Previous Years' Numericals

Coulomb's Law and System of Charges

1. Two identical metallic spheres, having unequal opposite charges are placed at a distance of 0.50 m apart in air. After bringing them in contact with each other, they are again placed at the same distance apart. Now the force of repulsion between them is 0.108 N. Calculate the final charge on each of them. ***[CBSE Delhi 2002]***

Sol. When identical spheres are brought in contact, then after separation they carry equal charges. Let q be the charge on each sphere, then Coulomb's force of repulsion between them will be

$$F = \frac{1}{4\pi\varepsilon_0} \frac{(q)(q)}{r^2}$$

Here $F = 0{\cdot}108$ N, $r = 0{\cdot}50$ m, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{C^2}$

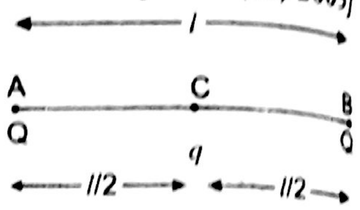
$\therefore \quad 0.108 = 9 \times 10^9 \dfrac{q^2}{(0.50)^2}$

$\Rightarrow \quad q^2 = \dfrac{0.108 \times (0.50)^2}{9 \times 10^9} = 3 \times 10^{-12}$

$q = \sqrt{3 \times 10^{-12}} = 1.732 \times 10^{-6}\ \text{C} = 1.732\ \mu\text{C}.$

2. A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium of $q = -\dfrac{Q}{4}$. *[CBSE (AI) 2005]*

Sol. Charge q is in equilibrium since charges A and B exert equal and opposite forces on it.



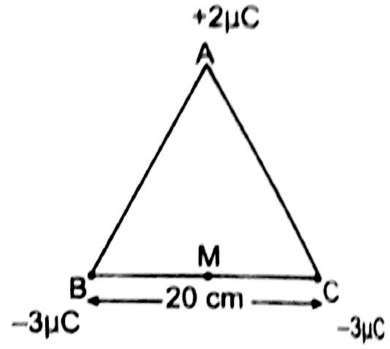
For equilibrium of charge Q at B:

$$F_{BC} + F_{AB} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(l/2)^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q.Q}{l^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{Q}{l^2}(4q + Q) = 0 \quad \Rightarrow \quad q = -\frac{Q}{4}$$

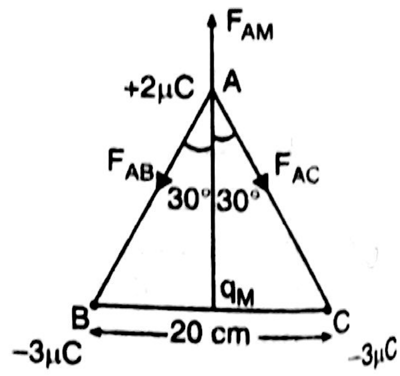
3. Three point charges of + 2 μC, – 3 μC and – 3 μC are kept at the vertices *A*, *B* and *C* respectively of an equilateral triangle of side 20 cm as shown in fig. What should be the sign and magnitude of charge to be placed at the mid point *M* of side *BC* so that charge at *A* remains in equilibrium ? *[CBSE Delhi 2005]*



Sol. Let charge placed at M be q_M. The forces acting on charge ($q_A = +2\mu$C) are F_{AB}, F_{AC} and F_{AM} as shown in fig.

$$\vec{F}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r_{AB}^2} \text{ along } \vec{AB}$$

$$= 9 \times 10^9 \times \frac{(2 \times 10^{-6})(3 \times 10^{-6})}{(0.20)^2} \text{ along } \vec{AB} = 1.35 \text{ N along } \vec{AB}$$



$$\vec{F}_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_C}{r_{AC}^2}$$

$$= 9 \times 10^9 \frac{(2 \times 10^{-6})(3 \times 10^{-6})}{(0.20)^2} = 1.35 \text{ N along } \vec{AC}$$

$$\vec{F}_{AM} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_M}{(r_{AM})^2} = 9 \times 10^9 \frac{(2 \times 10^{-6})(q_M)}{(\sqrt{3} \times 10^{-1})^2}$$

$$= 6 \times 10^5\, q_M \text{ N along } \vec{MA}$$

For equilibrium of charge q_A; the resultant of $\vec{F}_{AB}$ and $\vec{F}_{AC}$ must be equal and opposite to $\vec{F}_{AM}$.

i.e., $\quad F_{AB}\cos 30° + F_{AC}\cos 30° = F_{AM}$

$$\Rightarrow \quad 1.35 \times \frac{\sqrt{3}}{2} + 1.35\frac{\sqrt{3}}{2} = 6 \times 10^5\, q_M$$

$$\Rightarrow \quad q_M = \frac{1.35\sqrt{3}}{6\times10^5} = 0.225\sqrt{3}\times10^{-5}\ \text{C} = 2.25\sqrt{3}\ \mu\text{C}$$

4. Two point charges of $+5\times10^{-19}$ C and $+20\times10^{-19}$ C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.

[CBSE Delhi 2001]

Sol. Let charges $q_1 = +5\times10^{-19}$ C and $q_2 = +20\times10^{-19}$ C be placed at A and B respectively. Distance $AB = 2$ m.

As charges are similar, the electric field strength will be zero between the charges on the line joining them. Let P be the point (at a distance x from q_1) at which electric field intensity is zero. Then, $AP = x$ metre, $BP = (2-x)$ metre. The electric field strength at P due to charge q_1 is

$$\vec{\mathbf{E}}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2}, \text{ along the direction } A \text{ to } P.$$

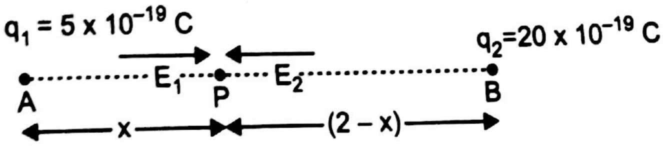
The electric field strength at P due to charge q_2 is

$$\vec{\mathbf{E}}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(2-x)^2}, \text{ along the direction } B \text{ to } P.$$

Clearly, $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ are opposite in direction and for net electric field at P to be zero, $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ must be equal in magnitude.

So, $\quad E_1 = E_2$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(2-x)^2}$$



Given $q_1 = 5\times10^{-19}$ C, $q_2 = 20\times10^{-19}$ C

Therefore, $$\frac{5\times10^{-19}}{x^2} = \frac{20\times10^{-19}}{(2-x)^2}$$

or $\quad \dfrac{1}{2} = \dfrac{x}{2-x} \quad$ or $\quad x = \dfrac{2}{3}$ m

Electric Dipole

5. An electric dipole of length 10 cm having charges 6×10^{-3} C, placed at 30° with respect to a uniform electric field, experience a torque of magnitude $6\sqrt{3}$ Nm. Calculate (*i*) magnitude of electric field and (*ii*) potential energy of electric dipole.

[CBSE Delhi 2006C]

Sol. Given, $q = 6\times10^{-3}$ C, $2l = 10$ cm $= 0.10$ m, $\theta = 30°$, $\tau = 6\sqrt{3}$ Nm.

(*i*) Electric dipole moment

$$p = q.(2l)$$
$$= 6\times10^{-3}\times0.10 = 6\times10^{-4}\ \text{Cm}$$

Torque, $\tau = pE\sin\theta$

$\Rightarrow$ Electric field strength, $E = \dfrac{\tau}{p\sin\theta}$

$$= \frac{6\sqrt{3}}{6\times10^{-4}\times\sin 30°}\ \text{NC}^{-1}$$

$$= \frac{6\sqrt{3}}{6 \times 10^{-4} \times 0.5} \text{ NC}^{-1} = 2\sqrt{3} \times 10^{4} \text{ NC}^{-1}$$

(*ii*) Potential energy of dipole

$$U = -pE\cos\theta$$

$$= -6 \times 10^{-4} \times 2\sqrt{3} \times 10^{4} \times \cos 30°$$

$$= -6 \times 10^{-4} \times 2\sqrt{3} \times 10^{4} \times \left(\frac{\sqrt{3}}{2}\right) = -\mathbf{18\ J}$$

6. **Calculate the amount of work done in rotating a dipole, of dipole moment 3×10^{-8} cm, from its position of stable equilibrium to the position of unstable equilibrium, in a uniform electric field of intensity 10^4 N/C.** *[CBSE (F) 2011]*

Sol. $P = 3 \times 10^{-8}$ cm ; $E = 10^4$ N/C

At stable equilibrium $(\theta_1) = 0°$

At unstable equilibrium $(\theta_2) = 180°$

Work done in rotating dipole is given by :

$$W = PE(\cos\theta_1 - \cos\theta_2)$$

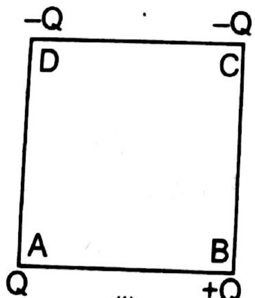
$$= (3 \times 10^{-8})(10^4)[\cos 0° - \cos 180°]$$

$$= 3 \times 10^{-4}[1-(-1)]$$

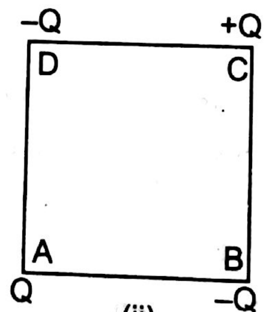
$$W = 6 \times 10^{-8} \text{ J}$$

Electric Potential and Potential Energy

7. **Four point charges are placed at the four corners of a square in the two ways (*i*) and (*ii*) as shown below. Will the (*i*) electric field (*ii*) Electric potential, at the centre of the square, be the same or different in the two configurations and why?**

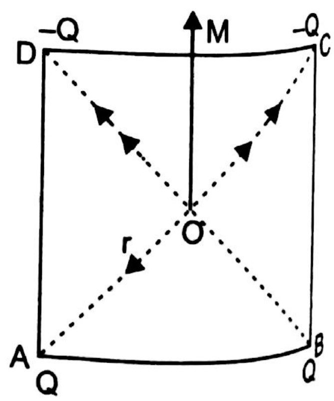


(i)



(ii)

Sol. (*i*) Electric field is a vector quantity. The electric field in first case will be finite and directed along *OM*, while electric field in second case is zero, because in second case it is cancelled due to charge (*Q*, *Q*) at *A* and *C* and (– *Q*, – *Q*) at *B* and *D*.



(*ii*) The electric potential is zero in both cases. In first case

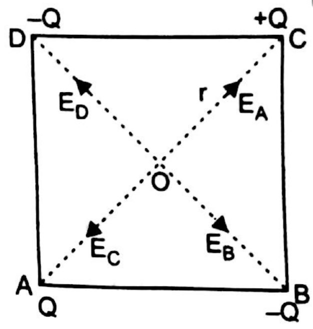
$$V_I = V_A + V_B + V_C + V_D$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} + \frac{Q}{r} - \frac{Q}{r} - \frac{Q}{r}\right] = 0$$

In second case,

$$V_{II} = V_A + V_B + V_C + V_D$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} - \frac{Q}{r} + \frac{Q}{r} - \frac{Q}{r}\right] = 0$$



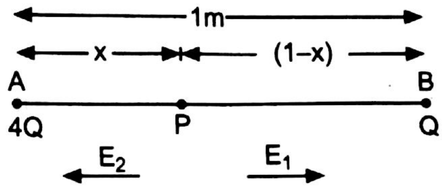
8. *(i)* **Two point charges 4*Q* and *Q* are separated by a distance 1 m in air. At what point on the line joining the charges is the electric field intensity zero ?**

(ii) **Also calculate the electrostatic potential energy of the charge if $Q = 2 \times 10^{-7}$ C.**

[CBSE (AI) 2008C]

Sol. *(i)* Let x be the distance of point P from charge $4Q$ where electric field intensity is zero, then

$$\vec{E} = \vec{E_1} + \vec{E_2} = 0$$



As E_1 and E_2 are oppositely directed.

In magnitude, $E_1 = E_2$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{4Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(1-x)^2} \quad \Rightarrow$$

$$\frac{4}{x^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow \quad x^2 = 4(1-x)^2 \quad \Rightarrow \quad x = \pm 2(1-x)$$

If we take (+) sign $x = 2(1-x)$.

This gives $\quad x = \frac{2}{3}$ m $=$ **66·7 cm**

If we take (–) sign, $x = -2(1-x) \quad \Rightarrow \quad x = \mathbf{2\ m}$

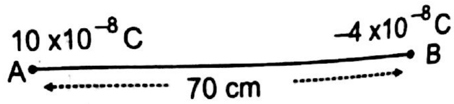
(ii) Electrostatic potential energy, $\quad U = \frac{1}{4\pi\varepsilon_0}\frac{(4Q)(Q)}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q^2}{r^2}$

$$= 9 \times 10^9 \times \frac{4 \times (2 \times 10^{-7})^2}{(1)^2} = \mathbf{1{\cdot}44 \times 10^{-3}\ J}$$

9. Two point charges 10×10^{-8} C and -4×10^{-8} C are separated by a distance of 70 cm in air as shown in figure.

(i) **Find at what distance from point *A* would the electric potential be zero.**

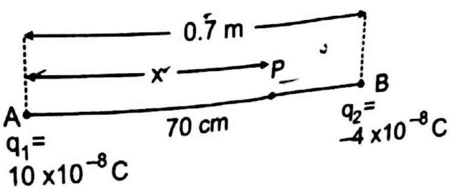
(ii) **Also calculate the electrostatic potential energy of the system.** ***[CBSE (AI) 2008C, 2004]***



Sol. *(i)* **Key idea:** The electric potential due to opposite charges is subtractive. To find the electric potential, the sign of the charge is retained in the formula.

Let x be the distance of point P from A at which q electric potential is zero

i.e., $\quad V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0{\cdot}70 - x)} = 0$

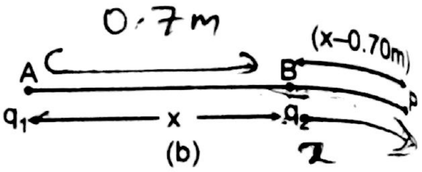


(a)

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{10 \times 10^{-8}}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-4 \times 10^{-8})}{(0{\cdot}70 - x)} = 0$$

$$x = \frac{17.0}{14} = 0{\cdot}5\ \text{m}$$

Also $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x} + \frac{q_2}{x - 0.70}\right) = 0$



$x = \frac{7}{6}$ m $= 1.167$ m

Thus electric potential is zero at distances $x = 0.5$ m and 1.167 m from charge $q_1 = 10 \times 10^{-8}$ C.

(*ii*) Electrostatic potential energy of system

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9 \times 10^9 \times \frac{(10 \times 10^{-8})(-4 \times 10^{-8})}{0.70} = 5.14 \times 10^{-5} \text{ J}$$

10. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown

$$q = 1.6 \times 10^{-10} \text{ C}$$

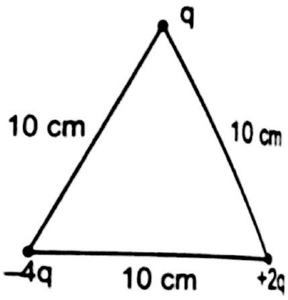
[CBSE Delhi 2008]

Sol. Potential energy of system *i.e.,* work done to assemble the system of charges

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q.(-4q)}{0.10} + \frac{q.(2q)}{0.10} + \frac{(-4q).(2q)}{0.10}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-10q^2}{0.10}\right] = -\frac{1}{4\pi\varepsilon_0}(100q^2)$$

$$= -9 \times 10^9 \times 100 \times (1.6 \times 10^{-10})^2 = -2.3 \times 10^{-8} \text{ J}$$



Work done to dissociate the system of charges

$$W = -U = \mathbf{2.3 \times 10^{-8}\ J}$$

11. Two point charges + 10 μC and – 10 μC are separated by a distance of 40 cm in air. Calculate the electrostatic potential energy of the system, assuming zero potential energy at infinity.

[CBSE Delhi 2004]

Sol. Potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9 \times 10^9 \times \frac{(10 \times 10^{-6})(-10 \times 10^{-6})}{2.0 \times 10^{-2}} = -45$ **J**

Electric Flux and Gauss's Theorem

12. A uniformly charged conducting sphere of diameter 2·5 m has a surface charge density 100 μC/m². Calculate

(*i*) charge on the sphere and

(*ii*) total electric flux passing through the sphere.

[CBSE Delhi 2008]

Sol. Given, $\sigma = 100\ \mu\text{C/m}^2 = 100 \times 10^{-6}$ C/m².

Diameter, $D = 2R = 2.5$ m

(*i*) Charge on sphere, $Q = \sigma . 4\pi R^2 = \sigma . \pi(2R)^2$

$$= (100 \times 10^{-6} \text{ C/m}^2) \times 3.14 \times (2.5 \text{ m})^2$$

$$= 19.625 \times 10^{-4} \text{ C}$$

$$= 1.96 \times 10^{-3} \text{ C} = \mathbf{1.96\ mC}$$

(*ii*) Electric flux passing through the sphere

$$\phi = \frac{1}{\varepsilon_0}(Q) = \frac{1}{8.86 \times 10^{-12}} \times (1.96 \times 10^{-3}) = 2.21 \times 10^8 \text{ Nm}^2 \text{ C}^{-1}$$

13. A point charge $17\cdot7\ \mu C$ is located at the centre of the cube of side $0\cdot03$ m. Find the electric flux through each face of cube.

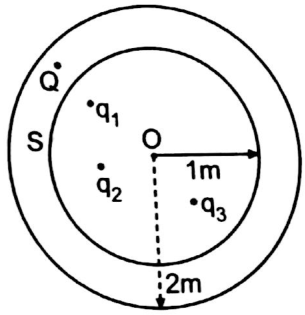
Sol. By Gauss's theorem electric flux through whole cube $= \dfrac{1}{\varepsilon_0} q$

$\therefore$ A cube has six identical faces and charge is placed at centre, so electric flux through each face of cube $= \dfrac{1}{6}\cdot\left(\dfrac{q}{\varepsilon_0}\right)$

$$= \frac{1\times 17\cdot7\times10^{-6}\ C}{6\times 8\cdot85\times 10^{-12}\ C^2\ N^{-1}\ m^{-2}} = \mathbf{3\cdot33\times 10^6\ N\ m^2\ C^{-1}}$$

14. The flux of electrostatic field through the closed spherical surface S' is found to be four times that through the closed spherical surface S. Find the magnitude of the charge Q. Given $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$, $q_3 = 9\cdot84\,\mu C$. *[CBSE Delhi 2004C]*

Sol. Given flux through $S' = 4\times$ flux through S



By Gauss theorem,

Net flux through a surface $= \dfrac{1}{\varepsilon_0}\times$ charge enclosed

Flux through $S' = \dfrac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + Q)$

Flux through $S = \dfrac{1}{\varepsilon_0}(q_1 + q_2 + q_3)$

$\therefore \qquad \dfrac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + Q) = 4\times\dfrac{1}{\varepsilon_0}(q_1 + q_2 + q_3)$

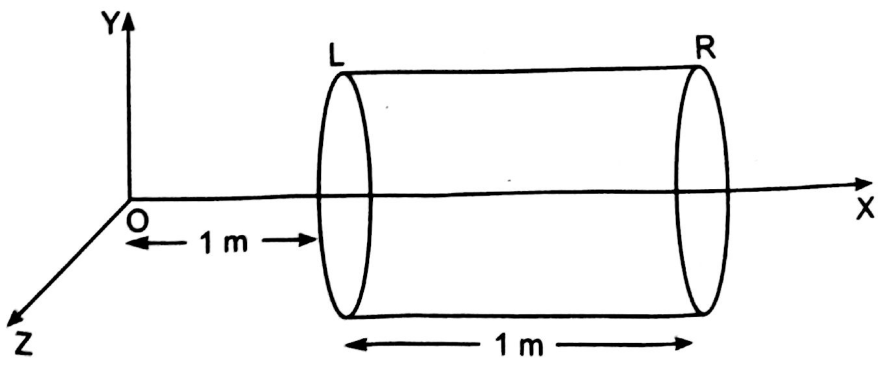
$\Rightarrow \qquad Q = 3(q_1 + q_2 + q_3) = 3(1 - 2 + 9\cdot84)\ \mu C$

$= \mathbf{26\cdot52\ \mu C}$

15. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x\hat{i}$, where E is in NC^{-1} and x is in metres. Find

(i) net flux through the cylinder.

(ii) charge enclosed by the cylinder.



[CBSE Delhi 2013]

Sol. (*i*) Electric flux through a surface $\varphi = \vec{E}.\vec{S}$

Flux through the left surface $\varphi_L = -|E|\,|S|$

$= -50x.\,|S|$

Since $x = 1$m,

$$\varphi_L = -50\times1\times25\times10^{-4}$$
$$= -1250\times10^{-4}$$
$$= -0.125\text{Nm}^2\,\text{C}^{-1}$$

Flux through the right surface

$$\varphi_R = |E|\,|S|$$

Since $x = 2$m,

$$\varphi_R = 50x\,|S|$$
$$= 50\times2\times25\times10^{-4}$$
$$= 2500\times10^{-4}$$
$$= 0.250\text{Nm}^2\,\text{C}^{-1}$$

Net flux through the cylinder

$$\varphi_{net} = \varphi_R + \varphi_L$$
$$= 0.250 - 0.125$$
$$= 0.125\text{Nm}^2\,\text{C}^{-1}$$

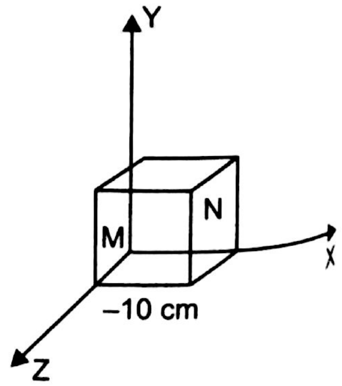
(*ii*) Charge inside the cylinder, by Gauss's Theorem

$$\varphi_{net} = \frac{q}{\varepsilon_0}$$
$$= q = \varepsilon_0\,\varphi_{Net}$$
$$= 8.854\times10^{-12}\times0.125$$
$$= 8.854\times10^{-12}\times\frac{1}{8}$$
$$= 1.107\times10^{-12}\text{ C}$$

16. Electric field in the given figure is directed along + X direction and given by $E_x = 5Ax + 2B$, where E is in NC^{-1} and x is in metre, A and B are constants with dimensions. Taking $A = 10\text{ NC}^{-1}\text{ m}^{-1}$ and $B = 5\text{ NC}^{-1}$, calculate

(*i*) the electric flux through the cube.

(*ii*) net charge enclosed within the cube.



Sol. (*i*) Given $E_x = 5Ax + 2B$

The electric field at face M where $x = 0$ is

$$E_1 = 2B$$

The electric field at face N where $x = 10\text{ cm} = 0{\cdot}10$ m is

$$E_2 = 5A\times0{\cdot}10 + 2B = 0{\cdot}5A + 2B$$

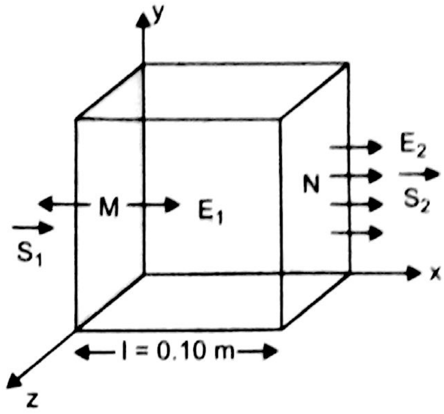
The electric flux through face M is

$$\phi_1 = \vec{E_1} \cdot \vec{S_1} = E_1 S_1 \cos \pi = -E_1 S_1$$

$$= -2B \times l^2 \text{ where } l = 10 \text{ cm} = 0.01 \text{ m}$$

The electric flux through face N

$$\phi_2 = \vec{E_2} \cdot \vec{S_2} = E_2 S_2 \cos 0 = (0.5A + 2B)\, l^2$$



Net electric flux

$$\phi = \phi_1 + \phi_2$$

$$= -2Bl^2 + (0.5A + 2B)\, l^2 = 0.5Al^2$$

$$= 0.5 \times 10 \times (0.10)^2$$

$$= 5 \times 10^{-2} \text{ Vm}$$

(*ii*) If q is net charge enclosed within the cube, then by Gauss's theorem

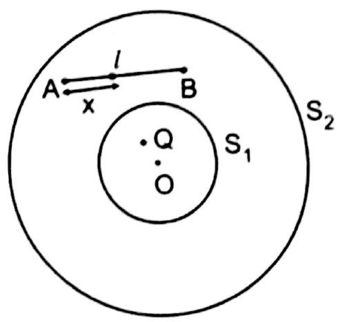
$$\phi = \frac{1}{\varepsilon_0} q$$

$$q = \varepsilon_0 \phi = 8.85 \times 10^{-12} \times 5 \times 10^{-2} \text{ C} = \mathbf{4.425 \times 10^{-13} \text{ C}}$$

17. In the figure shown, calculate the total flux of the electrostatic field through the spheres S_1 and S_2. The wire AB shown here has a linear charge density λ given by $\lambda = kx$ where x is distance measured along the wire, from the end A. ***[CBSE (AI) 2004C]***

Sol. Total charge on wire AB

$$Q_{AB} = \int_0^l \lambda dx = \int_0^l kx\, dx = k\left[\frac{x^2}{2}\right]_0^l = \frac{1}{2} kl^2$$



By Gauss theorem,

Total flux through $S_1 = \dfrac{Q}{\varepsilon_0}$

Total flux through $S_2 = \dfrac{Q + Q_{AB}}{\varepsilon_0} = \left(\dfrac{Q + \frac{1}{2}kl^2}{\varepsilon_0}\right)$

Capacitors

Q. 18. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the plates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of the capacitance of the capacitor in the second case. ***[CBSE (AI) 2006; Delhi 2005]***

Ans. Initial capacitance $C_1 = \dfrac{\varepsilon_0 A}{d} = 8 \text{ pF}$ ...(*i*)

In the second case, the separation is $d' = \dfrac{d}{2}$ and medium between the plates has dielectric constant $K = 5$,

$\therefore$ New capacitance $C_2 = \dfrac{K\varepsilon_0 A}{d'} = \dfrac{5\varepsilon_0 A}{d/2} = \dfrac{10\,\varepsilon_0 A}{d}$

Using (*i*), we have

$$C_2 = 10\left(\frac{\varepsilon_0 A}{d}\right) = 10 \times 8 \text{ pF} = \mathbf{80 \text{ pF}}$$

19. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC.

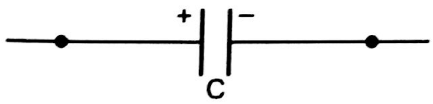
Calculate:

(*i*) The potential V and the unknown capacitance C.

(*ii*) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V?

[CBSE Delhi 2013]

Sol.



(*i*) If unknown capacitor of capacitance 'C' is connected to a battery of 'V' volts,

$$Q = CV$$

$$\Rightarrow \quad CV = 360\,\mu C \qquad (1)$$

On reducing the potential/voltage by 120V

So, $Q' = C(V - 120)$

$$\Rightarrow \quad C(V - 120) = 120\,\mu C \qquad (2)$$

On solving equation (1) and (2)

$$\frac{360\,\mu C}{V} = \frac{120\,\mu C}{V - 120}$$

$$\Rightarrow \quad V = 180V$$

Unknown capacitance from equation (1)

$$Q = CV$$

$$360\,\mu C = C \times 180V$$

$$\Rightarrow \quad C = \frac{360\,\mu C}{180V} = 2$$

$$C = 2\,\mu F$$

(*ii*) Charge on the capacitor, if voltage is increased by 120V

$$Q = C(V + 120)$$

$$= 2(180 + 120)$$

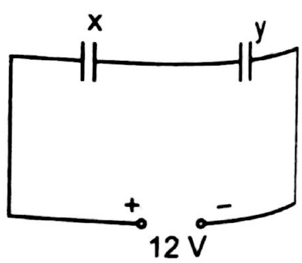
$$Q = 600\,\mu C$$

20. *X* and *Y* are two parallel plate capacitors having the same areas of plates and same separation between the plates. *X* has air between the plates and *Y* contains a dielectric medium $\varepsilon_r = 4$.

(*i*) Calculate the capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(*ii*) Calculate the potential difference between the plates of *X* and *Y*.

(*iii*) What is the ratio of electrostatic energy stored in *X* and *Y*?



[CBSE Delhi 2009]

Sol. (*i*) Capacitance of *X*, $C_X = \dfrac{\varepsilon_0 A}{d}$

Capacitance of Y, $C_Y = \frac{\varepsilon_r \varepsilon_0 A}{d} = 4\frac{\varepsilon_0 A}{d}$

$\therefore \quad \frac{C_Y}{C_X} = 4 \Rightarrow C_Y = 4C_X$ ...(i)

As X and Y are in series, so

$$C_{eq} = \frac{C_X C_Y}{C_X + C_Y} \Rightarrow 4\mu F = \frac{C_X \cdot 4C_X}{C_X + 4C_X} \Rightarrow C_X = 5\,\mu F$$

and $C_Y = 4C_X = 20\,\mu F$

(ii) In series charge on each capacitor is same, so

P.d. $\quad V = \frac{Q}{C} \propto \frac{1}{C}$

$\therefore \quad \frac{V_X}{V_Y} = \frac{C_Y}{C_X} = 4 \Rightarrow V_X = 4V_Y$ ...(ii)

Also $\quad V_X + V_Y = 12$ ...(iii)

From (1) and (2),

$$4V_Y + V_Y = 12 \Rightarrow V_Y = 2\cdot4 V$$

$\therefore \quad V_X = 4 \times 2\cdot4 = \mathbf{9\cdot6\ V}$

Thus potential difference across X, $V_X = \mathbf{9\cdot6\ V}$, P.d. across Y, $V_Y = \mathbf{2\cdot4\ V}$

(iii) $\frac{\text{Energy stored in } X}{\text{Energy stored in } Y} = \frac{Q^2/2C_X}{Q^2/2C_Y} = \frac{C_Y}{C_X} = \mathbf{4} \Rightarrow \frac{U_X}{U_Y} = 4$

21. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes $\frac{2}{3}$rd of its original value. What is the new distance between the plates ? ***[CBSE (AI) 2008]***

Sol. Initial capacitance, $C_0 = \frac{\varepsilon_0 A}{d}$

If d' is new distance, final capacitance, $C = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$

Given $C = \frac{2}{3}C_0$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d' - t\left(1-\frac{1}{K}\right)} = \frac{2}{3}\frac{\varepsilon_0 A}{d} \quad \Rightarrow \quad d' - t\left(1-\frac{1}{K}\right) = \frac{3}{2}d$$

$$\Rightarrow \quad d' = \frac{3}{2}d + t\left(1-\frac{1}{K}\right)$$

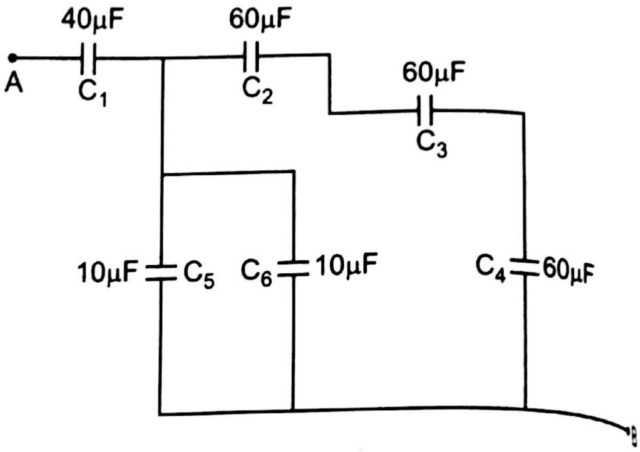
Given $d = 4$ mm, $t = 3$ nm, $K = 3$

$$\therefore \quad d' = \frac{3}{2} \times (4\text{ mm}) + 3\text{ mm}\left(1-\frac{1}{3}\right) = 6\text{ mm} + 2\text{ mm} = \mathbf{8\ mm}$$

That is new distance between the plates is **8 mm.**

22. Find the equivalent capacitance of the combination of capacitors between the points *A* and *B* as shown in fig. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and *B*.

[CBSE Delhi 2002]



Sol. The capacitors C_2, C_3, C_4 each of 60 μF are in series, their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$$

$\Rightarrow$ $C' = \mathbf{20\ \mu F}$

Now capacitors $C_5 = 10\ \mu F, C_6 = 10\ \mu F$, and $C' = 20\ \mu F$ are in parallel, so their equivalent capacitance

$$C'' = 10 + 10 + 20 = 40\ \mu F$$

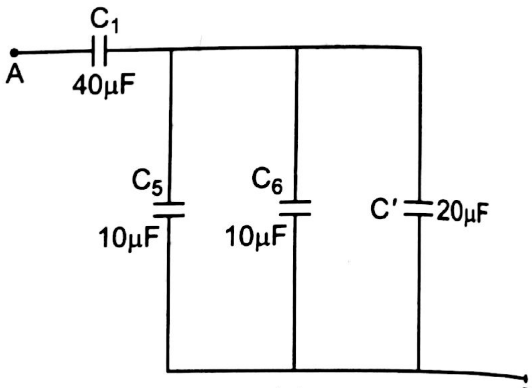
Now capacitors $C_1 = 40\ \mu F$ and $C'' = 40\ \mu F$ are connected in series; their equivalent capacitance

$$\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C''} = \frac{1}{40} + \frac{1}{40} = \frac{1}{20}$$

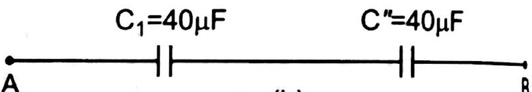
$\Rightarrow$ $\mathbf{C_{AB} = 20\ \mu F}$

Total charge flowing in circuit

$$Q = CV = (20\ \mu C) \times 100\ V = \mathbf{2000\ \mu C}$$

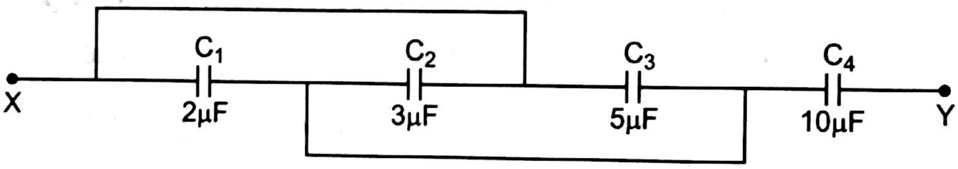


(a)



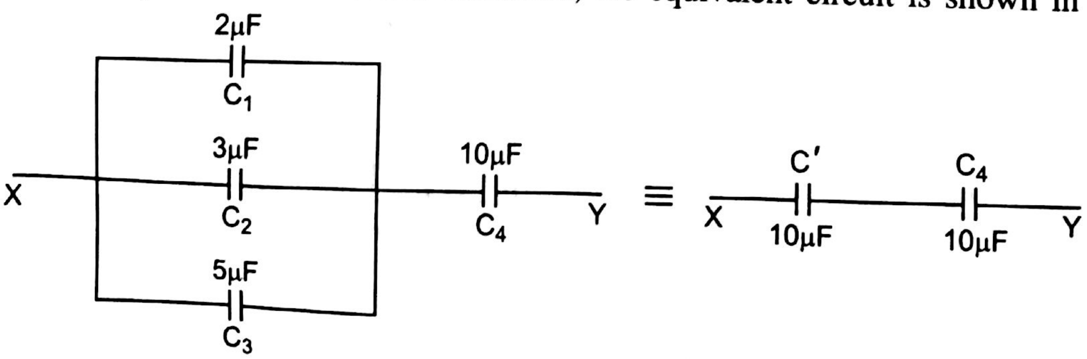
(b)

23. Four capacitors are connected as shown in fig. :



Calculate the equivalent capacitance between the points X and Y.

Sol. One of the plates of capacitors C_1, C_2 and C_3 is connected to one common point, while the second plate to another common point. Hence, C_1, C_2, C_3 are effectively connected in parallel and C_4 in series with this parallel combination. Therefore, the equivalent circuit is shown in figure.



The equivalent capacitance of C_1, C_2, C_3 connected in parallel is

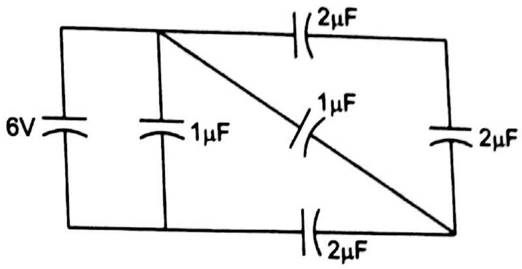
$$C' = C_1 + C_2 + C_3 = 2 + 3 + 5 = 10\ \mu F$$

C' and C_4 are in series, therefore, equivalent capacitance of combination between X and Y.

$$C_{XY} = \frac{C' C_4}{C' + C_4} = \frac{10 \times 10}{10 + 10} = \mathbf{5\ \mu F}$$

24. Find the total energy stored in the capacitors in the given network: ***[CBSE Delhi 2004]***

Sol. The equivalent capacitance of C_1 and C_2 in series $C' = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{2\times 2}{2+2} = 1\ \mu F$



C' is in parallel with C_3, so equivalent capacitance of C_1, C_2 and C_3 is

$$C'' = 1 + 1 = 2\ \mu F$$

C'' is in series with C_4; their equivalent capacitance

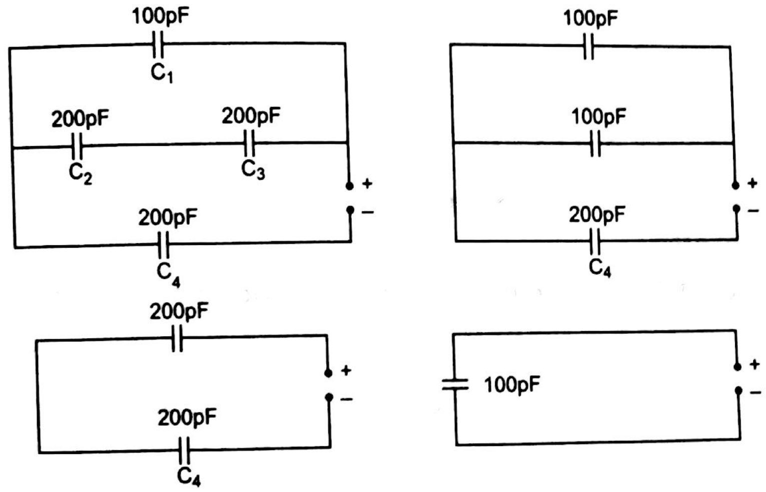
$$C''' = \frac{C_4C''}{C_4 + C''} = \frac{2\times 2}{2+2} = 1\ \mu F$$

This is in parallel with C_5; So equivalent capacitance across AB is $C_{AB} = 1 + 1 = 2\ \mu F$

Energy stored $V = \dfrac{1}{2} C_{AB}\ V^2 = \dfrac{1}{2} \times 2 \times 10^{-6} \times (6)^2 = 36 \times 10^{-6}$ J

25. Obtain the equivalent capacitance of the network given below. For a supply of 300 V, determine the charge and voltage across C_4. ***[CBSE (AI) 2008]***

Sol.



$\therefore \quad C_{eq} = 100$ pF

Now, $Q = C_{eq} \times V = 100 \times 10^{-12} \times 300 = 3 \times 10^{-8}$ coulomb

Potential difference across $C_4 = \dfrac{Q}{C_4} = \dfrac{3\times 10^{-8}}{200 \times 10^{-12}} = 1{\cdot}5 \times 10^2 =$ **150 V**

26. Keeping the voltage of the charging source constant, what would be the percentage change in the energy stored in a parallel plate capacitor if the separation between its plates were to be decreased by 10%?

Sol. Energy stored in a capacitor for a fixed voltage, $U = \dfrac{1}{2}CV^2$. Capacitor of parallel plate capacitor

$C = \dfrac{\epsilon_0 A}{d}$. If the separation between the plates is decreased by 10%, new separation, $d - \dfrac{10}{100}d = 0{\cdot}9$ d.

$\therefore$ New capacitance $C' = \dfrac{\epsilon_0 A}{0{\cdot}9d} = \dfrac{C}{0{\cdot}9} = \dfrac{10}{9}$ C

% change in energy is $\Delta U = \dfrac{C' - C}{C} \times 100\% = \left(\dfrac{C'}{C} - 1\right) \times 100\% = \left(\dfrac{10}{9} - 1\right) \times 100\% = \dfrac{100}{9} = \mathbf{11{\cdot}1\%}$

i.e., energy stored increases by **11·1%.**

27. A parallel plate capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 3 and dielectric strength 10^7 Vm^{-1}. For safety we would like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF ? *[CBSE (AI) 2005]*

Sol. The maximum electric field applied = 10% of dielectric strength

$$= \frac{10}{100} \times 10^7 \text{ Vm}^{-1} = 10^6 \text{ Vm}^{-1}$$

Potential difference across capacitor = 1kV = 1000 V

Capacitance $C = 50\ pF = 50 \times 10^{-12}$ F

The maximum charge on the plates

$$Q = CV = 50 \times 10^{-12} \times 1000 = 5 \times 10^{-8} \text{ C}$$

If σ is the surface charge density of plates

$$E = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = K\varepsilon_0 E$$

$\therefore$ $Q = \sigma A$

$\therefore$ Required area $= \frac{Q}{\sigma} = \frac{Q}{K\varepsilon_0 E} = \frac{5 \times 10^{-8}}{3 \times 8{\cdot}85 \times 10^{-12} \times 10^6} = 18{\cdot}8 \times 10^{-4} \text{ m}^2 = \mathbf{18{\cdot}8\ cm^2}$

28. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?
Find the ratio of energy stored in the two configurations if they are both connected to the same source. *[CBSE (AI) 2011]*

Sol. Let C be the capacitance of each capacitor, then in series $\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

or $C = 3C_s = 3 \times 1\ \mu\text{F} = 3\ \mu\text{F}$

When these capacitors are connected in parallel, net capacitance, $C_p = 3C = 3 \times 3 = 9\ \mu\text{F}$

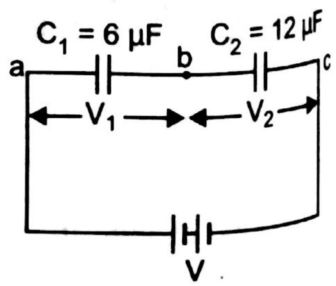
When these two combinations are connected to same source the potential difference across each combination is same.

Ratio of energy stored, $\frac{U_s}{U_p} = \frac{\frac{1}{2}C_sV^2}{\frac{1}{2}C_pV^2} = \frac{C_s}{C_p} = \frac{1\ \mu\text{F}}{9\ \mu\text{F}} = \frac{1}{9}$

$\Rightarrow$ $U_s : U_p = 1 : 9$

29. Two capacitor of capacitance 6 μF and 12 μF are connected in series with a battery. The voltage across the 6 μF capacitor is 2 V. Compute the total battery voltage. *[CBSE (AI) 2006]*

Sol. Given $C_1 = 6\ \mu F,\ C_2 = 12\ \mu F,\ V_1 = 2\ V$



Charge on capacitor C_1 is $q_1 = C_1V_1 = (6\ \mu F) \times 2\ (V) = 12\ \mu C$

In series arrangement charge on each capacitor remains the same; so charge on C_2 is also $q_2 = q_1 = 12\ \mu C$.

$\therefore$ P.d. across C_2 is

$$V_2 = \frac{q_2}{C_2} = \frac{12\ \mu C}{12\ \mu F} = 1\ \text{V}$$

Total battery voltage $V = V_1 + V_2 = 2 + 1 = \mathbf{3\ V}$

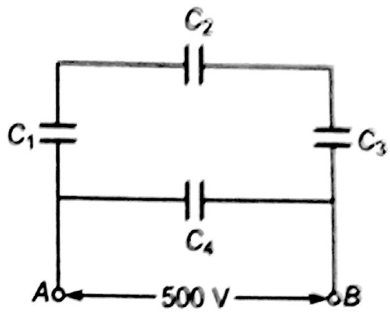
Aliter: Charge on each capacitor, $q = C_1V_1 = 12\ \mu C$

Equivalent capacitance, $C = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{6 \times 12}{6 + 12} = 4\ \mu F$

Total battery voltage $V = \dfrac{q}{C} = \dfrac{12\ \mu C}{12\ \mu F} = 3\ \text{V}$

30. A network of four capacitors each of 15 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor.

[CBSE (AI) 2010]



Sol. (a) C_1, C_2 and C_3 are in series, their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

$\Rightarrow \quad C' = 5\ \mu\text{F}$

C_4 is in parallel with C', so equivalent capacitance of network

$$C_{eq} = C' + C_4 = 5 + 15 = 20\ \mu\text{F}$$

(b) Charge on capacitor C_4 is

$$q_4 = C_4V = (15\ \mu\text{F}) \times 500\ \text{V} = 7500\ \mu\text{C} = 7.5\ \text{mC}$$

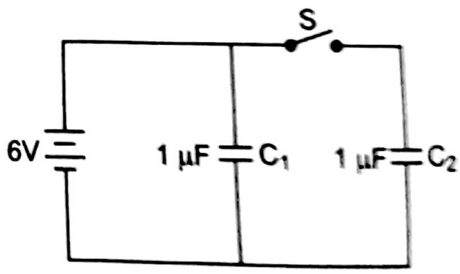
Charge on C_1, C_2 and C_3 is

$$q_1 = q_2 = q_3 = C'V$$

$$= \mathbf{5\ \mu F \times 500\ V = 2500\ \mu C = 2.5\ mC}$$

31. Figure shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch 'S' is closed. After sometimes 'S' is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted?

[CBSE Delhi 2011]



Sol. When switch S is closed, p.d. across each capacitor is 6V

$V_1 = V_2 = 6\ \text{V}$

$C_1 = C_2 = 1\ \mu\text{C}$

∴ Charge on each capacitor

$$q_1 = q_2\ (= CV) = (1\ \mu\text{F}) \times (6\ \text{V}) = 6\ \mu\text{C}$$

When switch S is opened, the p.d. across C_1 remains 6 V, while the charge on capacitor C_2 remains 6 μC. After insertion of dielectric between the plates of each capacitor, the new capacitance of each capacitor becomes

$$C_1' = C_2' = 3 \times 1\ \mu\text{F} = 3\ \mu\text{F}$$

(i) Charge on capacitor C_1, $q_1' = C_1'V_1 = (3\ \mu\text{F}) \times 6\ \text{V} = 18\ \mu\text{C}$

Charge on capacitor C_2 remains $6\ \mu\text{C}$

(ii) Potential difference across C_1 remains 6 V.

Potential difference across C_2 becomes

$$V_2' = \frac{q_2}{C_2'} = \frac{6\mu\text{C}}{6\ \mu\text{F}} = 2\ \text{V}$$