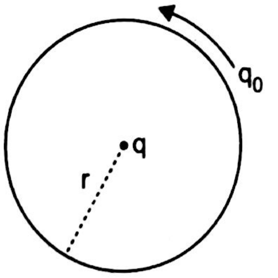
HOTS (Higher Order Thinking Skills)

Q.1. **If a point charge be rotated in an arc of radius *r* around a charge *q*, what will be the work done? Explain.**



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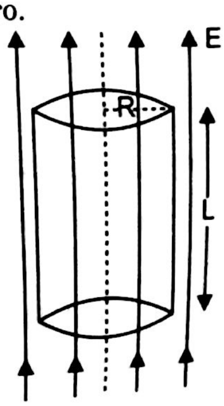
Ans. All points of circle of radius *r* are at same potential, hence work done is zero.

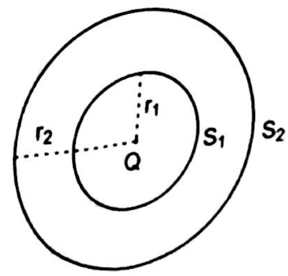
Q.2. **A cylinder of radius R and length L is placed in a uniform electric field parallel to the axis of the cylinder. What is the electric flux through the whole cylinder?**



Ans. The number of electric lines of force entering the cylinder is same as that leaving it. Therefore, the net electric flux through the cylinder is zero.

In fact, the flux associated with any type of closed surface in a uniform electric field is zero.

Q.3. **A sphere S_1 of radius r_1 encloses a total charge *Q*. If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) and there is no additional charge between S_1 and S_2; what is the ratio of electric flux through S_1 and S_2.**



Ans. By Gauss theorem, net electric flux $= \frac{1}{\epsilon_0} \times$ charge enclosed

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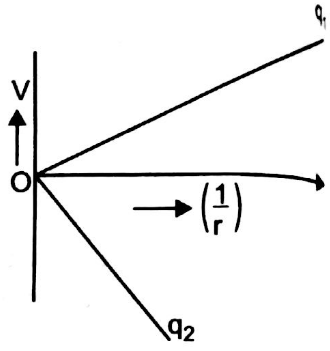
$\therefore \quad \phi_1 = \frac{1}{\epsilon_0} \cdot Q$ and $\phi_2 = \frac{1}{\epsilon_0} \cdot Q$, $\therefore \quad \frac{\phi_1}{\phi_2} = 1$

Q.4. The two graphs drawn below, show the variations of electrostatic potential (V) with $\frac{1}{r}$ (r being the distance of field point from the point charge) for two point charges q_1 and q_2.

(i) What are the signs of the two charges?

(ii) Which of the two charges has the larger magnitude and why?



Ans. (i) The potential due to positive charge is positive and due to negative charge, it is negative, so, q_1 is **positive** and q_2 is **negative.**

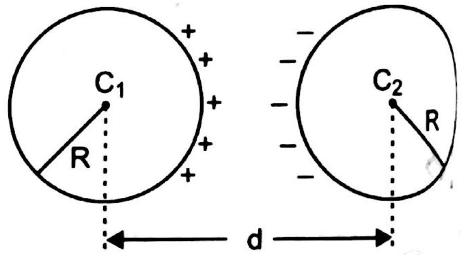
(ii) $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

The graph between V and $\frac{1}{r}$ is a straight line passing through the origin with slope $\frac{q}{4\pi\varepsilon_0}$.

As the magnitude of slope of the line due to charge q_2 is greater than that due to q_1, q_2 has larger magnitude.

Q.5. Two charged spherical conductors, each of radius R, are distance d apart such that d is slightly greater than $2R$. They carry charges $+q$ and $-q$. Will the force of attraction between them be exactly $\frac{q^2}{4\pi\varepsilon_0 d^2}$?

Ans. **No,** the force of attraction between the spherical conductors will be more than $\frac{1}{4\pi\varepsilon_0} \frac{q^2}{d^2}$, due to attraction of opposite charges, there will be redistribution of charges on spheres as shown in the fig. Obviously, the effective distance between the charges will be reduced and hence the effective force will be increased.

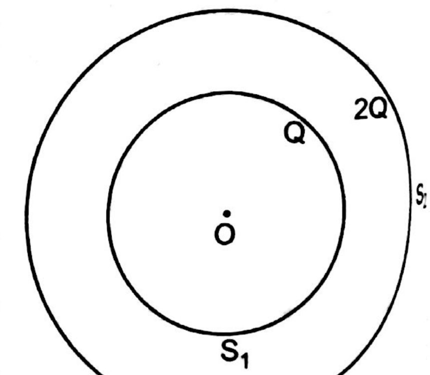


Q.6. S_1 and S_2 are two hollow concentric spheres enclosing charges Q and $2Q$ respectively as shown in the fig.

(i) What is the ratio of electric flux through S_1 and S_2?

(ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air?

[CBSE Delhi 2002]



Ans. (i) Surface S_1 encloses charge Q only, therefore, electric flux through S_1 is $\phi_1 = \frac{Q}{\varepsilon_0}$

Surface S_2 enclosed both charges Q and $2Q$ (*i.e.*, total charge $3Q$); therefore, electric flux through S_2 is $\phi_2 = \frac{3Q}{\varepsilon_0}$.

$$\text{Ratio } \phi_1 : \phi_2 = \frac{Q}{\varepsilon_0} : \frac{3Q}{\varepsilon_0} \Rightarrow \qquad \phi_1 : \phi_2 = 1:3.$$

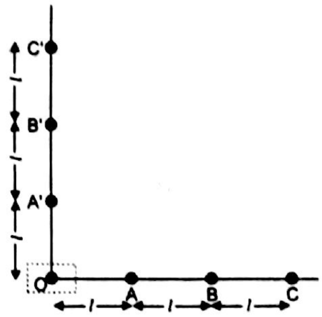
(ii) When a medium of dielectric constant $K = 5$ is introduced in the space inside S_1, the electric flux through S_1 will become

$$\phi_1{}' = \frac{Q}{K\varepsilon_0} = \frac{Q/\varepsilon_0}{K} = \frac{\phi_1}{5}$$

i.e., ***The electric flux through*** S_1 ***will become one-fifth of that in air.***

Q.7. The following data was obtained for the dependence of the magnitude of electric field, with distance, from a reference point *O*, within the charge distribution in the shaded region.

Field point	A	B	C	A′	B′	C′
Magnitude of electric field	E	E/8	E/27	E/2	E/16	E/64



(*i*) Identify the charge distribution and justify your answer.

(*ii*) If the potential due to this charge distribution, has a value *V* at the point *A*, what is its value at the point *A*′ ?

Ans. (*i*) We observe that the magnitude of electric field

(*a*) varies as the inverse cube of the distance of the field point along axial line.

(*b*) has a magnitude half of its magnitude (at an equidistant point) on the line perpendicular to this line.

[The field at axial point, $E_{axis} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$

The electric field at equatorial point, $E_{equatorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

∴ At same distance from electric dipole field at an equatorial point is half the value of field at axial point and follows the law $E \propto \frac{1}{r^3}$]

(*ii*) Potential at *A*, $\qquad V_A = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

The potential at any equatorial point is zero so potential at point *A*′, $V_{A'}$ = zero (as point *A*′ is an equatorial point).

Q.8. A charge *Q* located at a point $\vec{r}$ is in equilibrium under the combined electric field of three charges q_1, q_2, q_3. If the charges q_1, q_2 are located at points $\vec{r_1}$ and $\vec{r_2}$ respectively, find the direction of the force on *Q*, due to q_3 in terms of $q_1, q_2, \vec{r_1}, \vec{r_2}$ and $\vec{r}$.

Ans. $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1}) + \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{|\vec{r}-\vec{r_2}|^3}(\vec{r}-\vec{r_2}) + \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{|\vec{r}-\vec{r_3}|^3}(\vec{r}-\vec{r_3}) = 0$$

$$\Rightarrow \frac{q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1}) + \frac{q_2}{|\vec{r}-\vec{r_2}|^3}(\vec{r}-\vec{r_2}) = -\frac{q_3}{|\vec{r}-\vec{r_3}|^3}(\vec{r}-\vec{r_3})$$

$$\Rightarrow \quad (\vec{r} - \vec{r_3}) = -\frac{|r - r_3|^3}{q_3}\left[\frac{q_1 (\vec{r} - \vec{r_1})}{|\vec{r} - \vec{r_1}|^3} + \frac{q_2 (\vec{r} - \vec{r_2})}{|\vec{r} - \vec{r_2}|^3}\right]$$

Direction of force on Q due to q_3 along $(\vec{r} - \vec{r_3})$ is given by

$$\frac{q_1}{|\vec{r} - \vec{r_1}|^3}(\vec{r_1} - \vec{r}) + \frac{q_2}{|\vec{r} - \vec{r_2}|^3}(\vec{r_2} - \vec{r})$$

Q.9. A uniform electric field $\vec{E} = E_x \hat{i}$ N/C for $x > 0$ and $\vec{E} = -E_x \hat{i}$ N/C for $x < 0$ are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the X-axis. Find out the net outward flux. Using Gauss's law, write the expression for the net charge within the cylinder. *[CBSE Delhi 2008(C)]*

Ans. Electric flux through flat surface S_1

$$\phi_1 = \int_{S_1} \vec{E_1} \cdot d\vec{S_1}$$

$$= \int_{S_1} (E_x \hat{i}) \cdot (dS_1 \hat{i}) = E_x S_1$$

Electric flux through flat surface S_2

$$\phi_2 = \int_{S_2} \vec{E_2} \cdot d\vec{S_2} = \int_{S_2} (-E_x \hat{i}) \cdot (-dS_2 \hat{i}) = \int_{S_2} E_x dS_2 = E_x S_2$$

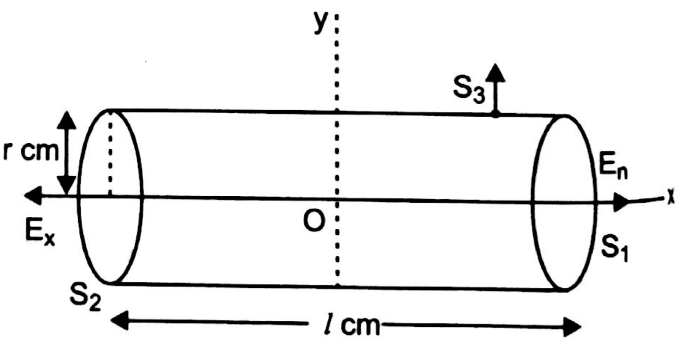
Electric flux through curved surface S_3

$$\phi_3 = \int_{S_3} (\vec{E_3} \cdot d\vec{S_3}) = \int_{S_3} E_3 \, dS_3 \cos 90° = 0$$

$\therefore$ Net electric flux, $\phi = \phi_1 + \phi_2 = E_x (S_1 + S_2)$

But $\quad S_1 = S_2 = \pi (r \times 10^{-2})^2 \text{ m}^2 = \pi r^2 \times 10^{-4} \text{ m}^2$

$\therefore \quad \phi = E_x . 2 (\pi r^2 \times 10^{-4})$ units



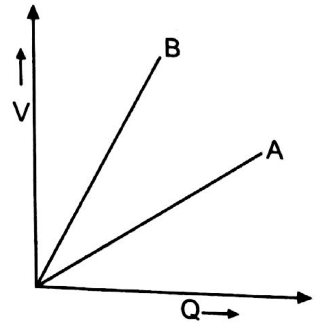
By Gauss's law

$$\phi = \frac{1}{\varepsilon_0} q \Rightarrow q = \varepsilon_0 \phi = \varepsilon_0 E_x (2\pi r^2 \times 10^{-4})$$

$$= 2\pi\varepsilon_0 E_x r^2 \times 10^{-4} = 4\pi\varepsilon_0 \left(\frac{E_x r^2 \times 10^{-4}}{2}\right)$$

$$= \frac{1}{9 \times 10^9}\left[\frac{E_x r^2 \times 10^{-4}}{2}\right] = 5.56 E_x r^2 \times 10^{-11} \text{ coulomb.}$$

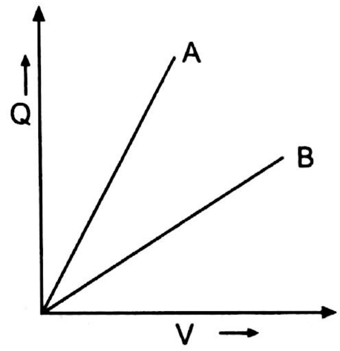
Q.10. The graph shows the variation of voltage 'V' across the plates of two capacitors A and B versus increase of charge 'Q' stored on them. Which of the two capacitors has higher capacitance? Give reason for your answer. *[CBSE Delhi 2004]*



Ans. $C = \frac{Q}{V} = \frac{1}{\text{Slope of line}}$

As slope of A is smaller, capacitance of A is higher.

Q.11. The given graph shows the variation of charge q versus potential difference V for capacitors C_1 and C_2. The two capacitors have the same plate separation, but the plate area of C_2 is double that of C_1. Which of the lines in the graph correspond to C_1 and C_2 and why? ***[CBSE Delhi 2006]***



$C_1 \propto A$
$C_2 \propto 2A$

Ans. Capacitance of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d} \propto A$

As plate area of C_2 is double that of C_1; $C_2 = 2C_1$

Slope of $q - V$ graph $= \frac{q}{V} = C$

As slope of A is greater than slope of B, A is corresponding to larger capacitance and B to smaller capacitance.

So lines corresponding to C_1 and C_2 are B and A respectively.

Q.12. Two charges 5 nC and –2 nC are placed at points (5 cm, 0, 0) and (23 cm, 0, 0) in a region of space where there is no other external field. Calculate the electrostatic potential energy of this charge system. ***(CBSE Delhi 2008C)***

Ans. Given $q_1 = 5\text{ nC} = 5 \times 10^{-9}$ C, $q_2 = -2\text{ nC} = -2 \times 10^{-9}$ C

The charges are placed on X-axis. The distance between the charges

$$x = x_2 - x_1 = (23 - 5)\text{ cm} = 18\text{ cm} = 0.18\text{ m}$$

∴ Electrostatic potential energy of charges

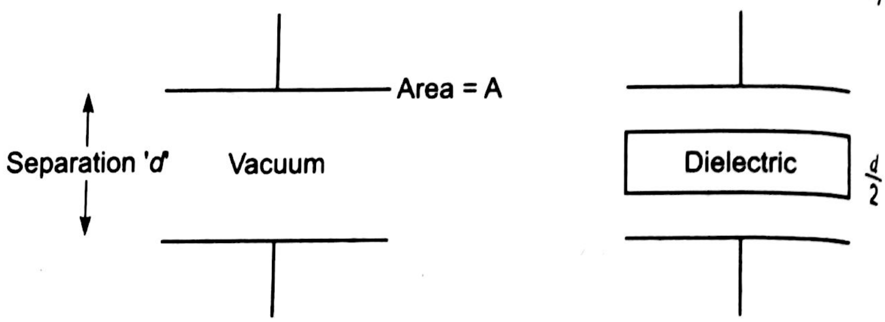
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x}$$

$$= \frac{9 \times 10^9 (5 \times 10^{-9})(-2 \times 10^{-9})}{0.18} = \mathbf{5 \times 10^{-7}\ J}$$

Q.13. **A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness *d*/2, where *d* is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.** *[CBSE (AI) 2013]*

Ans. Capacitance with dielectric of thickness 't'

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$



Put $t = \frac{d}{2}$

$$C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2} + \frac{d}{2K}}$$

$$= \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)} = \frac{2\varepsilon_0 A K}{d(K+1)}$$