SIMIL Reference numericals

NUMERICALS

NCERT Numericals

Magnetic Field due to a Straight Wire

1. A long straight wire carries a current of 35 A. What is the magnitude of magnetic field B at a point 20 cm from the wire.

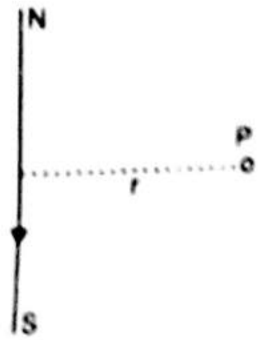
Sol. Magnetic field due to a current carrying straight wire at a distance r is

$$B = \frac{\mu_0 I}{2\pi r}$$

Given $A = 35$ A, $r = 20$ cm $= 0 \cdot 20$ m, $B = ?$

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 35}{2\pi \times 0.20} = 3.5 \times 10^{-5} \text{ T}$$

2. A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of B at a point 2.5 m east of the wire?

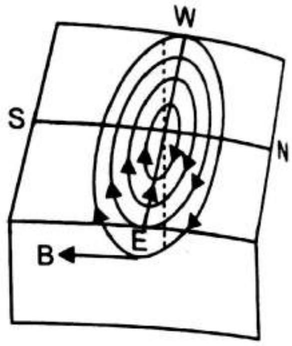


Sol. Given $I = 50$ A, $r = 2.5$ m

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 50}{2\pi \times 2.5} = \mathbf{4 \times 10^{-6}\ T}$$

By right hand palm rule the magnetic field is directed vertically upward.

3. **A horizontal overhead power line carries a current of 90 A in an east to west direction. What is the magnitude and direction of magnetic field due to the current at a distance 1.5 m below the line?**



Sol. The magnitude of magnetic field at a distance r is

$$B = \frac{\mu_0 I}{2\pi r}$$

Here $I = 90$ A, $r = 1.5$ m

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 90}{2\pi \times 1.5} = 1.2 \times \mathbf{10^{-5}\ tesla}$$

According to Right hand palm rule the magnetic field at a point vertically below the wire is directed along the south.

Magnetic Field due to a Circular Coil

4. **A circular coil of wire consisting of 100 turns, each of radius 8·0 cm carries a current of 0·40 A. What is the magnitude of magnetic field B at the centre of the coil?**

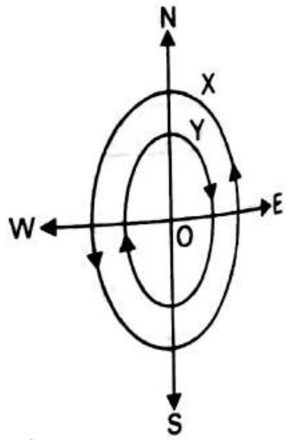
Sol. Given $N = 100$, $r = 8 \cdot 0$ cm $= 8 \cdot 0 \times 10^{-2}$ m, $I = 0 \cdot 40$ A

∴Magnetic field at centre of circular coil

$$B = \frac{\mu_0 NI}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 0 \cdot 40}{2 \times 8 \cdot 0 \times 10^{-2}}$$

$$= \pi \times 10^{-4}\ \text{T} = \mathbf{3 \cdot 14 \times 10^{-4}\ T}$$

5. **Two concentric circular coils X and Y of radius 16 cm and 10 cm respectively lie in the same vertical plane containing the north-south direction. Coil X has 20 turns and carries a current of 16 A, coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise and in Y is clockwise, for an observer looking at the coils facing west. Give the magnitude and the direction of the net magnetic field due to the coils at their centre.**



Sol. As currents in coils X and Y are opposite, the direction of magnetic field produced by them at the centre will be opposite.

The magnetic field produced at the centre due to a current carrying coil is

$$B = \frac{\mu_0 Ni}{2R}$$

Let B_1 and B_2 be magnetic fields at centre O due to coils X and Y respectively.

For coil X, $I_1 = 16$ A, $N_1 = 20$, $R_1 = 16$ cm $= 0 \cdot 16$ m

$$\therefore \quad B_1 = \frac{\mu_0 N_1 I_1}{2R_1} = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16} = 4\pi \times 10^{-4}\ \text{T, towards west}$$

For coil Y, $I_2 = 18$ A, $N_2 = 25$, $R_2 = 10$ cm $= 0 \cdot 10$ m

$\therefore \quad B_2 = \frac{\mu_0 N_2 I_2}{2R_2} = \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.10} = 9\pi \times 10^{-4}$ T, towards east.

$\therefore$ Net magnetic field $B = B_2 - B_1 = 9\pi \times 10^{-4} - 4\pi \times 10^{-4} = 5\pi \times 10^{-4}$ T

$= 5 \times 3.14 \times 10^{-4}$ T $=$ **15.7×10^{-4} T,** towards east.

Thus resultant magnetic field at centre has magnitude 15.7×10^{-4} T and is directed towards **east.**

6. A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at the most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is to be not ferromagnetic.

Sol. Given $B = 100$ G $= 100 \times 10^{-4}$ T $= 10^{-2}$ T, $I = 15$ A, $n = 1000$ turns/m.

We have $\quad B = \mu_0 n I$

$$\therefore \quad nI = \frac{B}{\mu_0} = \frac{10^{-2}}{4\pi \times 10^{-7}} = 8000$$

We may have $I = 10$ A, $n = 800$

The length of solenoid may be about 50 cm, number of turns about 400, so that $n = \frac{N}{l} = \frac{400}{05} = 8000.$

The area of cross-section of solenoid may be 10^{-3} m^2 or more; though these particulars are not unique, slight adjustments are possible.

Magnetic Field due to a Solenoid

7. A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of $\vec{B}$ inside the solenoid near its centre.

Sol. Given $l = 80$ cm $= 0.80$ m, $N = 5 \times 400 = 2000$, $I = 8.0$ A

Magnetic field inside the solenoid

$$B = \mu_0 n I = \frac{\mu_0 N I}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 8.0}{0.80}$$

$$= 8\pi \times 10^{-3} \text{ T} = \mathbf{2.5 \times 10^{-2} \text{ T}}$$

8. A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A; what is the magnetic field (*a*) outside the toroid (*b*) inside the core of toroid and (*c*) in the empty space surrounded by the toroid.

Sol. Mean radius of toroid $\quad r = \frac{r_1 + r_2}{2} = \frac{25 + 26}{2} = 25.5$ cm $= 25.5 \times 10^{-2}$ m

Total number of turns $N = 3500$, current $I = 11$ A

Number of turns per unit length, $\quad n = \frac{N}{2\pi r} = \frac{3500}{2\pi \times 25.5 \times 10^{-2}}$ turns/ metre

(*a*) Magnetic field outside the toroid is **zero.**

(*b*) Magnetic field inside the toroid

$$=\mu_0 nI = 4\pi\times10^{-7}\times\left(\frac{3500}{2\pi\times 25.5\times10^{-2}}\right)\times 11 = \mathbf{3.0\times 10^{-2}\ T}$$

(*c*) Magnetic field in empty space surrounded by toroid is zero.

Magnetic Force and Torque

9. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0·15 T?

Sol. Magnetic force $F = BIl \sin\theta$

Magnetic force per unit length, $f = \frac{F}{l} = BI \sin\theta = 0.15\times 8\times \sin 30° = \mathbf{0.6\ N/m}$

10. Two long and parallel straight wires A and B carrying currents of 8·0 A and 5·0 A in the same direction are separated by a distance of 4·0 cm. Estimate the force on a 10 cm section of wire A.

Sol. Given $I_1 = 8.0$ A, $I_2 = 5\cdot0$ A, $r = 4\cdot0$ cm $= 4\cdot0\times10^{-2}$ m

Currents in same direction attract each other; so magnetic force on $l = 10$ cm $= 10\times10^{-2}$ m length of wire *A* is

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r} = \frac{4\pi\times10^{-7}\times 8\cdot0\times5\cdot0\times10\cdot0\times10^{-2}}{2\pi\times4\cdot0\times10^{-2}} = \mathbf{2\cdot0\times10^{-5}\ N}$$

11. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and normal to the plane of the coil makes an angle of 30° with the direction of uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of the torque experienced by the coil?

Sol. Torque on coil $\tau = NIAB \sin\theta$

Here $N = 20; A = 10\text{ cm}\times 10\text{ cm} = 100\text{ cm}^2 = 100\times10^{-4}\text{ m}^2$

$I = 12$ A, $\theta = 30°$, $B = 0.80$ T

$\therefore$ $$\tau = (20)\times(12)\times(100\times10^{-4})\times 0.80 \sin 30°$$

$$= 24\times 0.8\times\left(\frac{1}{2}\right)\times10^{-1} = 0.96\ \text{Nm}$$

12. (*a*) **A circular coil of 30 turns and radius 8·0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal to the coil. Calculate the magnitude of counter-torque that must be applied to prevent the coil from turning.**

(*b*) **Would your answer change, if the circular coil in (*a*) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered).**

Sol. (*a*) Given $N = 30$, $A = \pi r^2 = \pi\times(8.0\times10^{-2})^2\ \text{m}^2$

$I = 6.0$ A, $B = 1.0$ T, $\theta = 60°$

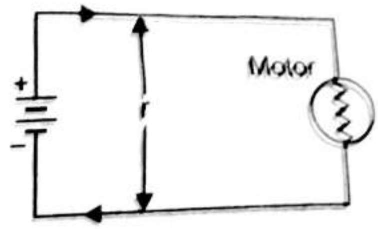
Torque $\tau = NIAB \sin\theta = 30\times 6.0\times\pi\times(8.0\times10^{-2})^2\times1.0\times\sin 60°$

$$= 30\times6.0\times3.14\times64\times10^{-4}\times\left(\frac{\sqrt{3}}{2}\right) = 3.13\ \text{Nm}$$

(*b*) As the expression for torque contains only area and not the shape of coil, so torque on a planar loop will remain the same provided magnitude of area is same.

13. The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short while). What is the force per unit length between the wires if they are 70 cm long and 1·5 cm apart? Is the force attractive or repulsive?

Sol. Force per unit length $f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ N/m



Here $\mu_0 = 4\pi \times 10^{-7}$ N/A^2, $I_1 = I_2 = 300$ A,

$r = 1{\cdot}5$ cm $= 1{\cdot}5 \times 10^{-2}$ m

$$\therefore \quad f = \frac{4\pi \times 10^{-7} \times 300 \times 300}{2\pi \times 15 \times 10^{-2}} = \mathbf{1.2\ N/m}$$

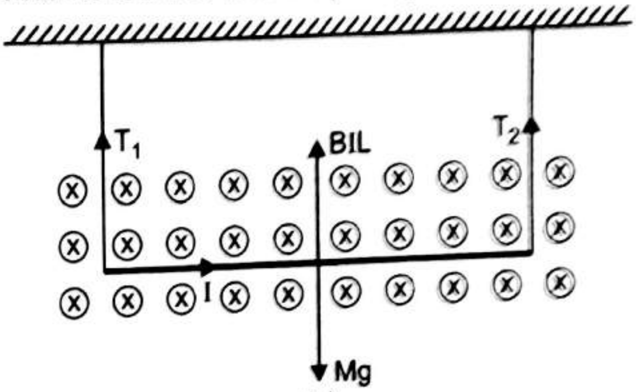
currents are in opposite directions, therefore the force is ***repulsive.***

Remark: The answer is approximate because the formula is true for infinitely long wires.

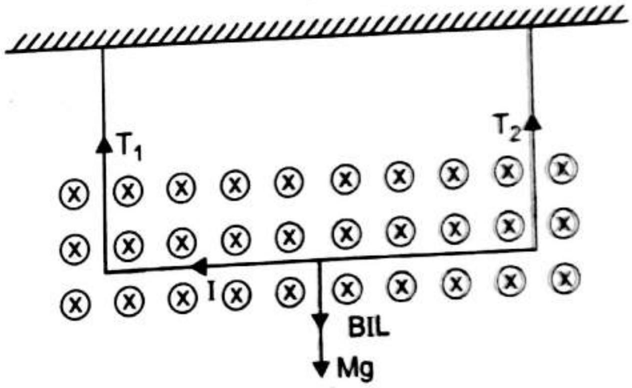
14. A straight horizontal conducting rod of length 0·45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5·0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field same as before? (Neglect the mass of wires, $g = 9{\cdot}8$ m/s^2). *[CBSE (AI) 2005]*



(a)



(b)

Sol. (*a*) **Key idea:** If tension in wires be zero, then the weight of rod and magnetic force on rod must be equal and opposite.

Weight of rod acts vertically downward. For magnetic force to act upward, the magnetic field should be normal to the length of current as shown in Fig. (a).

Magnetic force = weight of rod

$$BIL = Mg \qquad ...(i)$$

Magnetic field, $B = \frac{Mg}{IL}$

Given $M = 60\,\text{g} = 60 \times 10^{-3}$ kg, $I = 5{\cdot}0$ A, $L = 0{\cdot}45$ m

$$\therefore \quad B = \frac{60 \times 10^{-3} \times 9{\cdot}8}{5{\cdot}0 \times 0{\cdot}45} = 0{\cdot}26\ \text{T}$$

(*b*) **Key idea:** When the direction of current is reversed, the magnetic force also reverses the direction, so that weight Mg and magnetic force BIL act in the same direction.

$\therefore$ Total tension in wires $T = (T_1 + T_2) = Mg + BIL$

$= 2Mg$ since $Mg = BIL$ from (i)

$= 2 \times 60 \times 10^{-3} \times 9{\cdot}8 = \mathbf{1{\cdot}176\ N}$

Motion of Charged Particle in Magnetic Field

15. **(a) In a chamber a uniform magnetic field of 6·5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of $4 \cdot 8 \times 10^{-6}$ ms^{-1} normal to the field. Explain why the path of electron is a circle. Determine the radius of the circular orbit. ($e = 1 \cdot 6 \times 10^{-19}$ C, $m = 9 \cdot 1 \times 10^{-31}$ kg).**

(b) In Q. 15. (a), also find the frequency of revolution of the electron in its circular orbit. Does the answer depend upon the speed of the electron? Explain.

Sol. (a) The electron in transverse magnetic field experiences magnetic force $F_m = qvB$ which is perpendicular to $\vec{v}$ as well as $\vec{B}$; so magnetic force only changes the direction of path of electron, without changing its speed. This is only possible in circular path; the magnetic force provides the necessary centripetal force for circular path.

$$\frac{mv^2}{r} = evB$$

∴ Radius
$$r = \frac{mv}{eB} = \frac{9 \cdot 1 \times 10^{-31} \times 4 \cdot 8 \times 10^{6}}{1 \cdot 6 \times 10^{-19} \times 6 \cdot 5 \times 10^{-4}}$$
$$= 4 \cdot 2 \times 10^{-2} \text{ m} = \mathbf{4 \cdot 2 \text{ cm}}$$

(b) Time period of revolution of electron $T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{eB} = \frac{2\pi m}{eB}$

∴ Frequency
$$f = \frac{1}{T} = \frac{eB}{2\pi m}$$
$$= \frac{1 \cdot 6 \times 10^{-19} \times 6 \cdot 5 \times 10^{-4}}{2 \times 3 \cdot 14 \times 9 \cdot 1 \times 10^{-31}} = 18 \cdot 2 \times 10^{6} \text{ Hz} = \mathbf{18 \cdot 2 \text{ MHz}}$$

16. An electron emitted by a heated cathode and accelerated through a potential difference of 2·0 kV, enters a region with a uniform magnetic field of 0·15 T. Determine the trajectory of the electrons if the magnetic field (a) is transverse to its initial velocity. (b) makes an angle 30° with the initial velocity.

Sol. Velocity of electron accelerated through a potential difference V is given by

$$\frac{1}{2} mv^2 = eV \quad \Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

Given $V = 2 \cdot 0$ kV $= 2 \cdot 0 \times 10^3$ V, $e = 1 \cdot 6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg

∴
$$v = \sqrt{\frac{2 \times 1 \cdot 6 \times 10^{-19} \times 2 \cdot 0 \times 10^3}{9 \times 10^{-31}}} = \frac{8}{3} \times 10^7 \text{ m/s}$$

(a) When an electron enters the transverse magnetic field, its path is a circle of radius r, given by

$$\frac{mv^2}{r} = evB \quad \text{or} \quad r = \frac{mv}{eB}$$

Substituting given values

$$r = \frac{(9 \times 10^{-31}) \times \left(\frac{8}{3} \times 10^7\right)}{(1 \cdot 6 \times 10^{-19}) \times (0 \cdot 15)} = 10^{-3} \text{ m} = 1 \text{ mm}$$

(*b*) When electron enters at an angle 30° with magnetic fields, its path become **a helix** of radius

$$r=\frac{mv\sin 30°}{eB}=\left(\frac{mv}{eB}\right)\times\sin 30°=1\text{ mm}\times(0\cdot5)=\mathbf{0\cdot5\ mm}$$

Velocity component of along the field

$$v_{11}=v\cos 30°=\left(\frac{8}{3}\times10^7\ \text{m/s}\right)\times\frac{\sqrt{3}}{2}=\mathbf{2\cdot3\times10^7\ m/s}$$

17. A magnetic field set up using Helmholtz coils is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through the 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is $9\cdot0\times10^5\ \text{Vm}^{-1}$, make a simple guess as to what the beam contains. Why is the answer not unique?

Sol. Given $B=0\cdot75$ T, $E=9\times10^5\ \text{Vm}^{-1}$, $V=15\text{ kV}=15000$ V.

The velocity of electron v is given by

$$\frac{1}{2}mv^2=eV \qquad \text{or} \qquad v=\sqrt{\frac{2eV}{m}}$$

Substituting the value of V, we get

$$v=\sqrt{\frac{2e\times15000}{m}}=\sqrt{3\times10^4\ (e/m)}$$

If particles are undeflected in simultaneous transverse electric and magnetic field, $eB=evB$

$$v=\frac{E}{B} \qquad \Rightarrow \qquad \sqrt{3\times10^4\ e/m}=\frac{E}{B}$$

or $$\frac{e}{m}=\frac{E^2}{B^2}\times\frac{1}{3\times10^4}=\frac{(9\times10^5)^2}{(0\cdot75)^2}\times\frac{1}{3\times10^4}=4\cdot8\times10^7\ \text{C/kg}$$

This gives the value of *e/m* of charged particle and not any particular particle; the charged particle may be deuteron (D^+), He^{++} and Li^{+++} ions etc. Hence the answer is not unique.

18. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0·10 T normal to the plane of the coil. If the current in the coil is 5·0 A, what is the

(*a*) total torque on the coil

(*b*) total force on the coil

(*c*) average force on each electron in the coil due to the magnetic field?

[The coil is made of copper wire of cross-sectional area $10^{-5}\ \text{m}^2$ and the free electron density in copper is given to be about $10^{29}\ \text{m}^{-3}$].

Sol. Given $N=20$, $r=10\text{ cm}=0\cdot10$ m, $I=5\cdot0$ A, $B=0\cdot10$ T

Area of coil $A=\pi r^2=3\cdot14\times(0\cdot10)^2=3\cdot14\times10^{-2}\ \text{m}^2$

(*a*) Angle between normal to plane of coil and magnetic field is 0° *i.e.,* $\vec{\tau}=NI\vec{A}\times\vec{B}=0$

(*b*) Total force on a current carrying coil in a magnetic field is always zero.

(*c*) Average (magnetic) force on an electron $F_m=ev_dB$

But $\quad v_d = \dfrac{I}{neA}$

$\therefore \quad F_m = e\left(\dfrac{I}{neA}\right)B = \dfrac{IB}{nA} = \dfrac{5\cdot0 \times 0\cdot10}{10^{29} \times 10^{-5}} = 5 \times 10^{-25}$ N

19. A solenoid 60 cm long and radius 4·0 cm has 3 layers of windings of 300 turns each. A 2·0 cm long wire of mass 2·5 g lies inside the solenoid (near its centre) normal to its axis, both the wire and the axis of solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6·0 A in the wire. What value of current (with appropriate sense of circulation) in the winding of the solenoid can support the weight of the wire? $g = 9\cdot8$ ms^{-2}

Sol. For solenoid, $l_1 = 60$ cm $= 0\cdot60$ m, $N_1 = 3 \times 300 = 900$, $I_1 = ?$

For wire $l_2 = 2\cdot0$ cm $= 2\cdot0 \times 10^{-2}$ m, $m_2 = 2\cdot5$ g $= 2\cdot5 \times 10^{-3}$ kg, $I_2 = 6\cdot0$ A

Magnetic field produced by solenoid $B_1 = \mu_0\left(\dfrac{N_1}{l_1}\right)I_1$ (along the axis)

Magnetic force on wire, $\quad F_2 = I_2 l_2 B_1 = I_2 l_2 \mu_0 \left(\dfrac{N_1}{l_1}\right) I_1$

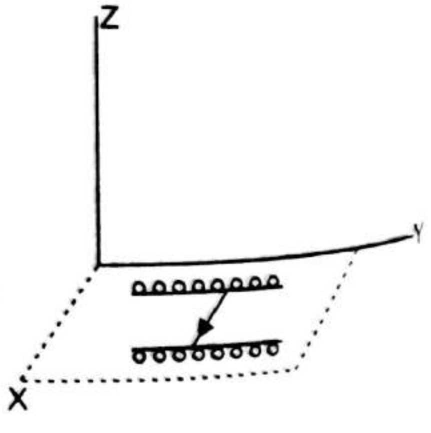
The weight of wire can be supported if this force acts vertically upward

i.e., $\quad mg = I_2 l_2 \, \mu_0 \left(\dfrac{N_1}{l_1}\right) I_1$

$$\Rightarrow \quad I_1 = \frac{mgl_1}{\mu_0 N_1 l_1 I_2} = \frac{2\cdot5 \times 10^{-3} \times 9\cdot8 \times 0\cdot60}{4\pi \times 10^{-7} \times 900 \times 2\cdot0 \times 10^{-2} \times 6\cdot0}$$

$$= \mathbf{108\cdot3\ A}$$

Let length of solenoid be along Y-axis and length of wire along X-axis. For upward magnetic force on wire the current in winding should be anticlockwise as seen from origin, so that magnetic field is along Y-axis and the current in wire should be along the positive X-axis mathematically.



$$\vec{F}_m = I\,\vec{l} \times \vec{B}$$

$$= Il\,\hat{i} \times B\,\hat{j} = IlB\,\hat{k}$$

$= IlB$ along the positive Z-axis

Weight is vertically downward (along the negative Z-axis)

Sensitivity of Galvanometer

20. Two moving coil meters M_1 and M_2 have the following particulars:

$$R_1 = 10\,\Omega,\ N_1 = 30,\ A_1 = 3\cdot6 \times 10^{-3}\ \text{m}^2,\ B_1 = 0\cdot25\ \text{T},$$

$$R_2 = 14\,\Omega,\ N_2 = 42,\ A_2 = 1\cdot8 \times 10^{-3}\ \text{m}^2,\ B_2 = 0\cdot50\ \text{T}$$

(The spring constants are identical for the two meters).

Determine the ratio of (*a*) current sensitivity and (*b*) voltage sensitivity of M_1 and M_2

Sol. Current sensitivity, $\quad S_C = \dfrac{NAB}{C}$

and voltage sensitivity, $S_V = \frac{NAB}{CR} = \frac{S_C}{R}$

The spring constant C is same for two meters.

(a) $\frac{(S_C)_{M_2}}{(S_C)_{M_1}} = \frac{N_2A_2B_2}{N_1A_1B_1} = \frac{42 \times 1.8 \times 10^{-3} \times 0.50}{30 \times 3.6 \times 10^{-3} \times 0.25} = \mathbf{1.4}$

(b) $\frac{(S_V)_{M_2}}{(S_V)_{M_1}} = \frac{(S_C)_{M_2}}{(S_C)_{M_1}} \times \frac{R_1}{R_2} = 1.4 \times \frac{10}{14} = \mathbf{1}$

21. A monoenergetic (18 keV) electron beam initially in the horizontal direction is subjected to a horizontal magnetic field of 0.04 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C).

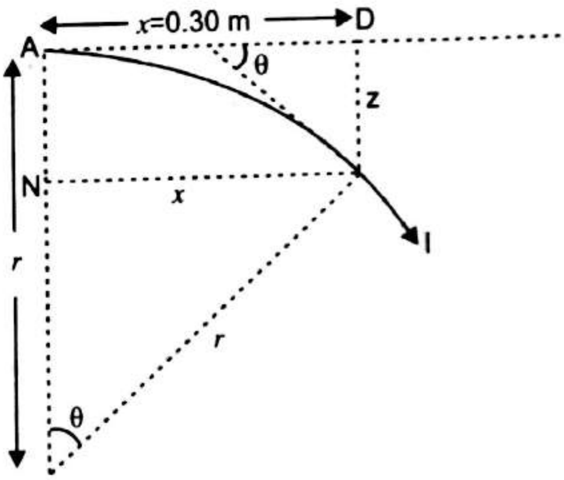
Sol. Given, $E_k = 18$ keV $= 18 \times 10^3 \times 1.6 \times 10^{-19} = 18 \times 1.6 \times 10^{-16}$ J,

$B = 0.04$ G $= 0.04 \times 10^{-4}$ T, $x = 30$ cm $= 0.30$ m

Momentum of beam, $mv = \sqrt{2mE_k}$

On entering the magnetic field normally beam is deflected up or down in a circle of radius r given by

$$r = \frac{mv}{eB} = \frac{\sqrt{2mE_k}}{eB} = \frac{\sqrt{2 \times 9.11 \times 18 \times 1.6 \times 10^{-16}}}{1.6 \times 10^{-19} \times 0.04 \times 10^{-4}} = 11.3 \text{ m}$$



Up or down deflection $z = AN = r - r\cos\theta$

$= r(1 - \cos\theta)$

From fig. $\sin\theta = \frac{x}{r} = \frac{0.30}{11.3}$

As θ is very small, $\sin\theta = \theta = \frac{0.33}{11.3}$

$$\cos\theta = 1 - \frac{\theta^2}{2} \Rightarrow 1 - \cos\theta = \frac{\theta^2}{2}$$

$\therefore$ Deflection

$$z = r \cdot \frac{\theta^2}{2} = \frac{11.3}{2} \times \left(\frac{0.33}{11.3}\right)^2 \text{ m} = 4 \times 10^{-3} \text{ m} = \mathbf{4\ mm}$$

Conversion of Galvanometer into Ammeter and Voltmeter

22. A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0–18 V?

Sol. For conversion of galvanometer into voltmeter a resistance R is connected in series with the coil.

Series resistance, $R = \frac{V}{I_g} - G$

Given, $V = 18$ V, $G = 12\,\Omega$, $I_g = 3$ mA $= 3 \times 10^{-3}$ A

$\therefore$ $R = \frac{18}{3 \times 10^{-3}} - 12 = 6000 - 12 = \mathbf{5988\,\Omega}$

Q. 23. A galvanometer has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A?

Sol. For conversion of galvanometer into an ammeter a shunt (a small resistance in parallel with coil) is connected. The value of shunt resistance 'S' is given by

$$I_g = \frac{S}{S+G} I \Rightarrow S = \frac{I_g}{I - I_g} G$$

Given, $I_g = 4 \text{ mA} = 4 \times 10^{-3}$ A, $I = 6$ A, $G = 15\,\Omega$

$$\therefore \qquad S = \frac{4 \times 10^{-3}}{6 - 4 \times 10^{-3}} \times 15 \approx \frac{4 \times 10^{-3}}{6} \times 15\,\Omega = 10 \times 10^{-3}\,\Omega = \mathbf{10\ m\Omega}$$

Magnetism

24. A short bar magnet placed with its axis at 30° with a uniform external magnetic field is 0·25 T experiences a torque of magnitude equal to $4 \cdot 5 \times 10^{-2}$ Nm. What is the magnitude of magnetic moment of the magnet?

Sol. Given $B = 0 \cdot 25$ T, $\tau = 4 \cdot 5 \times 10^{-2}$ Nm, $\theta = 30°$

We have $\tau = mB \sin\theta$

$\Rightarrow$ Magnetic moment $m = \frac{\tau}{B \sin\theta} = \frac{4 \cdot 5 \times 10^{-2}}{0 \cdot 25 \times \sin 30°}$

$$= \frac{4 \cdot 5 \times 10^{-2}}{0 \cdot 25 \times 0 \cdot 5} = \mathbf{0 \cdot 36\ Am^2} \text{ (or J/T)}$$

25. A short bar magnet of magnetic moment $M = 0 \cdot 32$ JT^{-1} is placed in a uniform magnetic field of 0·15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (*i*) stable and (*ii*) unstable equilibrium? What is the potential energy of the magnet in each case?

Sol. Given $m = 0 \cdot 32$ JT^{-1}, $B = 0 \cdot 15$ T

Potential energy of magnet in magnetic field

$$U = -mB\cos\theta$$

(*i*) In stable equilibrium the potential energy of magnet is the minimum; so

$$\cos\theta = 1 \text{ or } \theta = 0°$$

Thus in stable equilibrium position, the bar magnet is so aligned that its magnetic moment is along the direction of magnetic field ($\theta = 0°$).

$$U_m = -mB = -0 \cdot 32 \times 0 \cdot 15 = -4 \cdot 8 \times 10^{-2} \text{ J}$$

(*ii*) In unstable equilibrium, the potential energy of magnet is the maximum.

Thus in unstable equilibrium position, the bar magnetic is so aligned that its magnetic moment is opposite to the direction of the magnetic field *i.e.*, $\cos\theta = -1$ or $\theta = 180°$.

In this orientation potential energy, $U_{max} = +mB = +4 \cdot 8 \times 10^{-2}$ J.

26. (*a*) Closely wound solenoid of 800 turns and area of cross-section $2 \cdot 5 \times 10^{-4}$ m^2 carries a current of 3.0 A. Explain the sense in which solenoid acts like a bar magnet. What is the associated magnetic moment.

(*b*) If the solenoid is free to turn about the vertical direction in an external uniform horizontal magnetic field at 0.25 T, what is the magnitude of the torque on the solenoid when its axis makes an angle of 30° with the direction of the external field.

Sol. (*a*) If solenoid is suspended freely, it stays in N-S direction. The polarity of solenoid depends on the sense of flow of current. If to an observer looking towards an end of a solenoid, the current appears anticlockwise, the end of solenoid will be N-pole and other end will be S-pole.

Magnetic moment $m = NIA = 800 \times 3 \cdot 0 \times 2 \cdot 5 \times 10^{-4} = \mathbf{0 \cdot 60\ Am^2}$

(*b*) Torque an solenoid $\tau = mB \sin \theta$

$$= 0 \cdot 60 \times 0 \cdot 25 \sin 30° = 0 \cdot 60 \times 0 \cdot 25 \times 0 \cdot 5 = \mathbf{7 \cdot 5 \times 10^{-2}\ Nm}$$

27. A bar magnet of magnetic moment $1 \cdot 5\ JT^{-1}$ lies aligned with the direction of a uniform magnetic field of $0 \cdot 22$ T.

(*a*) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment.

(*i*) normal to the field direction, and (*ii*) opposite to the field direction?

(*b*) What is the torque on the magnet in cases (*i*) and (*ii*).

Sol. (*a*) Work done in aligning a magnet from orientation θ_1 to θ_2 is given by

$$W = U_2 - U_1 = -mB \cos \theta_2 - (-mB \cos \theta_1)$$
$$= mB (\cos \theta_1 - \cos \theta_2) \quad \ldots(i)$$

(*i*) Here $\theta_1 = 0°, \theta_2 = 90°$

$\therefore \quad W = mB (\cos 0° - \cos 90°) = mB (1 - 0) = mB = 1 \cdot 5 \times 0 \cdot 22 = \mathbf{0 \cdot 33\ J}$

(*ii*) Here $\theta_1 = 0°, \theta_2 = 180°$

$\therefore \quad W = mB (\cos 0° - \cos 180°) = 2\ mB = 2 \times 1 \cdot 5 \times 0 \cdot 22 = \mathbf{0 \cdot 66\ J}$

(*b*) Torque $\tau = mB \sin \theta$

In (*i*) $\cos \theta = 90° \quad \tau = mB \sin 90° = mB = 1 \cdot 5 \times 0 \cdot 22 = \mathbf{0 \cdot 33\ Nm}$

This torque tends to align the magnet along the direction of field.

In (*ii*) case $\theta = 180°$.

$$\tau = mB \sin 180° = \mathbf{0}$$

28. A closely wound solenoid of 2000 turns and area of cross-section $1 \cdot 6 \times 10^{-4}\ m^2$, carrying a current of $4 \cdot 0$ A is suspended through its centre allowing it to turn in a horizontal plane.

(*a*) What is the magnetic moment associated with the solenoid?

(*b*) What are the force and torque on the solenoid if a uniform magnetic field of $7 \cdot 5 \times 10^{-2}$ T is set up at an angle of 30° with the axis of the solenoid?

Sol. Given $N = 2000, \quad A = 1 \cdot 6 \times 10^{-4}\ m^2, \quad I = 4 \cdot 0$ A

(*a*) **Magnetic moment of solenoid,** $m = NIA = 2000 \times 4 \cdot 0 \times 1 \cdot 6 \times 10^{-4} = 1 \cdot 28\ \mathbf{Am^2}$

(*b*) Net force on current carrying solenoid (or magnetic dipole) in uniform magnetic field is always **zero**.

Torque $\tau = mB \sin \theta$.

Here $B = 7 \cdot 5 \times 10^{-2}$ T, $\theta = 30°$

$\therefore \quad \tau = 1 \cdot 28 \times 7 \cdot 5 \times 10^{-2} \times \sin 30°$

$$= 1 \cdot 28 \times 7 \cdot 5 \times 10^{-2} \times 0 \cdot 5 = \mathbf{4 \cdot 8 \times 10^{-2}\ Nm}$$

29. A short bar magnet has a magnetic moment $0 \cdot 48\ JT^{-1}$. Give the magnitude and direction of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on (*i*) the axis (*ii*) equatorial lines (normal bisector) of the magnet.

Sol. Given $m = 0 \cdot 48\ JT^{-1}, r = 10\ cm = 0 \cdot 10$ m (*i*) Magnetic field at axis, $B_1 = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$

$$=(10^{-7}) \times \frac{2 \times 0.48}{(0.10)^3} = 0.96 \times 10^{-4} \text{ T}$$

$$= \mathbf{0.96\ G} \quad \text{along } S\text{-}N \text{ direction}$$

(*ii*) Magnetic field at equatorial line

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{r^3} = 0.48 \times 10^{-4} \text{ T}$$

$$= 0.48 \text{ G along } N\text{-}S \text{ direction}$$

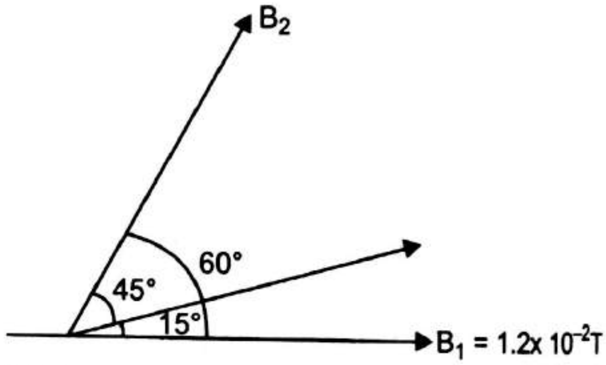
30. A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60° and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of other field?

Sol. **Key idea:** For equilibrium, the net torque on magnetic field must be zero. Therefore, the torques exerted by fields B_1 and B_2 on the dipole must be equal and opposite.

$$\tau_1 = \tau_2$$

$$mB_1 \sin\theta_1 = mB_2 \sin\theta_2$$

$$\Rightarrow \quad B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$



Given $B_1 = 1.2 \times 10^{-2}$ T, $\theta_1 = 15°$,

$\theta_2 = (60° - 15°) = 45°$

$$\therefore \quad B_2 = 1.2 \times 10^{-2} \times \frac{\sin 15°}{\sin 45°} = 1.2 \times 10^{-2} \times \frac{0.2588}{0.7071} = \mathbf{4.4 \times 10^{-3}\ T}$$

Earth's Magnetism

31. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at a place is known to be 0·35 G. Determine the magnitude of the earth's magnetic field at the place. (Given $\cos 22° = 0.927$, $\sin 22° = 0.375$).

Sol. By definition, angle of dip $\theta = 22°$

Given $\quad H = 0.35$ G

We have $\quad H = B_e \cos\theta \quad$ or $\quad B_e = \frac{H}{\cos\theta} = \frac{0.35}{\cos 22°}$ G

or

$$B_e = \frac{0.35}{0.927} = \mathbf{0.38\ G}$$

32. At a certain location in Africa, a compass points 12° west of the geographical north. The north tip of the magnetic needle of a dip circle placed in the plane of the magnetic meridian points 60° above the horizontal. The horizontal component of earth's magnetic field is measured to be 0.16 gauss. Specify the direction and magnitude of earth's magnetic field at the location.

Sol. This problem illustrates how the three elements of earth's field: angle of declination (α), angle of dip (θ) and horizontal component H, determine the earth's magnetic field completely.

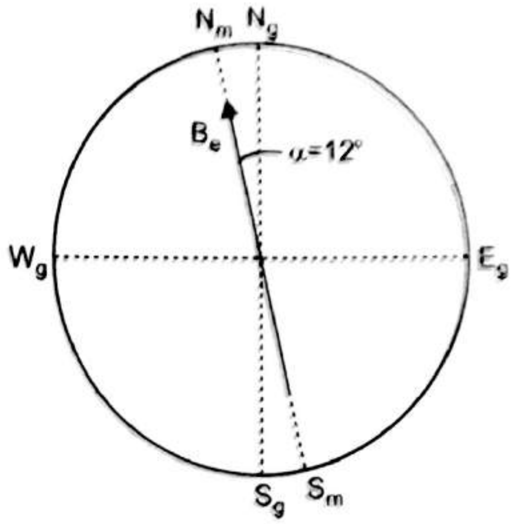
Here angle of declination $\alpha = 12°$

Angle of dip $\theta = 60°$

and Horizontal component,

$$H = 0{\cdot}16 \text{ gauss} = 0{\cdot}16 \times 10^{-4} \text{ T}$$

If B_e is the total earth's magnetic field, then the relation between B_e and H is $H = B_e \cos\theta$



$\Rightarrow$

$$B_e = \frac{H}{\cos\theta} = \frac{0{\cdot}16 \times 10^{-4}}{\cos 60°} = \frac{0{\cdot}16 \times 10^{-4}}{0{\cdot}5} = \mathbf{0{\cdot}32 \times 10^{-4}\ T}$$

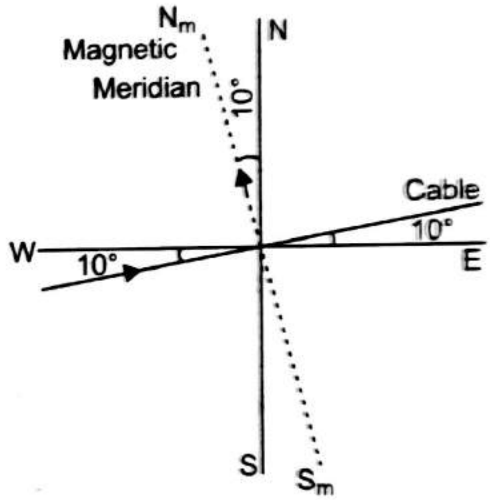
Thus, the magnitude of earth's field is $0{\cdot}32 \times 10^{-4}$ T $= 0{\cdot}32$ gauss and it lies in a vertical plane 12° west of geographical meridian making an angle of 60° (upwards) with the horizontal direction.

33. A long straight horizontal cable carries a current of 2·5 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographical meridian. The earth's magnetic field at the location is 0·33 G and the angle of dip is zero. Locate the line of neutral points (Ignore the thickness of the cable).

Sol. Given $B_e = 0{\cdot}33$ G, $I = 2{\cdot}5$ A

Angle of dip, $\theta = 0$

$\therefore H = B_e \cos 0° = 0{\cdot}33 \text{ G} = 0{\cdot}33 \times 10^{-4}$ T, $V = B_e \sin 0° = 0$



Magnetic field due to current carrying cable

$$B_c = \frac{\mu_0 I}{2\pi r}$$

The cable is perpendicular to the magnetic meridian. For neutral point, the magnetic field produced by the cable must be equal and opposite to earth's magnetic field *i.e.*,

$$B_c = H \Rightarrow \frac{\mu_0 I}{2\pi r} = H$$

$$r = \frac{\mu_0 I}{2\pi H} = \frac{4\pi \times 10^{-7} \times 2{\cdot}5}{2\pi \times 0{\cdot}33 \times 10^{-4}}$$

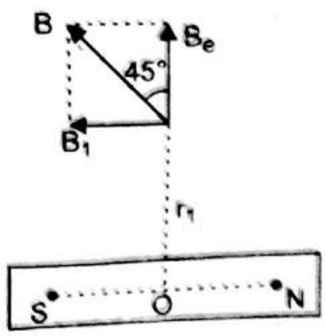
$$= 1{\cdot}5 \times 10^{-2} \text{ m} = \mathbf{1{\cdot}5\ cm}$$

That is, the line of neutral points is parallel to the cable at a distance 1·5 cm above the plane of paper.

34. A short bar magnet of magnetic moment $5{\cdot}25 \times 10^{-2}$ JT^{-1} is placed with its axis perpendicular to earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with the earth's field on (*a*) its normal bisector and (*b*) its axis. Magnitudes of the earth's field at the place is given to be 0·42 G. Ignore the length of the magnet in comparison to the distance involved.

Sol. Given $m = 5{\cdot}25 \times 10^{-2}$ JT^{-1}, $B_e = 0{\cdot}42 \text{ G} = 0{\cdot}42 \times 10^{-4}$ T

(*a*) Let B_1 be the magnetic field of magnet at its bisector.



$$\tan 45° = \frac{B_1}{B_e} \quad \Rightarrow \quad B_1 = B_e$$

At normal bisector

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{r_1^3}$$

$$\therefore \qquad \frac{\mu_0}{4\pi}\frac{m}{r_1^3} = 0.42 \times 10^{-4} \text{ T}$$

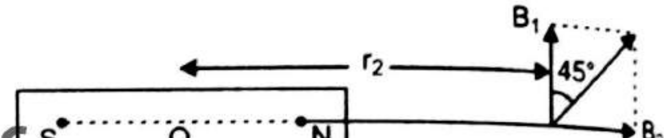
$$\Rightarrow \qquad 10^{-7} \times \frac{(5.25 \times 10^{-2})}{r_1^3} = 0.42 \times 10^{-4} \text{ T}$$

$$\text{or} \qquad r_1^3 = \frac{10^{-7} \times (5.25 \times 10^{-2})}{0.42 \times 10^{-4}} = \frac{52.5}{0.42} \times 10^{-6}$$

$$r_1 = \left(\frac{52.5}{0.42}\right)^{1/3} \times 10^{-2} \text{ m} = (125)^{1/3} \times 10^{-2} \text{ m} = 5 \times 10^{-2} \text{ m} = \mathbf{5\ cm}$$

(*b*) Let B_1 be the magnetic field of magnet at its axis, then

$$\tan 45° = \frac{B_2}{B_e} \quad \Rightarrow \quad B_2 = B_e$$



But at axis of magnet

$$B_2 = \frac{\mu_0}{4\pi}\frac{2m}{r_2^3}$$

$$\therefore \qquad \frac{\mu_0}{4\pi}\frac{2m}{r_2^3} = 0.42 \times 10^{-4}$$

$$\Rightarrow \qquad 10^{-7} \times \frac{2 \times 5.25 \times 10^{-2}}{r_2^3} = 0.42 \times 10^{-4}$$

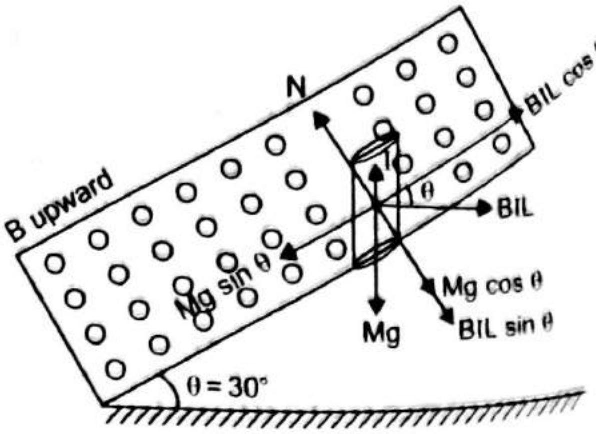
$$\Rightarrow \qquad r_2^3 = 10^{-7} \times \frac{2 \times 5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$

$$\text{or} \qquad r_2 = \left(\frac{2 \times 52.5}{0.42}\right)^{1/3} \times 10^{-2} \text{ m}$$

$$= 5 \times (2)^{1/3} \times 10^{-2} \text{ m} = 5 \times 1.26 \times 10^{-2} \text{ m}$$

$$= 6.3 \times 10^{-2} \text{ m} = \mathbf{6.3\ cm}$$

35. On a smooth inclined plane at 30° with the horizontal, a thin current carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0·15 T in the vertical direction. For what value of current can the rod remain stationary? The mass per unit length of the rod is 0·30 kg/m.



Sol. The forces acting on the rod are

(*i*) Weight Mg, vertically downward

(*ii*) Magnetic force BIL in horizontal direction

(*iii*) Normal reaction N

We resolve weight Mg and magnetic force $F_m = BIL$ along and normal to plane.

The component of weight along the plane $= Mg \sin\theta$

The component of weight normal to plane $= Mg \cos\theta$

The component of magnetic force along the plane $= BIL \cos\theta$

The component of magnetic force normal to plane $= BIL \sin\theta$

For rod to remain stationary, the normal reaction balances the normal components and the component of weight, $Mg \sin \theta$ is balanced by component $BIL \cos \theta$ of magnetic force.

i.e., $N = Mg \cos \theta + BIL \sin \theta$ ...(*i*)

and $Mg \sin \theta = BIL \cos \theta$ ...(*ii*)

Equation (*ii*) given

$$\text{current } I = \frac{Mg \sin \theta}{BL \cos \theta} = \left(\frac{M}{L}\right)\frac{g}{B} \tan \theta$$

Given $\frac{M}{L} = 0{\cdot}30 \text{ kg/m}, g = 9{\cdot}8 \text{ m/s}^2, B = 0{\cdot}15 \text{ T}, \theta = 30°$

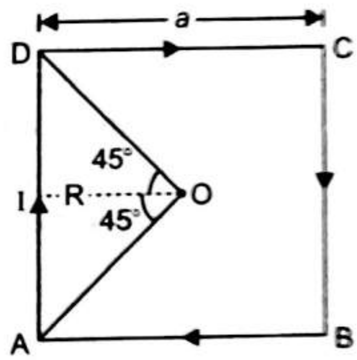
$$\therefore \quad I = 0{\cdot}30 \times \frac{9{\cdot}8}{0{\cdot}15} \times \tan 30° = 2 \times 9{\cdot}8 \times \frac{1}{\sqrt{3}}$$

$$= \frac{2 \times 9{\cdot}8\sqrt{3}}{3} = \frac{2 \times 9{\cdot}8 \times 1{\cdot}732}{3} = \mathbf{11{\cdot}3\ A}$$

Previous Years' Numericals

1. Find the magnetic field at the centre of square of side 'a' carrying current I A.

Sol. The current due to one side (say AD) of square at the centre O is



$$B_1 = \frac{\mu_0 I}{4\pi R}(\sin \alpha + \sin \beta)$$

Here $\alpha = \beta = 45°, R = \frac{a}{2}$

$$\therefore \quad B_1 = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}(\sin 45° + \sin 45°)$$

$$= \frac{2\mu_0 I}{4\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2\pi a}\frac{2}{\sqrt{2}} = \frac{\mu_0 I}{\pi a\sqrt{2}}$$

The magnetic field due to all sides is in same direction perpendicular to plane of the paper downward. Therefore, total magnetic field at the centre of the square is

$$B = 4B_1 = \frac{4\mu_0 I}{\pi a\sqrt{2}} = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}$$

2. A semi-circular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of magnetic field at the centre of the arc. *[CBSE Delhi 2002]*

Sol. The magnetic field due to a semi-circular arc of radius 'r' carrying current (I) at centre is given by

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\,\Delta l \sin 90°}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\Delta l}{r^2}$$

The net magnetic field due to whole length of arc l will be

$$B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\Sigma\,\Delta l$$

For semi-circular arc $\Sigma\,\Delta l = \pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}(\pi r) = \frac{\mu_0 I}{4r}$$

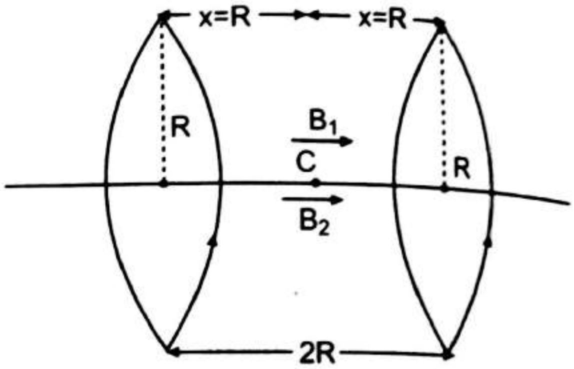
Given $I = 10 \text{ A}, r = 20 \text{cm} = 0{\cdot}20 \text{ m}$

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 0{\cdot}20} = \frac{4 \times 3{\cdot}14 \times 10^{-7} \times 10}{4 \times 0{\cdot}20} = 1{\cdot}57 \times 10^{-5} \text{ T}$$

3. Two parallel coaxial circular coils of equal radius '*R*' and equal number of turns '*N*', carry equal currents '*I*' in the same direction and are separated by a distance '2*R*'. Find the magnitude and direction of the net magnetic field produced at the mid-point of the line joining their centres.

Sol. Magnetic field due to a circular coil of radius '*R*' at a distance x from centre is

$$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$$



Here $x = R$

$$\therefore \; B = \frac{\mu_0 NIR^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 NIR^2}{2.2\sqrt{2}R^3} = \frac{\mu_0 NI}{4\sqrt{2}R}$$

Total magnetic field at centre C due to both coils

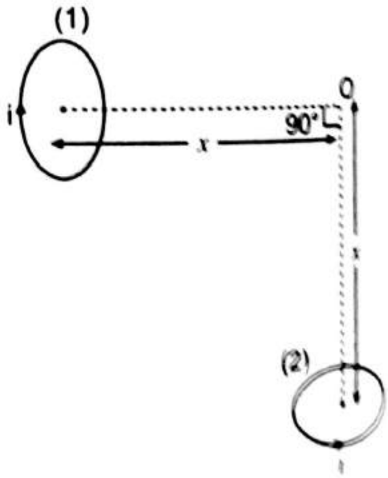
$$B = B_1 + B_2 = 2 \times \frac{\mu_0 NI}{4\sqrt{2}R} = \frac{\mu_0 NI}{2\sqrt{2}R}$$

4. Two small identical circular loops, marked (1) and (2), carrying equal currents, are placed with the geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point *O*.

[CBSE Delhi 2008, 2005, (F) 2013]

Sol. Magnetic field due to coil 1 at point O

$$\vec{B}_1 = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}} \text{ along } \vec{OC_1}$$



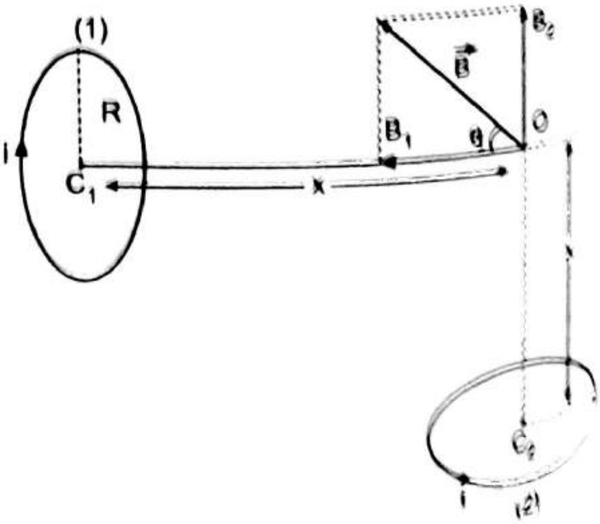
Magnetic field due to coil 2 at point O

$$\vec{B}_2 = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}} \text{ along } \vec{C_2O}$$

Both $\vec{B}_1$ and $\vec{B}_2$ are mutually perpendicular, so the net magnetic field at O is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{2}B_1 \qquad (\text{as } B_1 = B_2)$$

$$= \sqrt{2}\,\frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$$



As $R << x$

$$B = \frac{\sqrt{2}\mu_0 iR^2}{2.x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}.\mu_0 i(\pi R^2)}{x^3}$$

$$= \frac{\mu_0}{4\pi}\,\frac{2\sqrt{2}\,\mu_0 iA}{x^3}$$

where $A = \pi R^2$ is area of loop.

$$\tan\theta = \frac{B_2}{B_1}$$

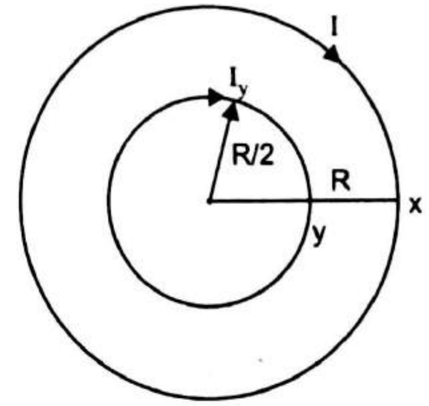
$$\Rightarrow \qquad \tan\theta = 1 \qquad (\because B_2 = B_1)$$

$$\Rightarrow \qquad \theta = \frac{\pi}{4}$$

$\therefore$ $\vec{B}$ is directed at an angle $\frac{\pi}{4}$ with the direction of magnetic field $\vec{B}_1$.

5. Two circular coils X and Y having radii R and $R/2$ respectively are placed in horizontal plane with their centres coinciding with each other. Coil X has current I flowing through it in the clockwise sense. What must be the current in coil Y to make the total magnetic field at the common centre of the two coils, zero?

With the same currents flowing in the two coils, if the coil Y is now lifted vertically upwards through a distance R, what would be the net magnetic field at the centre of coil Y?



Sol. For current at common centre O to be the zero; the current in coil Y must be anticlockwise.

Net magnetic field at centre O is $B_1 - B_2 = 0$.

$$\Rightarrow \qquad B_1 = B_2 \qquad \text{or} \qquad \frac{\mu_0 I}{2R} = \frac{\mu_0 I_y}{2(R/2)}$$

$$\Rightarrow \qquad I_y = \frac{I}{2} \qquad \textit{i.e.,} \qquad \text{current in coil } Y \text{ is } \frac{I}{2} \text{ in anticlockwise direction.}$$

Magnetic field at the centre of coil Y is

$$B_1 = \frac{\mu_0 (I/2)}{2(R/2)} = \frac{\mu_0 I}{2R} \text{ (upward)}$$

Now centre of coil Y is at a distance $x = R$ on the axis of coil X, so magnetic field due to X at the centre of coil Y is $B_2 = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{4\sqrt{2} R^3}$

$$\Rightarrow \qquad B_2 = \frac{\mu_0 I}{4\sqrt{2} R} \qquad \text{(downward)}$$

$\therefore$Net magnetic field, $\quad B = B_1 - B_2$

$$= \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{4\sqrt{2} R} = \frac{\mu_0 I}{R}\left[\frac{1}{2} - \frac{1}{4\sqrt{2}}\right]$$

$$= \frac{\mu_0 I}{R}\left(\frac{2\sqrt{2} - 1}{4\sqrt{2}}\right) = \frac{\mu_0 I}{R}\left(\frac{1 \cdot 828}{5 \cdot 657}\right) = 0 \cdot 323 \frac{\mu_0 I}{R}$$

This net field is in the direction of the field due to the coil Y, *i.e.*, perpendicular to its plane and directed vertically upwards.

6. A rectangular loop of sides 25 cm and 10 cm carrying a current of 15 A is placed with its longer side parallel to a long straight conductor 2·0 cm apart carrying a current of 25 A (fig). What is the net force on the loop? ***[CBSE (AI) 2005]***

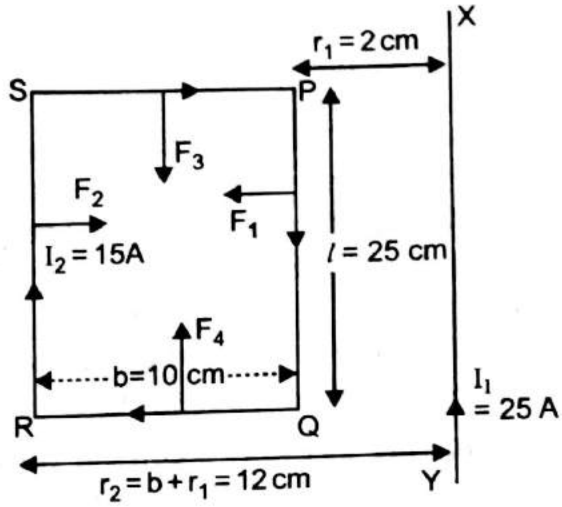
Sol. Rectangular loop $PQRS$ is placed near a long straight wire as shown.

$$l = PQ = RS = 25 \text{ cm} = 0 \cdot 25 \text{ m}$$

$$b = QR = PS = 10 \text{ cm} = 0 \cdot 10 \text{ m}$$

$$r_1 = 2 \text{ cm} = 0 \cdot 02 \text{ m},$$

$$r_2 = 12 \text{ cm} = 0 \cdot 12 \text{ m}$$



The currents in PQ and XY are antiparallel, so PQ is repelled away from wire XY. This repulsive force is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} l = \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0\cdot25}{2\pi \times 0\cdot02} = 9\cdot375 \times 10^{-4} \text{ N}$$

The currents in XY and RS are in the same direction, so wire RS is attracted towards from wire XY. This attractive force is

$$F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0\cdot25}{2\pi \times 0\cdot12} = 1\cdot563 \times 10^{-4} \text{ N}$$

The currents in PS and QR are equal and opposite. By symmetry they exert equal and opposite forces (F_3 and F_4) and hence net force on these sides is zero.

∴ Net force on rectangular loop

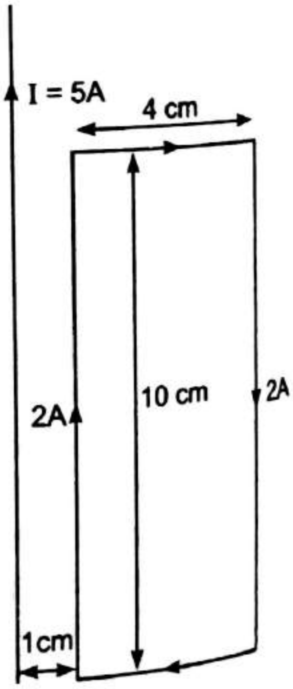
$$F = F_1 - F_2 = 9\cdot375 \times 10^{-4} - 1\cdot563 \times 10^{-4} = \mathbf{7\cdot812 \times 10^{-4}} \textbf{ N} \text{ (repulsive)}$$

The net force is directed away from long wire XY.

7. A rectangular loop of wire of size 4 cm × 10 cm carries a steady current of 2A. A straight long wire carrying 5A current is kept near the loop as shown. If the loop and the wire are coplanar, find

(*i*) the torque acting on the loop and

(*ii*) the magnitude and direction of the force on the loop due to the current carrying wire. ***[CBSE Delhi 2012]***



Sol. (*i*) Torque 'τ' = $MB \sin\theta$ where $\theta = 0°$

Therefore, $\tau = 0$ [∵ As M and B are parallel]

(*ii*) Force acting on the loop

$$|F| = \frac{\mu I_1 I_2}{2\pi} l \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 2 \times 10^{-7} \times 2 \times 5 \times 10^{-1} \left(\frac{1}{10^{-2}} - \frac{1}{5 \times 10^{-2}}\right)$$

$$= \frac{20 \times 10^{-8}}{10^{-2}} \left(1 - \frac{1}{5}\right) \text{N} = 20 \times 10^{-6} \times \frac{4}{5} \text{ N} = \mathbf{1.6 \times 10^{-5}} \textbf{ N}.$$

Direction: Towards the conductor or attractive

8. **Two long and parallel straight wires carrying currents of 2 A and 5 A in the opposite directions are separated by a distance of 1 cm. Find the nature and magnitude of the magnetic force between. them.** ***[CBSE (F) 2011]***

Sol. $I_1 = 2\text{ A}, I_2 = 5\text{ A}, a = 1\text{ cm} = 1 \times 10^{-2}\text{ m}$

Force between two parallel wires per unit length is given by

$$F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{a}$$

$$= 2 \times 10^{-7} \times \frac{2 \times 5}{1 \times 10^{-2}} = \mathbf{20 \times 10^{-5}\ N\ (Repulsive)}$$

9. **A solenoid of length 1·0 m, radius 1 cm and total turns 1000 wound on it, carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron were to move with a speed of 10^4 ms^{-1} along the axis of this current carrying solenoid, what would be the force experienced by this electron?** ***[CBSE Delhi 2008C]***

Sol. Magnetic field inside a solenoid,

$$B = \mu_0 nI$$

$$n = \frac{N}{l} = \frac{1000\text{ turns}}{1 \cdot 0\text{ m}} = 1000\text{ turns/m}$$

$$I = 5\text{ A}$$

$\therefore$ $B = (4\pi \times 10^{-7}) \times 1000 \times 5 = 20 \times 3 \cdot 14 \times 10^{-4}\text{ J} = \mathbf{6 \cdot 28 \times 10^{-3}\ T}$**,** along the axis

Force experienced by electron

$$F_m = qvB \sin\theta$$

Here $q = -e, v = 10^4$ m/s,

$$\theta = \text{angle between } \vec{v} \text{ and } \vec{B} = 0$$

$\therefore$ $F_m = -evB \sin 0° = \mathbf{0\ (zero)}$

10. **A magnetised needle of magnetic moment 4.8×10^{-2} J T^{-1} is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. Calculate the torque acting on the needle.** ***[CBSE (F) 2012]***

Sol. We have, $\tau = M\ B \sin\theta$

where $\tau \rightarrow$ torque acting on magnetic needle

$M \rightarrow$ Magnetic moment

$B \rightarrow$ Magnetic field strength

Then $\tau = 4.8 \times 10^{-2} \times 3 \times 10^{-2} \sin 30°$

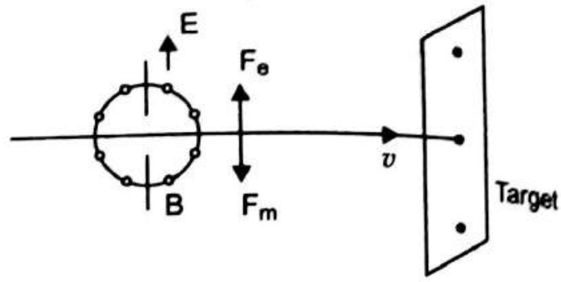
$$= 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \frac{1}{2}$$

$$\tau = 7.2 \times 10^{-4}\text{ Nm}$$

11. **A beam of protons passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of the beam. If the magnitudes of the electric and magnetic fields are 50 kV/m and 100 mT respectively; calculate the**
   - ***(i)*** **velocity v of the beam.**
   - ***(ii)*** **force with which it strikes a target on the screen, if the proton beam current is equal to 0·80 mA.** ***[CBSE (AI) 2008]***

Sol. (*i*) For a beam of charged particles to pass undeflected crossed electric and magnetic fields, the condition is that electric and magnetic forces on the beam must be equal and opposite *i.e.*,

$$eE = evB \quad \Rightarrow \quad v = \frac{E}{B}$$



Given, $E = 50\text{ kV/m} = 50 \times 10^3\text{ V/m}$

$B = 100\text{ mT} = 100 \times 10^{-3}\text{ T}$

$$\therefore \quad v = \frac{50 \times 10^3}{100 \times 10^{-3}} = \mathbf{5 \times 10^5\ ms^{-1}}$$

(*ii*) The beam strikes the target with a constant velocity, so force exerted on the target is **zero.** However, if proton beam comes to rest, it exerts a force on the target, equal to rate of change of linear momentum of the beam *i.e.*,

$$F = \frac{\Delta p}{\Delta t} = \frac{mv}{\Delta t} = \frac{mv}{q/i} = \frac{mvi}{q} = \frac{mvi}{ne}$$

where n is the number of protons striking the target per second.

12. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change?

Sol. Current sensitivity, $S_C = \frac{\theta}{I} = \frac{NAB}{C}$

Voltage sensitivity, $S_V = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{S_C}{R}$

When current sensitivity is increased by 50%, the resistance is made twice.

$\therefore$ New current sensitivity $S_C' = S_C + \frac{50}{100} S_C = 1{\cdot}5\, S_C$

New resistance $R' = 2R$

$\therefore$ New Voltage sensitivity, $S_V' = \frac{S_C'}{R'} = \frac{1{\cdot}5\, S_C}{2R} = 0{\cdot}75\, S_V$

Clearly, $S_V' < S_V$, *i.e.*, voltage sensitivity decreases

% decrease in **voltage sensitivity** $= \frac{S_V - S_V'}{S_V} \times 100\% = \frac{S_V - 0{\cdot}75 S_V}{S_V} \times 100\% = \mathbf{25\%}$

13. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip down at 60° with the horizontal. The horizontal component of the earth's magnetic field at the place is known at to be 0.4 G. Determine the magnitude of the earth's magnetic field at the place. ***[CBSE Delhi 2011]***

Sol. Angle of dip, $\theta = 60°$

$H = 0.4\text{ G} = 0.4 \times 10^{-4}\text{ T}$

If B_e is earth's magnetic field, then

$H = B_e \cos\theta.$

$$\Rightarrow \quad B_e = \frac{H}{\cos\theta} = \frac{0.4 \times 10^{-4}\text{ T}}{\cos 60°} = \frac{0.4 \times 10^{-4}\text{ T}}{0{\cdot}5} = 0.8 \times 10^{-4}\text{ T} = \mathbf{0.8\ G}$$

14. The horizontal component of earth's magnetic field at a given place is $0{\cdot}4 \times 10^{-4}$ Wb/m² and angle of dip is 30°. Calculate the value of (*i*) Vertical component (*ii*) Total intensity of earth's magnetic field. ***[CBSE Delhi 2003]***

Sol. (*i*) Given $H = 0.4 \times 10^{-4}$ Wb/m^2, $\theta = 30°$

$$\tan\theta = \frac{V}{H} \Rightarrow \text{vertical component } V = H\tan\theta$$

$$= 0.4 \times 10^{-4} \times \tan 30°$$

$$= \frac{0.4 \times 10^{-4}}{\sqrt{3}} = \mathbf{0.23 \times 10^{-4}\ Wb/m^2}$$

(*ii*) Total intensity of earth's magnetic field

$$B_e = \sqrt{H^2 + V^2} = \sqrt{(0.4 \times 10^{-4})^2 + \left(\frac{0.4 \times 10^{-4}}{\sqrt{3}}\right)^2} = \mathbf{0.46 \times 10^{-4}\ Wb/m^2}$$

15. A bar magnet of magnetic moment $1.5\ JT^{-1}$ lies aligned with the direction of a uniform magnetic field of 0·22 T. Calculate the amount of work done to turn the magnet so as to align its magnetic moment (i) normal to field direction and (ii) opposite to field direction.

[CBSE (AI) 2003]

Sol. (*i*) Given $M = 1.5$ J/T, $B = 0.22$ T

Work $W = MB(\cos\theta_1 - \cos\theta_2)$

Here $\theta_1 = 0°, \theta_2 = 90°$

$\therefore$ $W = MB(\cos 0 - \cos 90°) = MB$

(*ii*) Here $\theta_1 = 0°, \theta = \pi$

$W = MB(\cos 0 - \cos\pi) = 2MB$

$= 2 \times 1.5 \times 0.22 = \mathbf{0.66\ J}$

16. A galvanometer has a resistance 30 Ω and gives a full scale deflection for a current of 2 mA. How much resistance and in what way must be connected to convert it into:

(*i*) an ammeter of range 0·3 A **(*ii*) a voltmeter of range 0·2 V** *[CBSE (AI) 2007]*

Sol. Here, $G = 30\,\Omega$, $I_g = 2$ mA $= 2 \times 10^{-3}$ A

(*i*) $I = 0.3$ A

$\therefore$ **Shunt required,** $S = \dfrac{GI_g}{I - I_g} = \dfrac{30 \times 2 \times 10^{-3}}{0.3 - 2 \times 10^{-3}} = \mathbf{0.2\,\Omega}$

(*ii*) Here $V = 0.2$ V

$\therefore$ **Series resistance,** $R = \left(\dfrac{V}{I_g} - G\right) = \left(\dfrac{0.2}{2 \times 10^{-3}} - 30\right) = \mathbf{70\,\Omega}$

17. A galvanometer coil of 50 Ω resistance shows full scale deflection for a current of 5 mA. How will you convert this galvanometer into a voltmeter of range 0 to 15 V? *[CBSE (F) 2011]*

Sol. $G = 50\,\Omega$

$I_g = 5\ mA = 5 \times 10^{-3}$ A

$V = 15$ V

The galvanometer can be converted into a voltmeter when a high resistance R is connected in series with it.

Value of R is given by:

$$R = \frac{V}{I_g} - G = \frac{15}{5 \times 10^{-3}} - 50 = 3000 - 50 = 2950\,\Omega.$$

$R = \mathbf{2.95\ k\Omega}$.