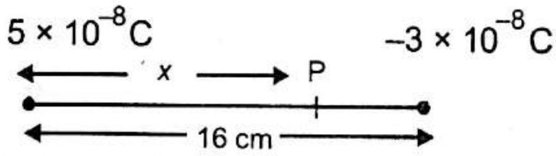
Electrostatic Potential and Capacitance

TEXTBOOK Questions

2.1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans.

- 

  Let potential be zero at P, x cm away from the positive charge. Then

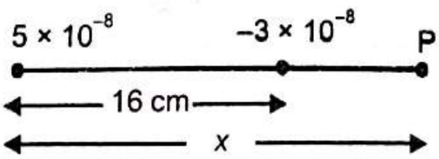
  $$\frac{k \times 5 \times 10^{-8}}{x} = \frac{k \times 3 \times 10^{-8}}{16 - x}$$

  or $\quad 80 - 5x = 3x$

  $\therefore \quad 8x = 80 \quad$ or $\quad x = 10$ cm.

- It is also possible that P lies x distance away from the positive charge on the R.H.S. of negative charge, then

  $$\frac{k \times 5 \times 10^{-8}}{x} = \frac{k \times 3 \times 10^{-8}}{x - 16}$$

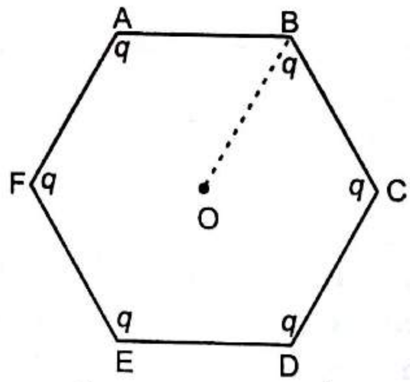
  

  or $\quad 5(x - 16) = 3x$

  $$5x - 80 = 3x$$
  $$2x = 80$$
  $$\therefore \quad x = 40 \text{ cm.}$$

2.2. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans. The given figure shows six equal amount of charges, each $q = 5\ \mu$C at the vertices of a regular hexagon.



So, $q = 5\ \mu\text{C} = 5 \times 10^{-6}$ C

Side of the hexagon, l = AB = BC = CD = DE = EF = FA = 10 cm

Distance of each vertex from centre O, d = 10 cm = 0.1 m

Total electric potential at point O,

$$V = \frac{6 \times q}{4\pi\varepsilon_0 d}$$

where, ε_0 = Permittivity of free space

We know that $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^{2}C^{-2}

$$\therefore \quad V = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1}$$
$$= 2.7 \times 10^6 \text{ V}$$

Therefore, the potential at the centre of the hexagon is 2.7×10^6 V.

2.3. Two charge 2 µC and –2 µC are placed at points *A* and *B*, 6 cm apart.

(*a*) Identify an equipotential surface of the system.

(*b*) What is the direction of the electric field at every point on this surface?

Ans. (*a*) The system represents an electric dipole of charge strength 2 µC and length 0.06 m, then electric dipole moment = 0.12×10^{-6} Cm.

Using, $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

(*i*) For points on axial line, $\theta = 0°$, $\cos\theta = 1$.

$$V \propto 1/r^2$$

The equipotential surfaces are spherical with centre at the centre of the dipole.

(*ii*) For points on equatorial line,

$$\theta = 90°, \cos\theta = \text{zero}.$$

The equatorial plane is a plane of zero potential.

(*b*) Direction of electric field at every point on the equipotential surface is normal to the surface.

2.4. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field.

(*a*) inside the sphere,

(*b*) just outside the sphere,

(*c*) at a point 18 cm from the centre of the sphere?

Ans. (*a*) Since charge resides only on the outer surface of the conductor, electric field inside the conductor is zero.

(*b*) $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$
$$E = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2}$$
$$= 10^5 \text{ NC}^{-1}$$

(*c*) $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$
$$E = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2}$$
$$= 4.44 \times 10^4 \text{ NC}^{-1}$$

2.5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1 pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Ans. Given, $C = 8$ pF $= 8 \times 10^{-12}$ F

$$C_0 = \frac{\varepsilon_0 A}{d}$$

If $d' = \frac{d}{2}$ and $K = 6$

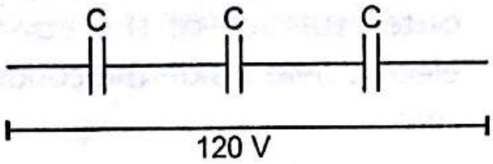
$$C = 12\frac{\varepsilon_0 A}{d} \qquad [\because C = KC_0]$$
$$= 12\, C_0 = 12 \times 8 \times 10^{-12}$$
$$= 96 \times 10^{-12} \text{ F}$$
$$= 96 \text{ pF}$$

2.6. Three capacitors each of capacitance 9 pF are connected in series.

(*a*) What is the total capacitance of the combination?

(*b*) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Ans. Given, $C = 9$ pF



(*a*)
$$\frac{1}{C_t} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$
$$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$
$$C_t = 3 \text{ pF}$$

(*b*)
$$V_t = V_1 + V_2 + V_3$$

As all the capacitors are of equal value

$$V_1 = V_2 = V_3$$

i.e. $3V = V_t = 120$

$\Rightarrow$ $V = 40$ V

2.7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(*a*) What is the total capacitance of the combination?

(*b*) Determine ... :harge on each capacitor i ... :ombination is connected tc ...) V supply.

Ans. $C_1 = 2$ pF, $C_2 = 3$ ... $= 4$ pF

(*a*) $C_p = C_1 + C_2 +$ ... 9 pF

(*b*) $V = 100$ V. In ... llel, potential difference is s... across each capacitor. Thus,

$$Q_1 = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10} \text{ C}$$
$$Q_2 = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10} \text{ C}$$
$$Q_3 = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10} \text{ C}$$

2.8. In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10^{-3} m² and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Ans. $A = 6 \times 10^{-3}$ m²

$d = 3$ mm $= 3 \times 10^{-3}$ m

$$C_0 = \frac{\varepsilon_0 A}{d}$$
$$= \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$
$$= 17.70 \times 10^{-12} \simeq 18 \text{ pF}$$
$$Q = C_0 V = 18 \times 10^{-12} \times 100$$
$$= 1.8 \times 10^{-9} \text{ C}$$

2.9. Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

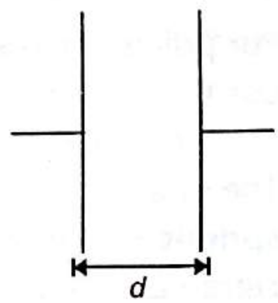
(*a*) while the voltage supply remained connected.

(*b*) after the supply was disconnected.

Ans. $C = 18$ pF, $V = 100$ volt, $t = 3$ mm,

$K = 6$, $d = 3$ mm

(*a*)
$$C_1 = 6 \times 18 \text{ pF} = 108 \text{ pF}$$
$$Q_1 = 108 \times 10^{-12} \times 100$$
$$= 1.08 \times 10^{-8} \text{ C}$$
$$V_1 = 100 \text{ V}$$



(*b*) $Q_2 = Q = 1.8 \times 10^{-9}$ C

(from ans. 2.8)

$$C_2 = 108 \text{ pF}$$
$$V_2 = \frac{100}{6} = 16.6 \text{ V}$$

2.10. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Ans. Use the relation

$$E_n = \frac{1}{2} C V^2$$
$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$
$$= 1.5 \times 10^{-8} \text{ J}$$

2.11. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

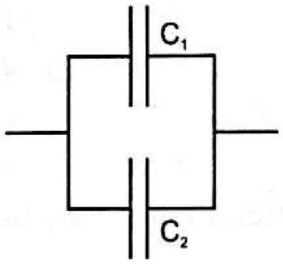
Ans.

$$C_1 = 600 \times 10^{-12} \text{ F}$$

$$V = 200 \text{ V}$$

$$Q = 600 \times 10^{-12} \times 200$$

$$= 12 \times 10^{-8} \text{ C}$$



Charge is same

$$C_1V = (C_1 + C_2)\, V'$$

$$V' = \frac{C_1V}{C_1 + C_2} = \frac{6 \times 10^{-10} \times 200}{12 \times 10^{-10}}$$

$$= \frac{12 \times 10^{-8}}{12 \times 10^{-10}} = 10^2 \text{ volt}$$

$$E_{ni} = \frac{1}{2}C_1V^2$$

$$= \frac{1}{2} \times 600 \times 10^{-12} \times 4 \times 10^4$$

$$= 12 \times 10^{-6} \text{ J}$$

$$E_{nf} = \frac{1}{2}(C_1 + C_2)\, V'^2$$

$$= \frac{1}{2}(12 \times 10^{-10})\, 10^4$$

$$= 6 \times 10^{-6} \text{ J}$$

$$\Delta E = (12 - 6) \times 10^{-6} \text{ J}$$

$$= 6 \times 10^{-6} \text{ J}$$

So, energy lost is 6×10^{-6} J.

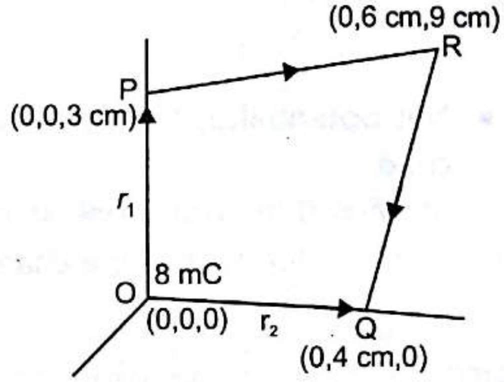
2.12. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Ans. Here, $q = 8 \times 10^{-3}$ C

$$q_o = -2 \times 10^{-9} \text{ C}$$

$$P.E._P = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r_1}$$

$$P.E._Q = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r_2}$$



$$W = \frac{q\,q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

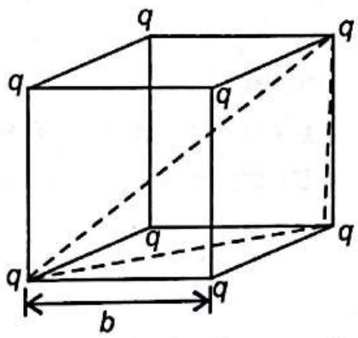
$$= -8 \times 10^{-3} \times 2 \times 10^{-9} \times 9 \times 10^9 \left[\frac{1}{0.04} - \frac{1}{0.03}\right]$$

$$W = 9 \times 10^9 \times 8 \times 10^{-3} \times 2 \times 10^{-9}\left[\frac{1}{0.03} - \frac{1}{0.04}\right]$$

$$W = 1.2 \text{ J}$$

2.13. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Ans.



$$d = \text{Diagonal of one of the face.}$$

$$d^2 = \sqrt{b^2 + b^2}$$

$$d = b\sqrt{2}$$

$$l = \text{length of diagonal of cube}$$

$$l^2 = \sqrt{d^2 + b^2}$$
$$= \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$
$$l = b\sqrt{3}$$
$$r = \frac{l}{2} = \frac{\sqrt{3}}{2} b$$

$\therefore$ Geometrical distance of centre of the cube from each vertex.

$$r = \frac{\sqrt{3}}{2} b$$

- Net potential at the centre of the cube

V = 8 × (potential at centre due to a single charge)

$$V = 8 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{\left(\frac{\sqrt{3}b}{2}\right)}$$

$$V = \frac{4q}{\pi\varepsilon_0(\sqrt{3}b)}$$

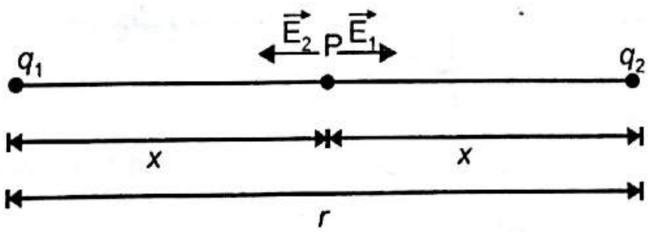
- Due to symmetry of charge around the centre of the cube, net electric field at the point is zero.

2.14. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.

Ans. q_1 = 1.5 μC, q_2 = 2.5 μC, r = 30 cm
x = 15 cm = 15 × 10^{-2} m



(a) (i) Electric field at a mid-point

$$E = E_2 - E_1$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{x^2} - \frac{1}{4\pi\varepsilon_0} \frac{q_1}{x^2}$$

$$= \frac{1}{4\pi\varepsilon_0 x^2}[q_2 - q_1]$$

$$= \frac{9 \times 10^9}{(15 \times 10^{-2})^2}[2.5 - 1.5] \times 10^{-6}$$

$$E = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 10^4}{225}$$

$= 0.04 \times 10^7$ V/m

$E = 4.0 \times 10^5$ Vm^{-1} directed towards charge q_1.

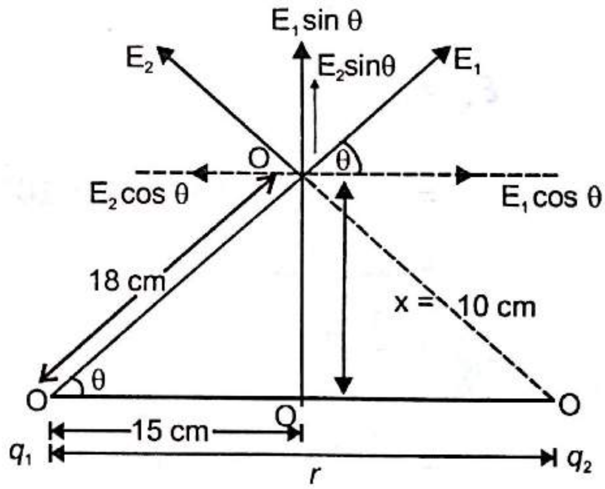
(ii) Potential at mid-point

$$V = \frac{1}{4\pi\varepsilon_0 x}[q_2 + q_1]$$

$$= \frac{9 \times 10^9 \times 4 \times 10^{-6}}{15 \times 10^{-2}}$$

$= 2.4 \times 10^5$ V

(b) (i)



$$E_1 = \frac{q_1}{4\pi\varepsilon_0(18)^2 \times 10^{-4}}$$

$$= \frac{9 \times 10^9 \times 1.5 \times 10^{-6}}{324 \times 10^{-4}}$$

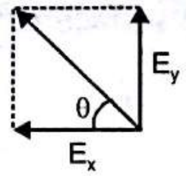
$= 4.17 \times 10^5$ Vm^{-1}

$$E_2 = \frac{9 \times 10^9 \times 2.5 \times 10^{-6}}{324 \times 10^{-4}}$$

$= 6.94 \times 10^5$ Vm^{-1}

$E_Y = (E_1 + E_2) \sin\theta$

$E_X = (E_2 - E_1) \cos\theta$



$$E = \sqrt{E_X{}^2 + E_Y{}^2}$$

$$E_Y = (11.11 \times 10^5) \times \frac{10}{18}$$

$$\left(\because \sin\theta = \frac{10}{18}\right)$$

$$= 6.17 \times 10^5 \text{ Vm}^{-1}$$

$$E_X = (2.77 \times 10^5) \times \frac{15}{18}$$

$$\left(\because \cos\theta = \frac{15}{18}\right)$$

$$= 2.30 \times 10^5 \text{ Vm}^{-1}$$

$$E = 6.58 \times 10^5 \text{ V/m}$$

$$\tan\phi = \frac{E_Y}{E_X} = \frac{6.17 \times 10^5}{2.30 \times 10^5}$$

$$\phi = \tan^{-1}(2.68)$$

$\phi = 69.5°$ with the axial line.

(*ii*) Electric potential

$$V = V_1 + V_2$$

$$V_1 = \frac{q_1}{4\pi\varepsilon_0(18 \times 10^{-2})} = \frac{1.5 \times 10^{-6} \times 9 \times 10^9}{18 \times 10^{-2}} = 7.5 \times 10^4 \text{ V}$$

$$V_2 = \frac{q_2}{4\pi\varepsilon_0(18 \times 10^{-2})} = \frac{2.5 \times 10^{-6} \times 9 \times 10^9}{18 \times 10^{-2}} = 12.5 \times 10^4 \text{ V}$$

$$V = 2.0 \times 10^5 \text{ V}$$

2.15. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(*a*) A charge *q* is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(*b*) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

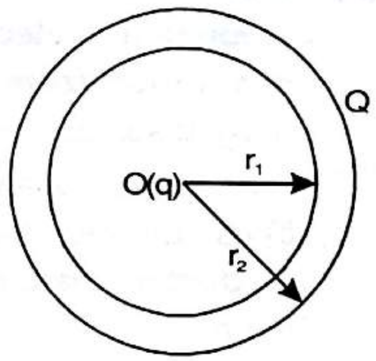
Ans. (*a*) If a charge $+q$ is placed at the centre of the shell, it induces $-q$ charge on the inner surface of the shell and $+q$ charge on the outer surface. Thus, Surface charge density on inner surface is

$$\sigma = -\frac{q}{4\pi r_1{}^2}$$

and on outer surface is

$$\sigma = \frac{Q+q}{4\pi r_2{}^2}$$

(*b*) By Gauss' law, the net charge on the inner surface enclosing the cavity (not having any charge) must be zero.



For a cavity of arbitrary shape, this is not enough to claim that electric field inside must be zero. The cavity may have positive and negative charges with total charge zero.

In order to clear this point, we take a closed loop, part of which is inside the cavity along a field line and remaining part inside the conductor. Since, field inside the conductor is zero. A certain amount of work is done by the field in carrying a test charge over the closed loop.

As this is not possible for an electrostatic field, no field lines are there inside the cavity (*i.e.* no field).

Thus, we conclude that there will be no charge on the inner surface of the conductor, whatever be its shape.

2.16. (*a*) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

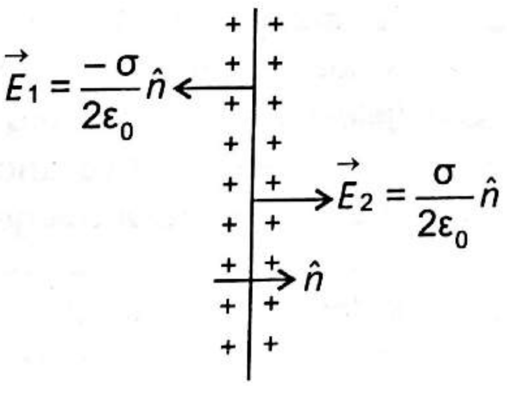
$$\left(\vec{E}_2 - \vec{E}_1\right) . \hat{n} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\sigma \hat{n} / \varepsilon_0$.

(*b*) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

[Hint: For (*a*), use Gauss's law. For, (*b*) use the fact that work done by electrostatic field on a closed loop is zero.]

Ans. (*a*)



Consider a charged surface having surface charge density σ, normal electric field is present on either side of surface but directions of electric fields $\vec{E}_1$ and $\vec{E}_2$ are mutually opposite as shown. Thus field has discontinuity from one side of charged surface to another.

As $\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

and $\vec{E}_1 = -\frac{\sigma}{2\varepsilon_0}\hat{n}$

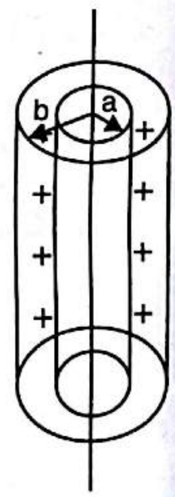
$$\therefore \left(\vec{E}_2 - \vec{E}_1\right)\hat{n} = \frac{\sigma}{\varepsilon_0}$$

This shows electric field just outside the conductor is $\frac{\sigma}{\varepsilon_0}$.

(*b*) When a charged particle is moved from one point to the other in a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

2.17. A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Ans. Imagine a Gaussian surface cylindrical) of radius '*r*' and length *L*, in between the common axis. Charge enclosed by the Gaussian surface is = λ*L*. At each point electric field vector and area vector are in the same direction. Thus,



$$\phi_E = E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

2.18. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(*a*) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite

separation of the electron from proton.

(*b*) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (*a*)?

(*c*) What are the answers to (*a*) and (*b*) above, if the zero of potential energy is taken at 1.06 Å separation?

Ans. $q_1 = -1.6 \times 10^{-19}$ C

$q_2 = +1.6 \times 10^{-19}$ C

$r = 0.53 \times 10^{-10}$ m

(*a*) When zero of potential energy is taken at infinite separation

$$P.E. = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= -\frac{9\times10^9 \times (1.6\times10^{-19})^2}{0.53\times10^{-10}} \text{ J}$$

$P.E. = -27.2$ eV

$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$

(*b*) $$K.E. = \frac{1}{2} P.E. = 13.6 \text{ eV}$$

Total energy $= K.E. + P.E.$

$= 13.6 \text{ eV} + (-27.2) \text{ eV}$

$= -13.6$ eV

Work required to free the electron

$= -(-13.6 \text{ eV}) = 13.6$ eV

(*c*) If the zero of *P.E.* is taken at 1.06 Å separation

P.E. at separation 1.06 Å

$$= -\frac{9\times10^9 \times (1.6\times10^{-19})^2}{1.06\times10^{-10}} \text{ J}$$

$= -13.6$ eV

Now *P.E.* at 0.53 Å w.r.t. the *P.E.* at 1.06 Å is

$= -27.2 - (-13.6) = -13.6$ eV

Required work $= -(-13.6)$

$= 13.6$ eV

2.19. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion (H_2^+). In the ground state of an (H_2^+), the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

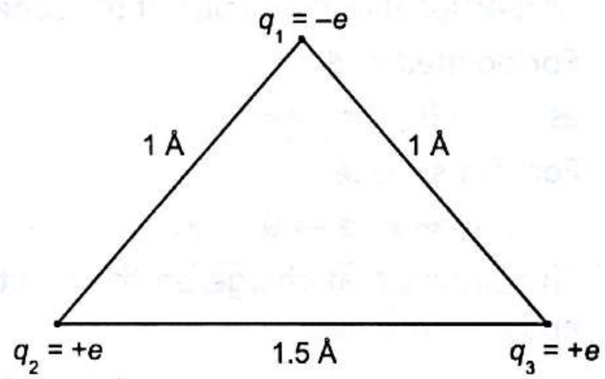
Ans. $q_1 = -1.6 \times 10^{19}$ C

$q_2 = q_3 = 1.6 \times 10^{-19}$ C

$r_{12} = r_{13} = 10^{-10}$ m

$r_{23} = 1.5 \times 10^{-10}$ m

When zero of *P.E.* is taken at infinity,



$$P.E. = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{13}}\right] \times \frac{1}{1.6\times10^{-19}} \text{ eV}$$

On substituting values we get

$P.E. = -19.2$ eV

2.20. Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Ans. Consider two spheres with surface density of charges σ_1 and σ_2 respectively.

Thus, $q_1 = 4\pi a^2 \sigma_1$ and $q_2 = 4\pi b^2 \sigma_2$

Also $V_1 = \frac{q_1}{a}$ and $V_2 = \frac{q_2}{b}$

$$E_1 = \frac{q_1}{a^2} \text{ and } E_2 = \frac{q_2}{b^2}$$

Since $V_1 = V_2$

$$4\pi a\sigma_1 = 4\pi b\sigma_2$$

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$$

i.e., electric field is inversely proportional to the radius of the sphere.

$$\therefore \quad \frac{a}{b} = \frac{\sigma_2}{\sigma_1}$$

i.e., surface charge density is inversely proportional to the radius of the sphere.

For pointed end

as $a \to 0$, $\sigma \to \infty$

For flat surface

$a \to \infty$, $\sigma \to 0$

This proves that charge on the pointed ends is maximum.

2.21. Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively.

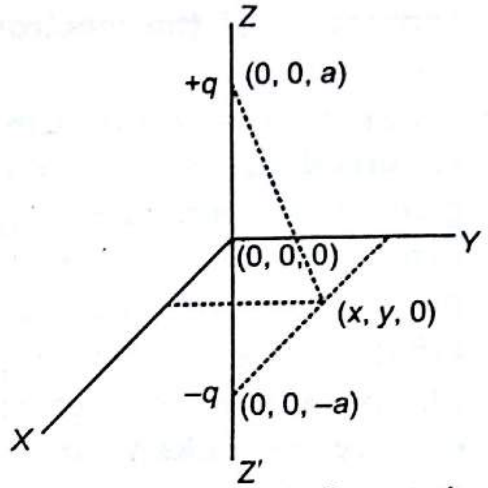
(*a*) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?

(*b*) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(*c*) How much work is done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Ans. Separation between the charges is $l = 2a$

Electric dipole moment is $p = q(2a)$



(*a*) (*i*) Point $(0, 0, z)$ lies on the axial line of the electric dipole. In case the point lies near the charge $+q$. Potential at the point due to charge $+q$ is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-a)}$$

Potential at the point due to charge $-q$ is

$$V_2 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(z+a)}$$

Net potential at the point is

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z-a} - \frac{1}{z+a}\right]$$

$$= \frac{p}{4\pi\varepsilon_0(z^2 - a^2)}$$

$(\because \ p = q \times 2a)$

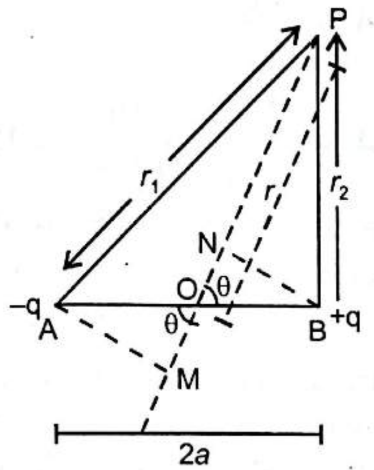
If the point lies near the charge $-q$ then the net potential would be

$$V = -\frac{p}{4\pi\varepsilon_0(z^2 - a^2)}$$

(*ii*) Point $(x, y, 0)$ lies on equitorial plane of the dipole. This point is equidistant from both the charges. (*i.e.*, $S = \sqrt{x^2 + y^2 + a^2}$). So, the potential at the point is zero.

(b) Potential at a point due to an electric dipole

$$OM = ON = a\cos\theta$$
$$r_1 = r + a\cos\theta$$
$$r_2 = r - a\cos\theta$$



Potential at '*P*' due to dipole

$$V = \frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_1}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0\left(r^2 - a^2\cos^2\theta\right)}$$

When $= \frac{r}{a} >> 1$ *i.e.*,

$r >> a$

$\cos\theta \approx 1$

and $V = \frac{p}{4\pi\varepsilon_0 r^2}$, So $V \propto \frac{1}{r^2}$

(c) Both the points being on equipotential surface, the work done would be zero.

No, because work done is independent of the path.

2.22. Figure 2.34 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on *r* for *r*/*a* >> 1, and contrast your results with that due to an electric dipole and an electric monopole (*i.e.*, a single charge).

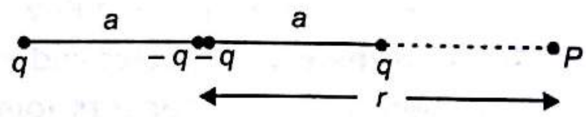


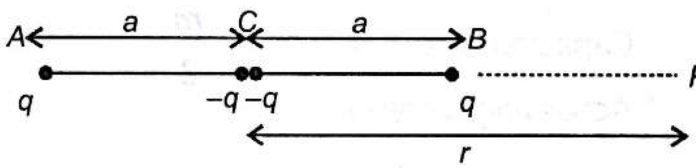
Figure 2.34

Ans.
$$V_{PA} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)}$$

$$V_{PC} = \frac{1}{4\pi\varepsilon_0}\frac{(-2q)}{r}$$

$$\Rightarrow \quad V_{PB} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r-a}$$

$$V = V_{PA} + V_{PC} + V_{PB}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2a^2q}{r(r^2 - a^2)}$$



As $\frac{r}{a} >> 1, r^2 >> a^2$

i.e., $V = \frac{1}{4\pi\varepsilon_0}\frac{2a^2q}{r^3}$

Hence, $V \propto \frac{1}{r^3}$ for quadrupole, for dipole $V \propto \frac{1}{r^2}$ and for monopole $V \propto \frac{1}{r}$.

2.23. An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Ans. Total capacitance required = 2 μF
Potential difference, V = 1000 V
Capacity of capacitors available = 1 μF
Maximum potential difference that a capacitor can withstand = 400 V
Suppose, there are 'm' rows and in each row there are 'n' capacitors joined in series.
Potential across each row = 1000 volt
Potential across each capacitor in a row

$$= \frac{1000}{n}$$

According to the condition

$$\frac{1000}{n} = 400$$
$$n = 2.5$$
or
$$n = 3$$

Total capacitance in a row = $\frac{1}{3}$

Capacitance of 'm' rows = $\frac{m}{3}$

According to requirement

$$\frac{m}{3} = 2 \Rightarrow m = 6$$

Total no. of capacitors required
$$= 3 \times 6 = 18$$

2.24. What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Ans. Capacitance of a parallel capacitor, C = 2F
Distance between the two plates, d = 0.5 cm = 0.5 × 10^{-2} m
Capacitance of a parallel plate capacitor is given by the relation,

$$C = \frac{\varepsilon_0 A}{d}$$

where, ε_0 = Permittivity of free space
$= 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$$A = \frac{Cd}{\varepsilon_0}$$

$$\therefore \quad A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}} = 1130\ km^2$$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is generally taken in the range of μF.

2.25. Obtain the equivalent capacitance of the network in figure 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.

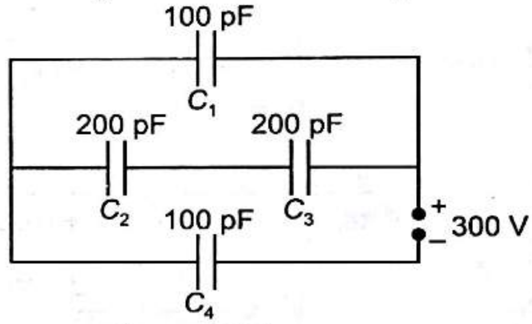


Figure 2.35

Ans. The network of capacitors can be redrawn as shown here with
$C_1 = C_4$ = 100 pF and $C_2 = C_3$ = 200 pF.
Clearly, C_2 and C_3 are in series. So their equivalent capacitance will be

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = 100\text{ pF.}$$

Now C_{23} is in parallel with C_1. So their net capacitance will be

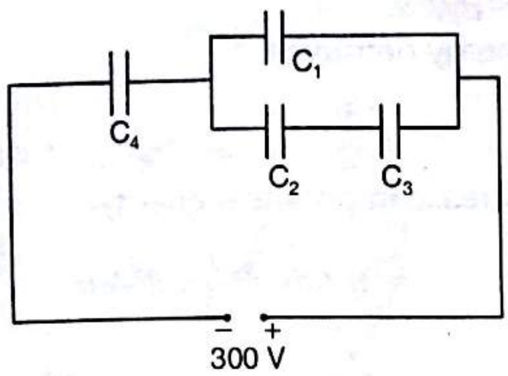
$$C_{123} = C_1 + C_{23} = 200\text{ pF.}$$

Now C_{123} is in series with C_4. So their net capacitance will be

$$C_{1234} = \frac{C_{123}.C_4}{C_{123} + C_4} = \frac{200}{3}\text{ pF}$$

Total charge drawn from the source,

$$q = C_{1234}.$$
$$V = \frac{200}{3} \times 300\text{pC} = 2 \times 10^4\text{pC}$$



This will also be the charge on C_4. So $q_4 = 2 \times 10^4$ pC

Since $C_1 = C_{23}$, therefore, q will divide equally among C_1 and C_{23}. Therefore,

$$q_1 = \frac{q}{2} = 10^4 \text{ pC and } q_2 = q_3 = \frac{q}{2} = 10^4 \text{ pC}$$

(since C_2 and C_3 are in series)

Now, if voltages across C_1, C_2, C_3 and C_4 are V_1, V_2, V_3 and V_4 respectively then

$$V_1 = \frac{q_1}{C_1} = \frac{10^4 \text{pC}}{100 \text{ pF}} = 100 \text{ volt},$$

$$V_2 = \frac{q_2}{C_2} = \frac{10^4 \text{pC}}{200 \text{ pF}} = 50 \text{ volt}$$

$$V_3 = \frac{q_3}{C_3} = \frac{10^4 \text{pC}}{200 \text{ pF}} = 50 \text{ volt},$$

$$V_4 = \frac{q_4}{C_4} = \frac{2 \times 10^4 \text{pC}}{100 \text{ pF}}$$

$$= 200 \text{ volt.}$$

2.26. The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(*a*) How much electrostatic energy is stored by the capacitor?

(*b*) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume *u*. Hence arrive at a relation between *u* and the magnitude of electric field *E* between the plates.

Ans. (*a*) $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$

$d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

$V = 400$ volt

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}}$$

$$C = 3.186 \times 10^{-11} \text{ F}$$

Hence, electrostatic energy,

$$E_n = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 3.186 \times 10^{-11} \times (400)^2$$

$$= 2.55 \times 10^{-6} \text{ J} = 2.55 \text{ μJ}$$

(*b*) Energy density,

$$u = \frac{E_n}{Ad} \quad ...(i)$$

$$u = \frac{2.55 \times 10^{-6}}{90 \times 10^{-4} \times 2.5 \times 10^{-3}}$$

$$u = 0.113 \text{ Jm}^{-3}$$

Relation between energy density (u) and electric field (E):

As we know, $E_n = \frac{1}{2}CV^2$

$$\therefore \quad E_n = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times E^2 d^2$$

$$\left[\because C \frac{\varepsilon_0 A}{d} \text{ and } V = Ed.\right]$$

$$= \frac{1}{2}\varepsilon_0 E^2 Ad$$

$$\frac{E_n}{Ad} = \frac{1}{2}\varepsilon_0 E^2 \quad \text{[From (i)]}$$

i.e. $u = \frac{1}{2}\varepsilon_0 E^2$

2.27. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Ans. Capacitance of charged capacitor,

$$C_1 = 4\mu F = 4 \times 10^{-6} F$$

Supply voltage $V_1 = 200$ V

Energy stored in C_1

$$E_1 = C_1 V_1^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2$$

$$= 8 \times 10^{-2} \text{ J}$$

Capacitance of uncharged capacitor

$$C_2 = 2\mu F = 2 \times 10^{-6} F$$

According to conservation of charge, initial charge of C_1 is equal to final charge on capacitors C_1 and C_2.

$$\therefore \quad V_2(C_1 + C_2) = C_1 V_1.$$

$$V_2 \times (4 + 2) \times 10^{-6} = 4 \times 10^{-6} \times 200$$

$$V_2 = \frac{400}{3} \text{ V.}$$

Energy for combination of two capacitor is given by

$$E_2 = \frac{1}{2}(C_1 + C_2)V_2^2$$

$$= \frac{1}{2}(2 + 4) \times 10^{-6} \times \left(\frac{400}{3}\right)^2$$

$$= 5.33 \times 10^{-2} \text{ J}$$

Amount of energy lost by capacitor

$$C_1 = E_1 = E_2$$

$$= 0.08 - 0.0533$$

$$= 0.0267 = 2.67 \times 10^{-2} \text{ J}$$

2.28. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (1/2) *QE*, where *Q* is the charge on the capacitor, and *E* is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.

Ans. Work done in increasing the separation between the plates by a distance dx against the force of attraction F between the plates is

$$dW = F\,dx \quad \text{...}(i)$$

Energy density,

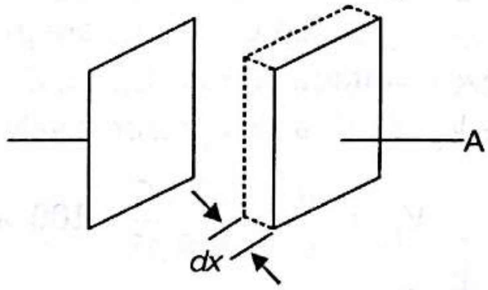
$$u = \frac{1}{2}\varepsilon_0 E^2$$

Increase in potential energy

$$= u\,Adx = \frac{1}{2}\varepsilon_0 E^2 Adx$$

$$= \frac{1}{2}(\varepsilon_0 E)AE\,dx = \frac{1}{2}\sigma AE\,dx \qquad \left(\because E = \frac{\sigma}{\varepsilon_0}\right)$$

$$= \frac{1}{2}QE\,dx \qquad \left(\because \sigma = \frac{Q}{A}\right) \text{...}(ii)$$



As work done is equal to change in potential energy.

Considering equations (*i*) and (*ii*) we get,

$$Fdx = \frac{1}{2}QEdx$$

$$F = \frac{1}{2}QE$$

The factor $\frac{1}{2}$ in the expression is due to the fact that plates are being moved against an average value $\frac{E}{2}$.

2.29. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Figure 2.36). Show that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

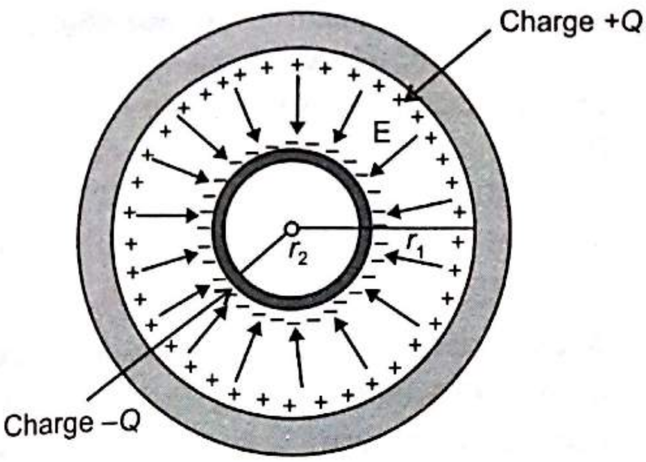


Figure 2.36

Ans. Radius of the outer shell = r_1

Radius of the inner shell = r_2

Charge on the inner surface of the outer shell is $+Q$.

Induced charge on the outer surface of the inner shell is $-Q$.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

where, ε_0 = Permittivity of free space

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}$$

Capacitance of the given system is

$$C = \frac{\text{Charge }(Q)}{\text{Potential difference }(V)}$$

$$= \frac{Q \cdot 4\pi\varepsilon_0 r_1 r_2}{Q(r_1 - r_2)} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

Hence, proved.

2.30. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(*a*) Determine the capacitance of the capacitor.

(*b*) What is the potential of the inner sphere?

(*c*) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Ans. Radius of the outer sphere,

r_1 = 13 cm = 0.13 m

Radius of the inner sphere,

r_2 = 12 cm = 0.12 m

Charge on the inner sphere,

$Q = 2.5\ \mu C = 2.5 \times 10^{-6}$ C

Dielectric constant of a liquid, $\varepsilon_r = 32$

(*a*) Capacitance of this spherical capacitor is given by the relation,

$$C = \frac{4\pi\varepsilon_0 \varepsilon_r r_1 r_2}{r_1 - r_2}$$

where, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$$\therefore \quad C = \frac{32 \times 0.12 \times 0.13}{9 \times 10^9 \times (0.13 - 0.12)}$$

$$\approx 5.5 \times 10^{-9} \text{ F}$$

Hence, the capacitance of the capacitor is 5.5×10^{-9} F.

(*b*) Potential of the inner sphere is given by,

$$V = \frac{Q}{C} = \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}}$$

$$= 4.5 \times 10^2 \text{ V}$$

Hence, the potential of the inner sphere is 4.5×10^2 V.

(*c*) Radius of an isolated sphere,

$r = 12 \times 10^{-2}$ m

Capacitance of the sphere is given by the relation,

$$C' = 4\pi\varepsilon_0 r$$

$$= 4\pi \times 8.85 \times 10^{-12} \times 12 \times 10^{-2}$$

$$= 1.33 \times 10^{-11} \text{ F}$$

The capacitance of the isolated sphere is smaller than the capacitance of concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential difference is less and its capacitance is more than the isolated sphere.

2.31. Answer carefully:

(*a*) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1Q_2/4\pi\varepsilon_0 r^2$, where *r* is the distance between their centres?

(*b*) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(*c*) A small test charge is released from at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(*d*) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(*e*) We know that electric field is discontinous across the surface of a charged conductor. Is electric potential also discontinuous there?

(*f*) What meaning would you give to the capacitance of a single conductor?

(*g*) Guess a possible reason why water has a much greater dielectric constant (=80) than say, mica (=6).

Ans. (*a*) No, because charge distributions on the spheres will not be uniform.

(*b*) No.

(*c*) Not necessarily. (True only if the field line is a straight line.) The field line gives the direction of acceleration, not that of velocity, in general.

(*d*) Zero, no matter what the shape of the complete orbit be.

(*e*) No, potential is continuous.

(*f*) A single conductor is a capacitor with one of the 'plates' at infinity.

(*g*) Because water molecule has permanent dipole moment.

2.32. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (*i.e.*, bending of field lines at the ends).

Ans. Radius of outer cylinder,

r_1 = 1.5 cm = 0.015 m

Radius of inner cylinder,

r_2 = 1.4 cm = 0.014 m

Length of a co-axial cylinder,

l = 15 cm = 0.15 m

Charge given to the inner cylinder,

$Q = 3.5\ \mu C = 3.5 \times 10^{-6}$ C

Capacitance of a co-axial cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi\varepsilon_0 l}{\log_e \frac{r_1}{r_2}}$$

where, ε_0 = Permittivity of free space $= 8.85 \times 10^{-12}\ N^{-1}\ m^{-2}\ C^2$

$$\therefore \quad C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \log_{10}\left(\frac{0.015}{0.014}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299}$$

$= 1.2 \times 10^{-10}$ F

Potential difference of the inner cylinder is given by,

$$V = \frac{Q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}}$$

$= 2.9 \times 10^4$ V

2.33. A parallel plate capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1} (Dielectric strength is the maximum electric field a material can tolerate without breakdown, *i.e.*, without starting to conduct electricity through partial ionisation.). For safety we would like the field never to exceed say, 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Ans. Here $V = 1\text{ kV} = 10^3\text{V}$

$K = 3$

$C = 50\text{ pF} = 50 \times 10^{-12}\text{ F}$

and dielectric strength = $10^7\ Vm^{-1}$

Maximum field

$= 10\%$ of dielectric strength

$= 10\%$ of 10^7

$= 10^6\ Vm^{-1}$

Also $E = \frac{V}{d}$

$\therefore \quad d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}\text{ m}$

Now $C = \frac{KA\varepsilon_0}{d}$

$\Rightarrow \quad A = \frac{Cd}{K\varepsilon_0}$

$= \frac{50\times10^{-12}\times10^{-3}}{3\times8.85\times10^{-12}}$

$= 1.883 \times 10^{-3}\text{ m}^2 \approx 19\text{ cm}^2$

2.34. Describe schematically the equipotential surfaces corresponding to

(*a*) a constant electric field in the z-direction,

(*b*) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(*c*) a single positive charge at the origin, and

(*d*) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Ans. (*a*) Planes parallel to *x–y* plane.

(*b*) Planes parallel to *x–y* plane, except that planes which is differing by a fixed potential get closer as field increases.

(*c*) Concentric spheres with centre at the origin.

(*d*) A periodically varying shape near the grid which gradually reaches the shape of the planes parallel to the grid at far distances.

2.35. In a Van de Graff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is $5 \times 10^7\ Vm^{-1}$. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Ans. $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R},$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \Rightarrow \frac{V}{E} = R$

$R = \frac{15\times10^6}{5\times10^7}$

$= 3 \times 10^{-1}\text{ m} \Rightarrow R = 30\text{ cm}$

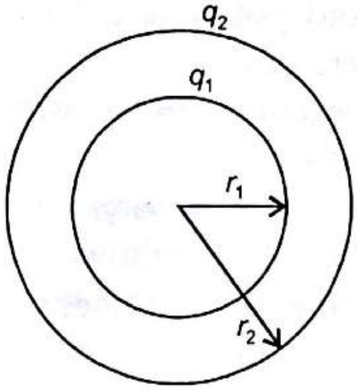
2.36. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when two are connected by a wire) no matter what the charge q_2 on the shell is.

Ans. According to the problem $q_1 > 0$.
Total potential on the sphere is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

Potential on the shell is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$



Potential difference between the sphere and the shell is

$$V = V_1 - V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Which is independent of the charge q_2, *i.e.*, charge on shell.

As q_1 is +ve, potential difference between the sphere and the shell is always positive. So, the charge will always flow from the sphere to the shell.

2.37. Answer the following:

(*a*) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(*b*) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m^2. Will he get an electric shock if he touches the metal sheet next morning?

(*c*) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(*d*) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

(Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = – 10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Ans. (*a*) As we step out of our house, the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential. Hence, we do not get an electric shock.

(*b*) Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminium sheet gradually and raises its voltage. This rise in the voltage depends on the capacitance of the capacitor formed by the aluminium sheet, slab and the ground.

(*c*) The atmosphere is being charged by thunderstorms and lightning occurring around the globe and discharged through the regions of normal weather. On an average, the two opposing currents are in equilibrium and the atmosphere remains electrically neutral.

(*d*) During a lightning, light energy is dissipated and during a thunderstorm, heat energy and sound energy are dissipated in the atmosphere.

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Selected NCERT Exemplar Problems

VSA

1. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Ans. Charge density of smaller sphere will be more than larger one.

SA

2. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

Ans. Suppose this was not true. The potential just inside the surface would be different from the Potential at surface resulting in a potential gradient. This would mean that there are field lines pointing inwards or outwards from the surface. These lines cannot be on the surface again at the other end, since the surface is equipotential. Thus, this is possible only if the other end of the lines are at charges inside, contradicting the premise. Hence, the entire volume inside must be at the same potential.

3. A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Ans. Capacitance will decrease.

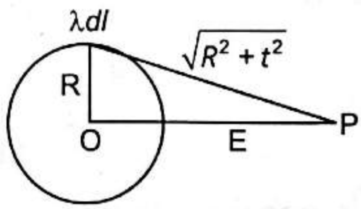
Energy stored $= \frac{1}{2}CV^2$, and so will increase.

Electric field too will increase. But the charge stored will remain the same.

Voltage will also be increased.

4. Calculate potential on the axis of a ring due to charge Q, uniformly distributed along the ring of radius R.

Ans. Suppose linear charge density is 'λ' on the ring. So, potential at a point P and at distant 't' from its centre O due to charge segment



$$dV_P = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{\sqrt{R^2+t^2}}$$

$$V_P = \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{2\pi R}{\sqrt{R^2+t^2}}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{R^2+t^2}}$$

5. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $\alpha = 2V^{-1}$. A similar capacitor with no dielectric is charged to U_0 = 78 V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.

Ans. Let the final voltage be U. If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor will be

$$Q_1 = CU$$

The capacitor with the dielectric has a capacitance εC. So, the charge on the capacitor is given as

$$Q_2 = \varepsilon U = \alpha C U^2$$

The initial charge on the capacitor that was charged is given as

$$Q_0 = CU_0$$

From the conservation of charges, we have

$$Q_0 = Q_1 + Q_2$$

or, $CU_0 = CU + \alpha CU^2$

$\Rightarrow \alpha U^2 + U - U_0 = 0$

$$\therefore \quad U = \frac{-1 \pm \sqrt{1+4\alpha U_0}}{2\alpha}$$

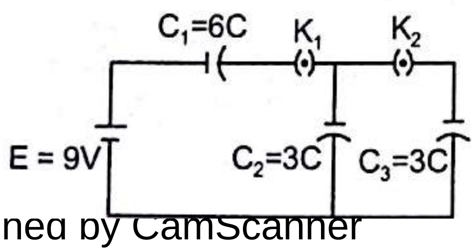
$$= \frac{-1 \pm \sqrt{1+624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} \text{ volts}$$

As U is positive, then

$$U = \frac{\sqrt{625}-1}{4} = \frac{24}{4} = 6 \text{ V}$$

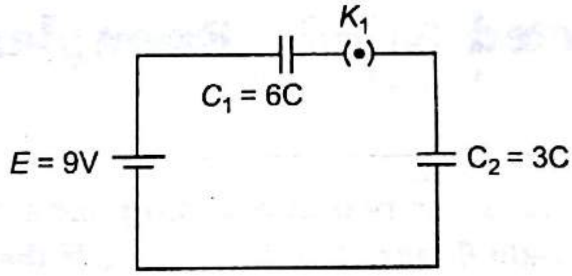
6. In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors?



Then K_1 was opened and K_2 was closed (order is important), What will be the charge on each capacitor now ? [C = 1μF]

Ans. When K_1 is closed and K_2 is open, circuit becomes

Equivalent capacitance



$$C' = \frac{C_1 C_2}{C_1 + C_2}$$

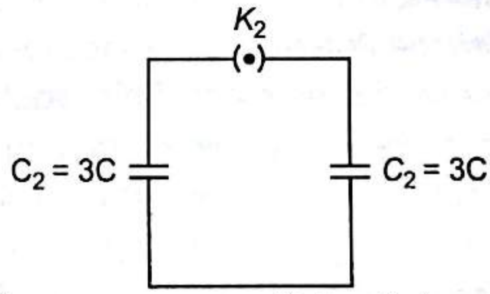
$$C' = \frac{18C^2}{9C} = 2\,C$$

Charge drawn by the circuit $Q' = C'E = (2C)9 = 18$ C

As the capacitance C_1 and C_2 are in series, charge stored by them will be

$$Q'_1 = 18 \text{ C}; \; Q'_2 = 18 \text{ C}$$

When K_1 is opened and K_2 is closed. Supply get disconnected and then we have the circuit as



Common potential attained by the capacitors is

$$V' = \frac{Q'_2}{6C} = \frac{18\text{ C}}{6\text{ C}}$$

Now charge on capacitor C_2, $Q''_2 = 3$ C × 3 = 9 C

On capacitor C_3, $Q'_3 = 3$ C × 3 = 9 C

On capacitor C_1, $Q''_1 = 18$ C