} = 10 \times 1.41 = 14.1 \text{ A}$$

15. (a) A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of current in the circuit.

(b) What is the net power absorbed by the circuit in a complete cycle ?

Sol. (a) Given $L = 44$ mH $= 44 \times 10^{-3}$ H, $V_{rms} = 220$ V, $f = 50$ Hz

Inductive reactance of current $X_C = \omega L$

$\therefore$ RMS value of current, $I_{rms} = \frac{V_{rms}}{\omega L} = \frac{V_{rms}}{2\pi f L}$

$$= \frac{220}{2 \times \left(\frac{22}{7}\right) \times 50 \times 44 \times 10^{-3}} = \frac{220 \times 7 \times 10^3}{2 \times 22 \times 50 \times 44} \quad \frac{700}{44} = 15.9 \text{ A}$$

(*b*) $P_{ar} = V_{rms} \cdot I_{rms} \cdot \cos\phi$

In pure inductor circuit $\phi = \frac{\pi}{2}$ radians $\Rightarrow \cos\frac{\pi}{2} = 0$

As such Net Power consumed $= V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0$

16. (*a*) A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of current in the circuit.

(*b*) What is the net power absorbed by the circuit in a complete cycle.

Sol. (*a*) Given $C = 60\,\mu F = 60 \times 10^{-6}$ F, $V_{rms} = 110$ V, $f = 60$ Hz

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

RMS value of current, $I_{rms} = \frac{V_{rms}}{X_C} = 2\pi f C V_{rms}$

$$= 2 \times 3.14 \times 60 \times (60 \times 10^{-6}) \times 110 \text{ A} = \mathbf{2.49\ A}$$

(*b*) In a purely capacitive circuit, the current leads the applied p.d. by an angle $\frac{\pi}{2}$, therefore,

$$\cos\phi = \cos\frac{\pi}{2} = 0$$

$$\therefore\ P_{av} = V_{rms}\, I_{rms} \cos\phi = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0$$

i.e., In purely capacitive circuit the power absorbed by the circuit is zero.

LR Circuit

17. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

(*a*) What is the maximum current in the coil?

(*b*) What is the time lag between the voltage maximum and the current maximum?

Sol. Given $L = 0.50$ H, $R = 100\,\Omega$, $V = 240$ V, $f = 50$ Hz

(*a*) Maximum (or peak) voltage $V_0 = V\sqrt{2}$

Maximum current, $I_0 = \frac{V_0}{Z}$

Inductive reactance, $X_L = \omega L = 2\pi f L$

$$= 2 \times 3.14 \times 50 \times 0.50 = 157\,\Omega$$

Impedance of circuit

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(100)^2 + (157)^2} = 186\,\Omega$$

$\therefore$ Maximum current $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{240 \times 1.41}{186} = 1.82$ A

(*b*) Phase (lag) angle ϕ is given by

$$\tan\phi = \frac{X_L}{R} = \frac{157}{100} = 1.57$$

$\therefore \quad \phi = \tan^{-1}(1.57) = \mathbf{57 \cdot 5^\circ}$

$$\text{Time lag } \Delta T = \frac{\phi}{2\pi} \times T = \frac{\phi}{2\pi} \times \frac{1}{f} = \frac{57.5}{360} \times \frac{1}{50} \text{ s}$$

$$= 3.2 \times 10^{-3} \text{ s} = \mathbf{3.2 \text{ ms}}$$

RC Circuit

18. In Prob. 17, if the circuit is connected to a high frequency supply (240 V, 10 kHz); find :

(*a*) the maximum current in the coil.

(*b*) the time lag between the voltage maximum and the current maximum.

(*c*) Hence explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Sol. Here $R = 100\,\Omega$, $L = 0.50$ H, $V = 240$ V, $f = 10 \times 10^3$ Hz

(*a*) Inductive reactance $X_L = \omega L$

$$= 2\pi f L = 2 \times 3.14 \times (10 \times 10^3) \times 0.50 \text{ ohm}$$

$$= 3.14 \times 10^4\ \Omega$$

Impedance of circuit $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{(100)^2 + (3.14 \times 10^4)^2} \approx \mathbf{3.14 \times 10^4\ \Omega}$$

Maximum current, $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{240 \times 1.41}{3.14 \times 10^4}$ A

$$= 107 \times 10^{-4} \text{ A} = \mathbf{10.7 \text{ mA}}$$

(*b*) Phase lag $\phi = \tan^{-1} \frac{X_L}{R} = \frac{3.14 \times 10^4}{100} = 314 = 89.8^\circ \approx \frac{\pi}{2}$

Maximum current in high frequency circuit is much smaller than that in low frequency circuit: this implies that at high frequencies an inductor behaves like an open circuit.

In a dc circuit after steady state $\omega = 0$, so $X_L = \omega L = 0$; *i.e.*, inductor offers no hindrance and hence it acts as a pure conductor.

LC Circuit

19. (*a*) A charged 30 μF capacitor having initial charge 6 mC is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

(*b*) What is the total energy stored in the circuit initially? What is the total energy at later time?

Sol. Given $C = 30\,\mu\text{F} = 30 \times 10^{-6}$ F, $L = 27$ mH $= 27 \times 10^{-3}$ H.

Initial Charge $q_0 = 6$ mC $= 6 \times 10^{-3}$ C

(*a*) Angular frequency of free oscillations

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(27 \times 10^{-3} \times 30 \times 10^{-6})}}$$

$$= \frac{10^4}{9} = 1.1 \times 10^3 \text{ rad / s}$$

(*b*) Initial energy stored in circuit

$$= \text{Initial energy stored in capacitor} = \frac{q_0^2}{2C} = \frac{(6 \times 10^{-3})^2}{2 \times 30 \times 10^{-6}} = 0.6 \text{ J}$$

Energy is lost only in resistance.

As circuit is free from ohmic resistance; so the total energy at later time remains 0.6 J.

20. A radio can tune over the frequency range of a portion of medium wave (MW) broadcast band (800 kHz to 1200 kH). If its LC circuit has an effective inductance of 200 μH, what must be the range of variable capacitor?

Sol. Given $f_1 = 800 \text{ kHz} = 800 \times 10^3 \text{ Hz}, f_2 = 1200 \text{ kHz} = 1200 \times 10^3 \text{ Hz}$

$$L = 200 \,\mu\text{H} = 200 \times 10^{-6} \text{ H}$$

$$C_1 = ?, C_2 = ?$$

The natural frequency of LC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

i.e.,

$$C = \frac{1}{4\pi^2 f^2 L}$$

For $f = f_1 = 800 \times 10^3$ Hz,

$$C_1 = \frac{1}{4 \times (3.14)^2 \times (800 \times 10^3)^2 \times 200 \times 10^{-6}} \text{ F}$$

$$= 197.8 \times 10^{-12} \text{ F} \approx 198 \text{ pF}$$

For $f = f_2 = 1200 \times 10^3$ Hz

$$C_2 = \frac{1}{4 \times (3.14)^2 \times (1200 \times 10^3 \times 200 \times 10^{-6})} \approx 88 \text{ pF}$$

The variable capacitor should have a range of about 88 pF to 198 pF.

21. An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant when the circuit is closed be $t = 0$.

(*a*) What is the total energy stored initially? Is it conserved during LC oscillations?

(*b*) What is the natural frequency of the circuit?

(*c*) At what time is the energy stored (*i*) completely electrical (*i.e.*, stored in the capacitor)? (*ii*) completely magnetic *i.e.*, stored in the inductor?

(*d*) At what times is the total energy stored equally between the inductor and the capacitor?

(*e*) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Sol. Given $L = 20 \text{ mH} = 20 \times 10^{-3} \text{ H}, C = 50\,\mu\text{F} = 50 \times 10^{-6} \text{ F}, q_0 = 10 \text{ mC} = 10 \times 10^{-3} \text{ C}$

(*a*) Total energy stored initially $= \frac{q_0^2}{2C} = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}}$ J $= \mathbf{1.0\ J}$

Yes, the total energy is conserved during LC oscillations (because circuit is free from ohmic resistance).

(*b*) Angular frequency of circuit, $\omega = \frac{1}{\sqrt{LC}} = \mathbf{10^3\ rad/s}$

Natural linear frequency $f = \frac{\omega}{2\pi} = \frac{10^3}{2 \times 3.14} = \mathbf{159\ Hz}$

(*c*) When circuit is closed at $t = 0$, then equation of charge on capacitor is $q = q_0 \cos \omega t$

(*i*) Energy is completely electrical when $q = q_0$ *i.e.*, when $\cos \omega t = \pm 1$ or $\omega t = r\pi$ where $r = 0, 1, 2, 3, \dots$

$t = \frac{r\pi}{\omega}$, $T = \frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T}$,

$t = \frac{r\pi}{2\pi/T} = r \cdot \frac{T}{2}$, $(r = 0, 1, 2, 3, \dots)$

i.e., $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \dots,$

(*ii*) Energy is completely magnetic when electrical energy is zero,

i.e., when $\cos \omega t = 0$ or $\omega t = (2r + 1)\frac{\pi}{2}$, $r = 0, 1, 2, \dots$

$$t = (2r+1)\frac{\pi}{2\omega} = (2r+1)\frac{\pi}{2(2\pi/T)} = (2r+1)\frac{T}{4} \qquad (r = 0, 1, 2., \dots)$$

or $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots.$

(*d*) Energy is equally divided between inductor and capacitor, when half the energy is electrical. Let charge, in this state, be q, then

$$\frac{q^2}{2C} = \frac{1}{2}\frac{q_0^2}{2C} \quad \Rightarrow \quad q = \pm \frac{q_0}{\sqrt{2}}$$

$\Rightarrow$ $q_0 \cos \omega t = \pm \frac{q_0}{\sqrt{2}}$ or $\cos \omega t = \pm \frac{1}{\sqrt{2}}$

or $\omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \dots.$

or $\frac{2\pi}{T} t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{8}, \dots$ or $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \dots$

(*e*) When R is inserted in the circuit, the oscillations become damped and in each oscillation some energy is dissipated as heat. As time passes, the whole of the initial energy (1·0 J) is eventually dissipated as heat.

LCR Circuit

22. A series LCR circuit with R = 20Ω, L = 1.5 H and C = 35μF is connected to a variable frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle.

Sol. When frequency of supply is equal to natural frequency of circuit, then resonance is obtained. At resonance $X_C = X_L$

$\Rightarrow$ Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2} = R = 20\,\Omega$

Current in circuit, $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{200}{20} = 10$ A

Power factor $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1$

$\therefore$ Average power $\overline{P} = V_{rms}\, I_{rms} \cos\phi = V_{rms}\, I_{rms} = 200 \times 10 = 2000$ W $= 2$ kW

23. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(*a*) **Obtain the current amplitude and rms values.**

(*b*) **Obtain the rms values of potential drops across each element.**

(*c*) **What is the average power transferred to the inductor?**

(*d*) **What is the average power transferred to the capacitor?**

(*e*) **What is the total average power absorbed by the circuit? (Average implies average over one cycle).**

Sol. Given $V = 230$ V, $f = 50$ Hz, $L = 80$ mH $= 80 \times 10^{-3}$ H, $C = 60\,\mu\text{F} = 60 \times 10^{-6}$ F

(*a*) Inductive reactance $X_L = \omega L = 2\pi f L$

$$= 2 \times 3.14 \times 50 \times 80 \times 10^{-3} = \mathbf{25.1\,\Omega}$$

Capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.1\,\Omega$$

Impedance, Z = Net reactance $= \left| \dfrac{1}{\omega C} - \omega L \right|$

$$= 53.1 - 25.1 = 28.0\,\Omega$$

Current amplitude $I_0 = \dfrac{V_0}{Z} = \dfrac{V\sqrt{2}}{Z} = \dfrac{230 \times 1.41}{28.0} = \mathbf{11.6\ A}$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{11.6}{1.41} = \mathbf{8.23\ A}$$

(*b*) RMS value of potential drops across L and C are $V_L = X_L\, I_{rms} = 25.1 \times 8.23 = 207$ V

$$V_C = X_C\, I_{rms} = 53.1 \times 8.23 = 437 \text{ V}$$

Net voltage $= V_C - V_L = \mathbf{230\ V}$

(*c*) The voltage across L leads the current by angle $\dfrac{\pi}{2}$, therefore, average power

$$P_{av} = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0 \text{ (zero).}$$

(*d*) The voltage across C lags behind the current by angle $\dfrac{\pi}{2}$,

$$\therefore \quad P_{av} = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0 \text{ (zero)}$$

(*e*) As circuit contains pure L and pure C, average power consumed by LC circuit is **zero.**

24. A circuit containing a 80 mH inductor, a 60 μF capacitor and a 15 Ω resistor are connected to a 230 V, 50 Hz supply. Obtain the average power transferred to each element of the circuit and total power absorbed.

Sol. Given $L = 80 \text{ mH} = 80 \times 10^{-3}$ H, $C = 60\,\mu\text{F} = 60 \times 10^{-6}$ F, $R = 15\,\Omega$, $V_{rms} = 230$ V, $f = 50$ Hz

Inductive reactance $\quad X_L = \omega L = 25.1\,\Omega$

Capacitive reactance $\quad X_C = \dfrac{1}{\omega C} = 53.1\,\Omega$

Impedance of circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(15)^2 + (53.1 - 25.1)^2}$$

$$= \sqrt{(15)^2 + (28)^2} = \mathbf{31.8\,\Omega}$$

RMS current, $\quad I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{230}{31 \cdot 8} = \mathbf{7.23\ A}$

Average power transferred to resistance $= I_{rms}^2 R = (7.23)^2 \times 15 = \mathbf{785\ W}$

Average power transferred to inductor = Average power transferred to capacitor

$$= V_{rms}\, I_{rms} \cos\frac{\pi}{2} = \textbf{zero}$$

Total power absorbed $\cong$ 785 W

25. A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum, obtain the maximum value?

(b) What is the source frequency for which average power observed by the circuit is maximum? Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q-factor of the given circuit?

Sol. Given : $L = 0.12$ H, $C = 480 \text{ nF} = 480 \times 10^{-9}$ F, $R = 23\,\Omega$, $V_{rms} = 230$ V

(a) Current amplitude $= \dfrac{V_0}{Z} = \dfrac{V_{rms}\sqrt{2}}{\sqrt{R^2 + (X_C - X_L)^2}}$

Clearly current amplitude is maximum when $X_C - X_L = 0$

$\Rightarrow \quad X_C = X_L$

$\Rightarrow \quad \omega_C = \omega_L$ or $\omega = \dfrac{1}{\sqrt{LC}}$ · This is resonant frequency.

Resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0 \cdot 12 \times 480 \times 10^{-9})}} = \dfrac{10^5}{24} = 4.167 \times 10^3$ rad/s

Resonant linear frequency, $f_r = \dfrac{\omega_r}{2\pi} = \dfrac{4.167 \times 10^3}{2 \times 3.14} = 663$ Hz

At resonant frequency $Z = R$

(*b*) Average power, $\overline{P} = V_{rms} I_{rms} \cos\phi$

For maximum power, $\cos\phi = 1; I_{rms} = \frac{V_{rms}}{R} = \frac{230}{23} = 10$ A

$$\therefore \qquad \overline{P}_{\max} = V_{rms} I_{rms} = 230 \times 10 = 2300 \text{ watt}$$

(*c*) Power absorbed, $P = \frac{1}{2} \times$ maximum power

$$I^2 R = \frac{1}{2} I_0^2 R \Rightarrow I = \frac{I_0}{\sqrt{2}}$$

$$\frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} = \frac{1}{\sqrt{2}} \frac{V_{rms}}{R}$$

$$\Rightarrow \qquad R^2 + \left(\frac{1}{\omega C} - \omega L\right) = 2R^2 \Rightarrow \frac{1}{\omega C} - \omega L = \pm R$$

If $\omega_1 < \omega_r$, then $\frac{1}{\omega_1 C} - \omega_1 L = +R$ ...(*i*)

If $\omega_2 > \omega_r$, then $\frac{1}{\omega_2 C} - \omega_2 L = -R$ ...(*ii*)

Adding (*i*) and (*ii*)

$$\frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) - (\omega_1 + \omega_2) L = 0$$

$$\frac{\omega_1 + \omega_2}{C\, \omega_1 \omega_2} - (\omega_1 + \omega_2) L = 0 \Rightarrow \omega_1 \omega_2 = \frac{1}{LC} \qquad \text{...(iii)}$$

As $\omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \sqrt{\omega_1 \omega_2} = \frac{1}{\sqrt{LC}}$ resonant frequency.

Subtracting (*ii*) from (*i*), $\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{1}{C} + (\omega_2 - \omega_1) L = 2R$

$$\frac{\omega_2 - \omega_1}{\omega_1 \omega_2} \cdot \frac{1}{C} + (\omega_2 - \omega_1) L = 2R$$

Using (*iii*), we get

$$(\omega_2 - \omega_1) L + (\omega_2 - \omega_1) L = 2R$$

$$\Rightarrow \qquad \omega_2 - \omega_1 = \frac{R}{L}$$

If $\Delta\omega$ is the difference of ω_1 and ω_2 from ω_r, then $\omega_r + \Delta\omega - (\omega_r - \Delta\omega) = \frac{R}{L}$

$$\Rightarrow \qquad 2\Delta\omega = \frac{R}{L}$$

or $\qquad \Delta\omega = \frac{R}{2L} = \frac{23}{2 \times 0.12} = 95.8$ rad/s.

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{95.8}{2 \times 3.14} = 15.2 \text{ Hz}$$

$\therefore \quad f_1 = f_r - \Delta f = 663 - 15.2 = 648.2$ Hz

$f_2 = f_r + \Delta f = 663 + 15.2 = 678.2$ Hz

Thus power absorbed is half the power at resonant frequency at frequencies 648.8 Hz an 678.2 Hz.

(*d*) *Q*–value of given circuit,

$$Q = \frac{\omega_r L}{R} = \frac{4167 \times 10^3 \times 0.12}{23} = \mathbf{21.7}$$

26. Obtain the resonant frequency ω_r of a series LCR circuit with L = 2·0 H, C = 32 μF and R = 10Ω. What is the quality factor (Q) of this circuit?

Sol.

Resonant frequency, $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = \frac{1}{8} \times 10^3 = \mathbf{125\ rad\ s^{-1}}$

$$Q\text{-value of circuit} = \frac{\omega_r L}{R} = \frac{125 \times 2.0}{10} = \mathbf{25}$$

Transformer

27. A power transmission line needs input power at 2300 V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary windings in order to get output power at 230 V?

Sol. Given $V_p = 2300$ V, $N_p = 4000$ turns, $V_s = 230$ V, $N_s = ?$

We have $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\Rightarrow \quad N_s = \frac{V_s}{V_p} \times N_p = \frac{230}{2300} \times 4000 = \mathbf{400\ turns}$

28. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of two wire line carrying power is 0.5Ω per km. The town gets power from the line through a 4000V – 220 V step down transformer at a sub-station in the town. Calculate (*i*) the line power loss in the form of heat (*ii*) how much power must the plant supply, assuming there is negligible power loss due to leakage (*iii*) characterise the step up transformer at the plant.

Sol. Length of wire line $= 15 \times 2 = 30$ km

Resistance of wire line, $r = 30 \times 0.5 = 15\,\Omega$

(*i*) Power to be supplied $P = 800$ kW $= 800 \times 10^3$ W

Voltage at which power is transmitted = 4000 V

$P = VI \Rightarrow I = \frac{P}{V} = \frac{800 \times 10^3}{4000} = 200$ A

$\therefore$ Line power loss $= I_p^2 \times R = (200)^2 \times 15 = 6 \times 10^5$ watt = 600 kW

(*ii*) Power that must be supplied = 800 kW + 600 kW = 1400 kW

voltage drop across to wire line $= IR = 200 \times 15 = 3000$ volt

The plant generates power at 440 V and it has to be stepped up so that after a voltage drop of 3000 V, across the line, the power at 4000 V is received at the sub-station in the town. Therefore the output voltage is

$$3000 + 4000 = 7000 \text{ V}$$

Here step up transformer at the plant is

$$440 \text{ V} \longrightarrow 7000 \text{ V}$$

Previous Years' Numericals

1. **A small piece of metal wire is dragged across the gap between the pole piece of a magnet in 0.5 s. The magnetic flux between the pole pieces is known to be 8×10^{-4} Wb. Estimate the induced emf in the wire.** ***[CBSE Delhi 2004]***

Sol. Induced emf $E = -\dfrac{\Delta\phi}{\Delta t} = \dfrac{\Delta\phi}{\Delta t}$ (numerically)

$$= \frac{8 \times 10^{-4}}{0.5} = 1.6 \times 10^{-3} \text{ V} = \mathbf{1.6 \text{ mV}}.$$

2. **A circular copper disc 10 cm in radius rotates at 20 π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc.**

   (*i*) Calculate the potential difference developed between the axis of the disc and the rim.

   (*ii*) What is the induced current in the circuit whose terminals are connected between centre of disc and point of rim and the resistance of the circuit is 2 Ω. ***[CBSE (F) 2007, 2001]***

Sol. (*i*) Given normal magnetic field, $B_n = 0.2$ T

Angular speed, $\omega = 20\,\pi$ rad / s

Radius of disc, $r = 10 \text{ cm} = 0.10$ m

Induced emf, $\varepsilon = \dfrac{B_n\, \omega r^2}{2}$

$$= \frac{0.2 \times 20\,\pi \times (0.10)^2}{2} = 2 \times 3.14 \times 10^{-2} \text{ V}$$

$$= \mathbf{6.28 \times 10^{-2} \text{ V}}$$

(*ii*) Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{6.28 \times 10^{-2}}{2}$

$$= \mathbf{3.14 \times 10^{-2} \text{ A}}$$

3. **A jet plane is travelling west at 450 ms^{-1}. If the horizontal component of earth's magnetic field at that place is 4×10^{-4} T and the angle of dip is 30°, find the emf induced between the ends of wings having a span of 30 m.** ***[CBSE (AI) 2008]***

Sol. The wings of jet plane will cut the vertical component of earth's magnetic field, so emf is induced across the wing. The vertical component of earth's magnetic field.

$$V = H \tan\theta$$

Given $H = 4.0 \times 10^{-4}$ T, $\theta = 30°$

$$\therefore \quad V = (4.0 \times 10^{-4} \text{ T}) \tan 30° = 4 \times 10^{-4} \times \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times 10^{-4} \text{ T}$$

Induced emf across the wing

$$\varepsilon = Vvl$$

Given $v = 450 \text{ ms}^{-1}, l = 30 \text{ m}$

$$\therefore \qquad \varepsilon = \left(\frac{4}{\sqrt{3}} \times 10^{-4}\right) \times (450) \times 30 \text{ V} = \mathbf{3.12 \text{ V}}$$

4. The magnetic flux linked with a large circular coil, of radius *R*, is 0.5×10^{-3} Wb when a current of 0.5 A, flows through a small neighbouring coil of radius *r*. Calculate the coefficients of mutual inductance for the given pair of coils.

If the current through the small coil suddenly falls to zero, what would be the effect in the larger coil? *[CBSE 2008C]*

Sol. $\phi_2 = mi_4$

Given $\phi_2 = 0.5 \times 10^{-3}$ Wb, $i_1 = 0.5$ A

$$\therefore \qquad \text{Mutual inductance, } M = \frac{\phi_2}{i_1} = \frac{0.5 \times 10^{-3}}{0.5} = \mathbf{10^{-3} \text{ H} = 1 \text{ mH}}$$

If the current through small coil suddenly falls to zero, $[\rho_2 = M \frac{di_1}{dt}, \frac{di_1}{dt}$ is larger so emf induced is large]; so initially *the large current is induced in larger coil, which soon becomes zero.*

5. Current in a circuit falls steadily from 5.0 A to 0.0 A in 100 ms. If an average emf of 200 V is induced, calculate the self-inductance of the circuit. *[CBSE (F) 2011]*

Sol. Change in current $(\Delta I) = (0.0 - 5.0) \text{ A} = -5.0 \text{ A}$

Time taken $(\Delta t) = 100 \times 10^{-3}$ S

Induced emf $(e) = 200$ V

Induced emf (e) is given by

$$e = -\frac{\Delta\phi}{\Delta t}$$

$$= -\frac{\Delta(LI)}{\Delta t} \qquad (\phi = LI)$$

$$e = -L\frac{\Delta I}{\Delta t}$$

or $$L = -e.\frac{\Delta t}{\Delta I} = -\frac{(200).(100 \times 10^{-3})}{(-5 \cdot 0)}$$

$$L = 4 \cdot 0 \text{ H}$$

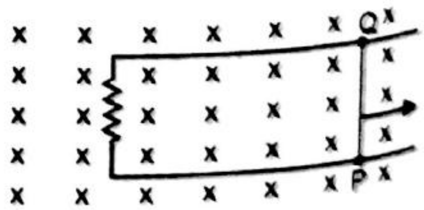
6. A 0.5 m long metal rod *PQ* completes the circuit as shown in the figure. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 *T*. If the resistance of the total circuit is 3 Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms^{-1}. *[CBSE (AI) 2006]*

Sol. Given: $l = 0.5$ m, $B = 0.15$ T, $R = 3\,\Omega$, $v = 2 \text{ ms}^{-1}$

Emf., $\varepsilon = vBl$

Current, $I = \frac{\varepsilon}{R} \Rightarrow \frac{vBl}{R}$

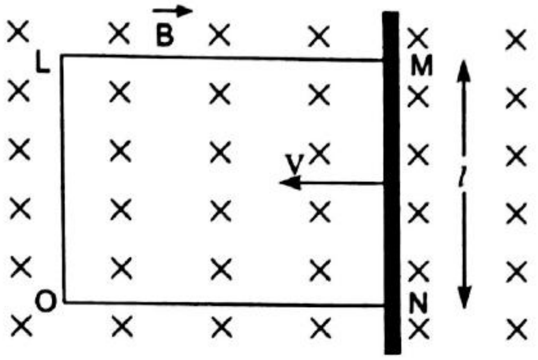
Force needed to move the rod,



$$F = BIL = B \times \frac{vBl}{R} \times l = \frac{vB^2l^2}{R}$$

$$= \frac{2 \times (0.15)^2 \times (0.5)^2}{3} = 3.75 \times 10^{-3} \text{ N.}$$

7. A rectangular conductor LMNO is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20 cm is moved towards left with a velocity of 10 ms^{-1}, calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm. *[CBSE (AI) 2013]*



Sol. Induced emf in a moving rod in a magnetic field is given by

$$\varepsilon = -Blv$$

Since the rod is moving to the left so

$$\varepsilon = +Blv$$
$$= 0.5 \times 0.2 \times 10$$
$$= 1v$$

Current in the rod $I = \frac{\varepsilon}{R} = \frac{1}{5} = 0.2\text{A}$

8. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the plane is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased? *[CBSE Delhi 2013]*

Sol. If a rod of length 'l' rotates with angular speed ω in uniform magnetic field 'B'

$$\varepsilon = \frac{1}{2} Bl^2 \omega$$

In case of earth's magnetic field $B_H = |B_e| \cos \delta$

and $B_V = |B_e| \sin \delta$

∴ $$\varepsilon = \frac{1}{2} |B_e| \cos \delta . l^2 \omega$$

$$= \frac{1}{2} \times 0.4 \times 10^{-4} \cos 60° \times (0.5)^2 \times 2\pi\nu$$

$$= \frac{1}{2} \times 0.4 \times 10^{-4} \times \frac{1}{2} \times (0.5)^2 \times 2\pi \times \left(\frac{120 \text{ rev}}{60\, s}\right)$$

$$= 10^{-5} \times 0.25 \times 2 \times 3.14 \times 2 \quad = 3.14 \times 10^{-5} \text{ volt}$$

Induced emf is independent of the number of spokes *i.e.*, it remain same.

9. A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox. *[CBSE (AI) 2008C]*

Sol. Given $R = 200\,\Omega$, $C = 15.0\,\mu F = 15\cdot 0 \times 10^{-6}$ F, $V = 220$ V, $f = 50$ Hz

Capacitance reactance $\quad X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times 3.14 \times 50 \times 15.0 \times 10^{-6}} = 212\,\Omega$$

Impedance of circuit, $\quad Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{(200)^2 + (212)^2} = 291.5\,\Omega$$

RMS current, $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{220}{291.5}$ A $= 0.75$ A

Voltage across resistance, $V_R = RI = 200 \times 0.75 = \mathbf{150\ V}$

Voltage across capacitor, $V_C = X_C I = 212 \times 0.75 = 159$ V

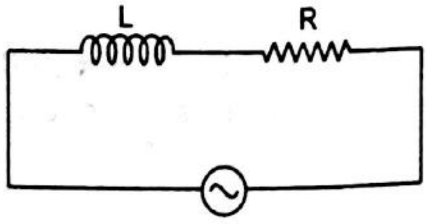
Algebraic sum of V_R and $V_C = V_R + V_C$

$$= 150 + 159 = 309\text{ V} > 212\text{ V}$$

This is because these voltage are not in same phase but V_C lags behind V_R by an angle $\pi/2$

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2} = \sqrt{(150)^2 + (159)^2} \approx \mathbf{220\ V}$$

10. In the given circuit, the potential difference across the inductor L and resistor R are 200 V and 150 V respectively and the rms. value of current is 5 A. Calculate (*i*) the impedance of the circuit and (*ii*) the phase angle between the voltage and the current. *[CBSE Delhi 2004]*



Sol. Voltage applied $\quad V = \sqrt{V_L^2 + V_R^2}$

$$= \sqrt{(200)^2 + (150)^2} = \mathbf{250\ V}$$

Impedance of circuit, $Z = \dfrac{V}{I} = \dfrac{250}{5} = \mathbf{50\,\Omega}$

Phase angle between voltage and current $\tan\phi = \dfrac{X_L}{R} = \dfrac{V_L}{V_R} = \dfrac{200}{150} = \dfrac{4}{3}$

$$\phi = \tan^{-1}\left(\frac{4}{3}\right) = \mathbf{53^\circ}$$

11. An ac voltage of 100 V, 50 Hz is connected across a 20 ohm resistor and 2 mH inductor in series. Calculate (*i*) impedance of the circuit (*ii*) rms current in the circuit. *[CBSE Delhi 2007]*

Sol. Given $V = 100$ V, $f = 50$ Hz, $R = 20\,\Omega$, $L = 2\text{ mH} = 2 \times 10^{-3}$ H

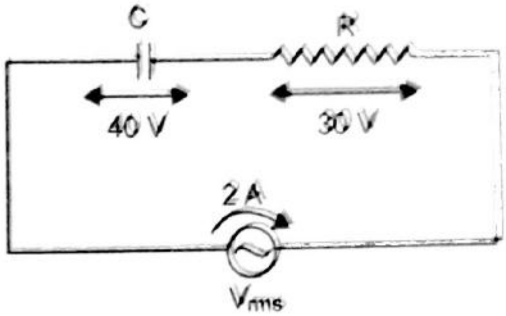
(*i*) Impedance of *R*-*L* circuit, $Z = \sqrt{R^2 + (\omega L)^2}$

$$= \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(20)^2 + (2 \times 3.14 \times 50 \times 2 \times 10^{-3})^2}$$

$$= \sqrt{400 + (0.628)^2} = 20\,\Omega$$

(*ii*) rms current $I_{rms} = \dfrac{V}{Z} = \dfrac{100}{20} = \mathbf{5\ A}$

12. **Calculate the (*i*) impedance (*ii*) wattless current of the given ac circuit.** *[CBSE (AI) 2008]*



Sol. (*i*) Potential difference across capacitance, $V_C = X_C I$

$\therefore$ Capacitive reactance,

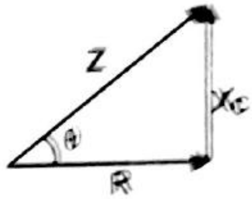
$$X_C = \frac{V_C}{I} = \frac{40}{2} = 20\Omega$$

Resistance, $R = \frac{V_R}{I} = \frac{30}{2} = 15\,\Omega$

Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{(15)^2 + (20)^2}$

$$= \sqrt{225 + 400} = \sqrt{625}\,\Omega = 25\,\Omega$$

(*ii*) The phase lead (ϕ) of current over applied voltage is



$$\tan\phi = \frac{X_C}{R}$$

Wattless Current, $I_{wattless} = I \sin\phi = I \cdot \left(\frac{X_C}{Z}\right)$

$$= 2 \times \frac{20}{25}\,\text{A} = \mathbf{1.6\,A}$$

13. **An inductor 200 mH, capacitor 500 μ F, resistor 10 Ω are connected in series with a 100 V, araible frequency ac source. Calculate the**

(*i*) **frequency at which the power factor of the circuit is unity.**

(*ii*) **current amplitude at this frequency.**

(*iii*) ***Q*-factor.** *[CBSE Delhi 2008]*

Sol. Given $L = 200\text{ mH} = 200 \times 10^{-3}$ H, $C = 500\,\mu\text{F} = 500 \times 10^{-6}$ F,

$R = 10\,\Omega$, $V_{rms} = 100$ V

(*i*) Angular (resonant) frequency ω_r at which power factor of the circuit is unity, is given by

$$\omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{200 \times 10^{-3} \times 500 \times 10^{-6}}} = \mathbf{100\,rad\,s^{-1}}$$

Linear Resonant Frequency

$$f_r = \frac{\omega_r}{2\pi} = \frac{100}{2 \times 3.14} = \frac{100}{6.28}\,\text{Hz} = \mathbf{15.9\,Hz}$$

(*ii*) At resonant frequency f_r impedance, $Z = R$

$\therefore$ Current amplitude, $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{R} = \frac{100\sqrt{2}}{10} = 10\sqrt{2}A = \mathbf{14 \cdot 1A}$

(*iii*) $Q\text{-factor} = \frac{\omega_r L}{R} = \frac{100 \times 200 \times 10^{-3}}{10} = \mathbf{2}$

14. **An inductor of unknown value, a capacitor of 100 μF and a resistor of 10 Ω are connected in series to a 200 V, 50 Hz ac source. It is found that the power factor of the circuit is unity. Calculate the inductance of the inductor and current amplitude.** *[CBSE Delhi 2008]*

Sol. For power factor unity, $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

$$\Rightarrow \qquad L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C} = \frac{1}{4\pi^2 f^2 C}$$

Given $f = 50$ Hz, $C = 100\ \mu F = 100 \times 10^{-6}$ F $= 10^{-4}$ F

$$\therefore \qquad L = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 10^{-4}} \text{ H} = \mathbf{0.10\ H}$$

Current amplitude, $\qquad I_0 = \dfrac{V_0}{Z}$

At resonance, $\qquad Z = R$

$$\therefore \qquad I_0 = \frac{V_0}{R} = \frac{200\sqrt{2}}{10} = \mathbf{20\sqrt{2}\ A}$$

$$= 20 \times 1.414 \text{ A} = 28.3 \text{ A}$$

15. Calculate the quality factor of a series *LCR* circuit with *L* = 2-0 H, *C*= 2 μF and *R* = 10 Ω. Mention the significance of quality factor in *LCR* circuit. ***[CBSE (F) 2012]***

Sol. We have, $\qquad Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{2}{2\times 10^{-6}}} = 100$

It signifies the sharpness of resonance.

16. Determine the current quality factor at resonance for a series *LCR* circuit with L = 1.00 mH, 1.00 nF and *R* = 100 Ω connected to an ac source having peak voltage of 100 V.

[CBSE (F) 2011]

Sol. $I_v = ?,\ Q = ?$

$L = 1.00 \text{ mH} = 1 \times 10^{-3}$ H, $C = 1.00 \text{ nF} = 1 \times 10^{-9}$ F, $R = 100\,\Omega$,

$E_0 = 100$ V

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E_0}{Z} \qquad \left\{\begin{array}{l}\text{at resonance } \omega L = \dfrac{1}{\omega C} \\ \text{Hence} \quad Z = R\end{array}\right\}$$

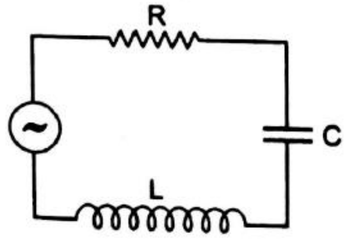
$$\therefore \qquad I = \frac{V}{R} = \frac{100}{100}$$

$$I_0 = 1 \text{ A}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.44}{2} = 0.707\ A$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{1.0\times 10^{-3}}{1.0 \times 10^{-9}}} = \frac{1}{100} \times 10^3 = 10$$

17. The figure shows a series LCR circuit with L = 5.0 H, C = 80 μF, R = 40 Ω connected to a variable frequency 240V source. Calculate. ***[CBSE Delhi 2012]***



(*i*) The angular frequency of the source which drives the circuit at resonance.

(*ii*) The current at the resonating frequency.

(*iii*) **The rms potential drop across the capacitor at resonance.**

Sol. (*i*) We know

ω_r = Angular frequency at resonance $= \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50$ rad/s

(*ii*) Current at resonance, $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{240}{40} = 6$ A

(*iii*) V_{rms} across capacitor

$$V_{rms} = I_{rms}\ X_C \qquad = 6 \times \frac{1}{50 \times 80 \times 10^{-6}} = \frac{6 \times 10^6}{4 \times 10^3} = 1500 \text{ V}$$

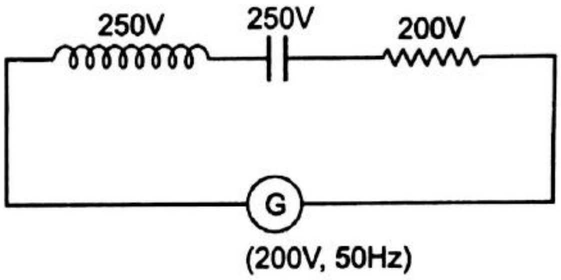
18. A series LCR circuit is connected to an ac source (200 V, 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200 V, 250 V and 250 V.

(i) The algebraic sum of the voltages across the three elements is greater than the voltage of the source. How is this paradox resolved?

(ii) Given the value of the resistance of R is 40Ω, calculate the current in the circuit.

[CBSE (F) 2013]

Sol. (i) From given parameters $V_R = 200V$, $V_L = 250$ V and $V_C = 250$ V



V_{eff} should be given as

$V_{eff} = V_R + V_L + V_C = 200 \text{ V} + 250\text{V} + 250 \text{ V} = 700$ V

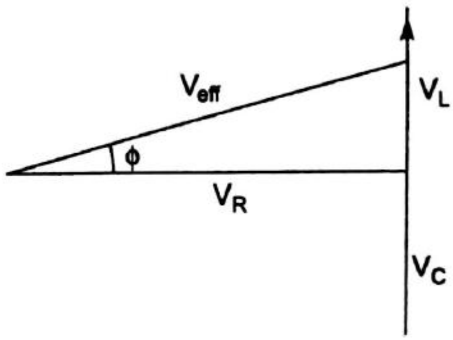
However, $V_{eff} > 200$ V of the ac source.

This paradox can be solved only by using phasor gram, as given below:

$(V_{eff}) + \sqrt{{V_R}^2 + (V_L - V_C)^2}$

Since $V_L – V_C$ so $V_{eff} = V_R = 200V$

(ii) Given R = 40Ω, so current in the LCR circuit.



$I_{eff} = \dfrac{V_{eff}}{R}$ $\qquad [X_L = X_C \text{ or } Z = R]$

$= \dfrac{200}{40} = 5$ A

19. How much current is drawn by the primary coil of a transformer which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω. ***[CBSE (AI) 2008]***

Sol. Current is secondary coil, $I_S = \dfrac{V_S}{Z} = \dfrac{22}{220}$ A $= \mathbf{0.1}$ **A**

For an ideal transformer

$$V_S\ I_S = V_p\ I_p$$

∴ Current in primary coil, $I_p = \dfrac{V_S I_S}{V_p} = \dfrac{22 \times 01}{220} = \mathbf{0.01}$ **A**

20. The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are 220 V and 1100 W respectively. Calculate

(*a*) **the number of turns in the secondary coil.**

(*b*) **the current in the primary coil.**

(*c*) **the voltage across the secondary coil.**

(*d*) **the current in the secondary coil.**

(*e*) **the power in the secondary coil.** *[CBSE Delhi 2006]*

Sol. (*a*) Transformation ratio $r = \frac{\text{Number of turns in secondary coil } (N_S)}{\text{Number of turns in primary coil } (N_P)}$

Given $N_P = 100, r = 100$

$\therefore$ Number of turns in secondary coil, $N_S = rN_P = 100 \times 100 = \mathbf{10{,}000}$

(*b*) Input voltage $V_P = 220$ V, Input power $P_{in} = 1100$ W

Current in primary coil $I_P = \frac{P_{in}}{V_P} = \frac{1100}{220} = \mathbf{5\ A}$

(*c*) Voltage across secondary coil (V_S) is given by

$$r = \frac{V_S}{V_P}$$

$$\Rightarrow \quad V_S = rV_P = 100 \times 220 = 22{,}000 \text{ V} = \mathbf{22\ kV}$$

(*d*) Current in secondary coil (I_S) is given by

$$r = \frac{I_P}{I_S}$$

$$\Rightarrow \quad I_S = \frac{I_P}{r} = \frac{5}{100} = \mathbf{0.05\ A.}$$

(*e*) Power in secondary coil, $P_{out} = V_S I_S = 22 \times 10^3 \times 0.05 = \mathbf{1100\ W}$

Obviously power in secondary coil is same as power in primary. This means that the transformer is ideal.

21. Calculate the current drawn by the primary coil of a transformer which steps down 200 V to 20 V to operate a device of resistance 20Ω. Assume the efficiency of the transformer to be 80%. *[CBSE (AI) 2007]*

Sol. Current in secondary, $I_S = \frac{V_S}{R_L} = \frac{20}{20} = \mathbf{1\ A}$

Efficiency of transformer, $\eta = \frac{\text{Power in secondary}}{\text{Power in primary}} = \frac{V_S I_S}{V_p I_p}$

$\therefore$ Current in Primary $I_p = \frac{V_S I_S}{V_p \eta}$

Given $V_S = 20$ V, $I_S = 1$ A, $V_P = 200$ V,

$$\eta = \frac{80}{100} = 0{\cdot}80$$

$$I_P = \frac{20 \times 1}{200 \times 0{\cdot}8} = \mathbf{0{\cdot}125\ A = 125\ mA}$$