Chapter–10:

Mechanical Properties of Fluids

CBSE CLASS XI NOTES

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MECHANICS OF FLUIDS

Smil..

A fluid is a substance which can flow.
→ both liquids & gases

*** Thrust**

Thrust on a surface is the total normal force acting on it.

*** Pressure**

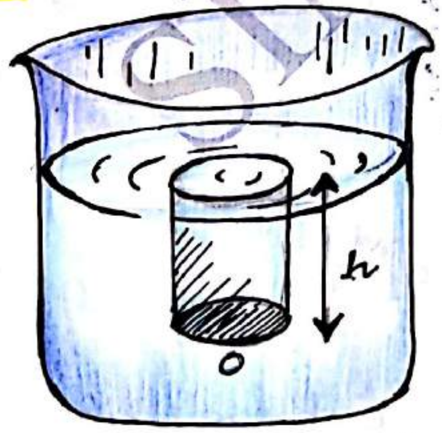
All fluids exert pressure.

$$\boxed{\text{pressure} = \frac{\text{Thrust}}{\text{area}}}$$

→ SI unit of pressure is N/m^2 or Pa

→ Dimensional formula $ML^{-1}T^{-2}$

Derive an expression for the pressure at a point inside a liquid at rest.



consider a liquid of density ρ contained in a vessel. Let us find the pressure P at a point O inside the liquid at a depth 'h' below the surface of the liquid.

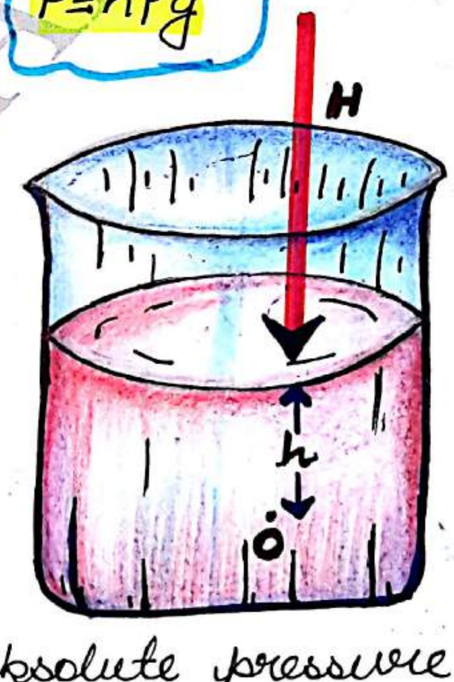
Imagine a horizontal area 'A' around the point O.

Thrust = weight of vertical column of liquid over area A

$$\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} = \frac{mg}{A}$$

$$= \frac{V\rho g}{A} = \frac{Ah\rho g}{A}$$

$$\boxed{P = h\rho g}$$

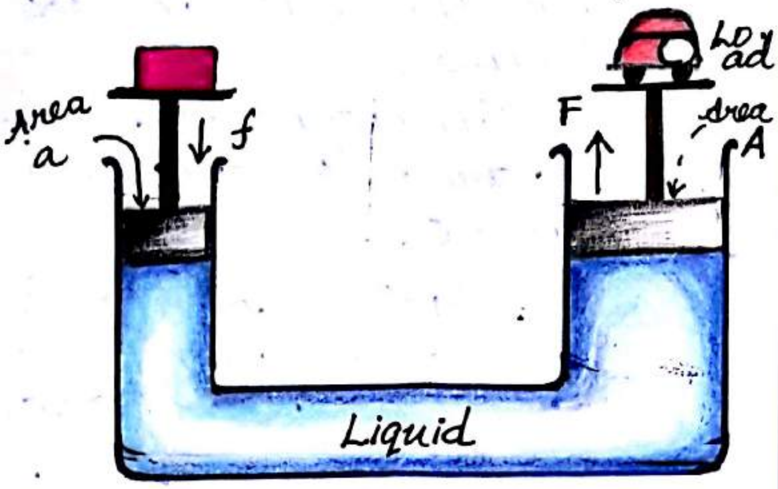


Absolute pressure = $H + h\rho g$

where H is the atmospheric pressure.

Pascals Law

pascals law states that, in a continuous fluid in equilibrium, the pressure applied at any point is transmitted equally to every other point in the fluid.



Hydraulic lift is used to lift heavy loads.

* car lifts and jacks, hydraulic brakes, dentist chairs etc.

→ pressure exerted on the liquid

$$p = \frac{f}{a}$$

→ According to Pascal's law, the same pressure P is also transmitted to the larger piston of cross-sectional area A.

$$F = P \times A$$

$$F = \frac{f}{a} \times A$$

$$F = \frac{A}{a} \times f$$

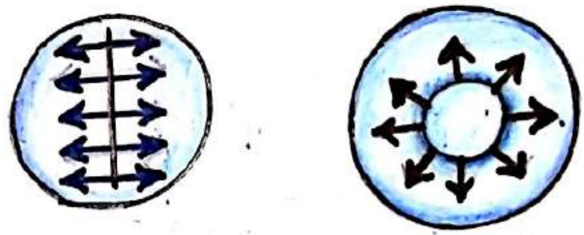
SURFACE TENSION

Define surface Tension. Give its units and dimensions?

Surface tension is the property by virtue of which the free surface of a liquid at rest behaves like an elastic streched membrane tending to contract so as to occupy minimum surface area.

Surface tension can be measured as the force acting tangential to liquid surface and perpendicular to its unit length of an imaginary line drawn on the surface of the liquid.

$$\text{surface tension} = \frac{\text{Force}}{\text{length}}$$



What are the types of molecular forces?

There are two types of molecular forces

1. cohesive force
2. Adhesive force.

1, cohesive force

The force of attraction between molecules of same substance is called co-hesive force.

$F_{solid} > F_{liquid} > F_{gas}$

2, adhesive force

The force of attraction between the molecules of different substances is called force of adhesion.

Eg; Force of Adhesion between paper and gum molecules.

Surface Energy.

* Define surface energy. prove that it is numerically equal to the surface tension?

Surface energy of a liquid is the potential energy per unit area of the liquid surface.

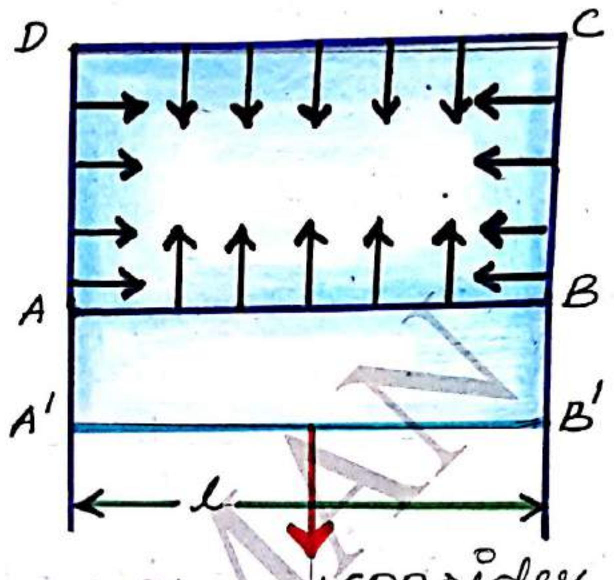
It is numerically equal to surface tension

∴ work done in producing an area = S.T × area

or

$$\text{surface Energy} = \frac{\text{work done}}{\text{Increase in surface area}}$$

* The SI unit of surface energy is Jm^{-2}



consider a rectangular wire frame ABCD. The AB is movable Dip the wire frame in soap solution, so that the film is formed over the frame. Due to surface tension the film has a tendency to shrink and thereby AB is pulled inward direction. AB can be kept in the same position by giving equal and opposite force (downward direction)

Let 'l' be the length of the wire.

T – force due to surface tension.

Then $F = T \times 2l$

2 appears here due to two surfaces. If the wire AB is pulled downward by a

small distance 'x' to the position $A'B'$. Then

work done $= W = F \times x$

$= T \times 2l \times x$

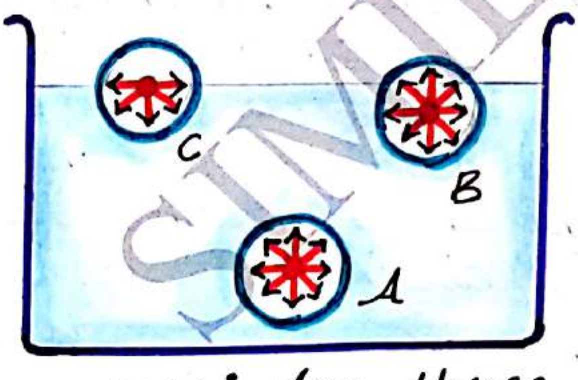
$$\boxed{W = T2lx}$$

surface energy = $\dfrac{\text{Work done in increasing surface area}}{\text{Increase in surface area}}$

surface energy $= \dfrac{T2lx}{2lx} = \dfrac{T2lx}{2lx} = T$

surface energy $= T =$ surface tension

* Explain Surface Tension on the basis of Molecular theory.?



consider three molecules A, B and C of a liquid. The circles indicate their sphere of influence

Molecule A:- It is well inside the liquid and is attracted equally in all directions by other molecules. Therefore no resultant force acting on it.

Molecule B:- more than half of its sphere of influence is below the liquid surface. More molecules are attracting it in the downward direction than in the upward direction. So it experience a net Inward pull.

Molecule C:- The inward pull is maximum in molecule 'C' as it lies on surface. This is because lower half is full of molecules, upper half is empty.

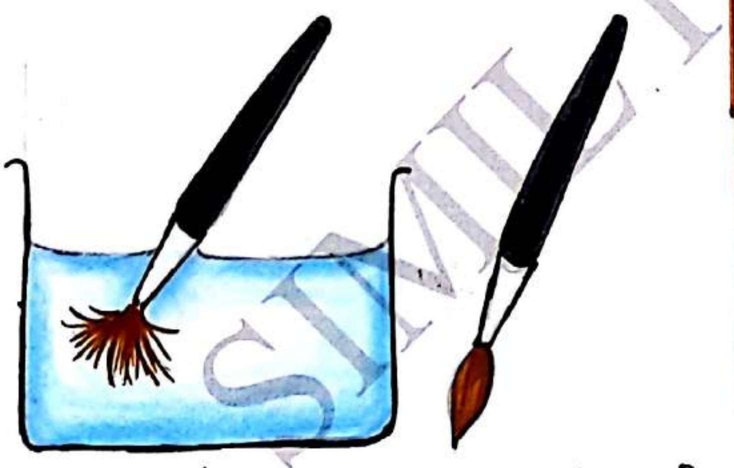
If a molecule is brought to the surface from the interior, work has to be done against the inward pull. There fore molecules on the surface have additional potential energy. Any stable system tries to have minimum energy. ∴ we should have minimum number of molecules on the surface to have minimum energy.

So the surface

area should be minimum
∴ the liquid tries to have minimum area and thereby, it behaves like a streched elastic membrane.

Explain some examples which illustrate the existence of surface tension?

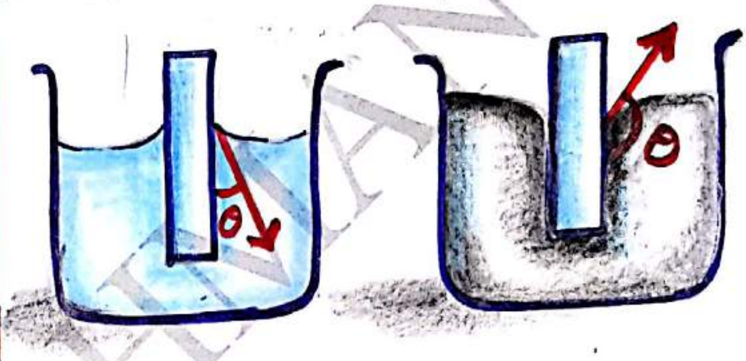
(1) Hair of a painting brush, when dipped in water spread out, but when taken out its hair cling together due to surface tension, water film formed on them trend to contract to minimum area.



(2) If a sewing needle is placed carefully on water surface, the needle rest there without sinking although the density is several times greater than that of water. It shows that water behaves like a streched membrane.

Angle of Contact

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.



Note:

1, angle of contact for a pure ~~water~~ and clean glass is ZERO.

2, For ordinary water and glass $\theta = 8^\circ$ (acute angle)

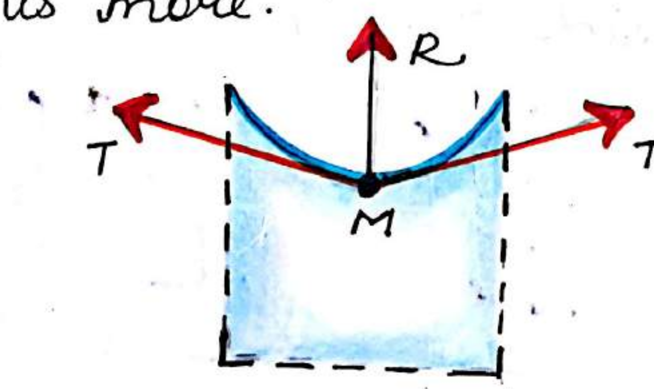
3, For mercury and glass $\theta = 140^\circ$ (obtuse)

Pressure difference across a Liquid Surface

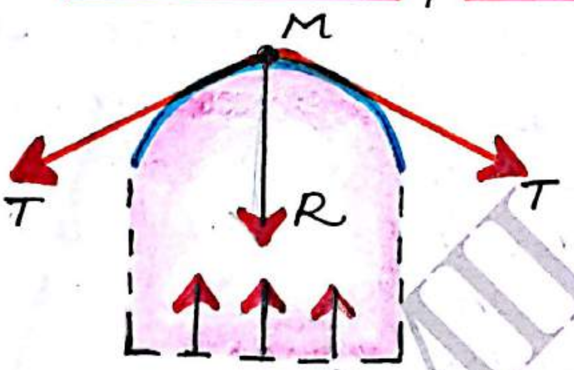
The surface of a liquid may be concave, convex or plane.

(a) concave surface

If the surface is concave the resultant force R on on molecule M due to surface tension T acts in upward direction. molecules – experiencing a net upward force. The molecule will be in equilibrium if pressure on concave side is more.

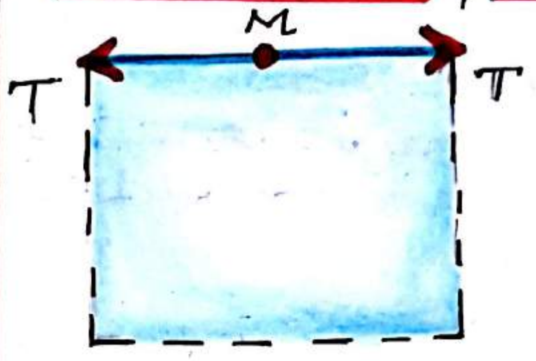


(b) convex surface



If the surface is convex, the resultant force R on molecule M due to surface tension T acts in downward direction. molecules – experiencing net downward force. The molecules will be in equilibrium if pressure on concave side is more.

(c) Plane surface.



If the surface is plane molecule pulled equally in all directions, No net force acts on it. No excess pressure, inside or outside the liquid surface.

Note:– A curved surface will be in equilibrium, only if there is an excess of pressure on the concave side of the curved surface.

Give expressions for excess of pressure P

(a) Inside a drop

(b) Inside a bubble in free space

(c) Inside a bubble in a liquid.

(a) Inside a drop

Excess pressure $P = \frac{2T}{R}$

where
T – surface tension
R – Radius of the drop.

(b) Inside a bubble in free space

Excess pressure $\boxed{P = \frac{4T}{R}}$

(c) Inside a bubble in a liquid

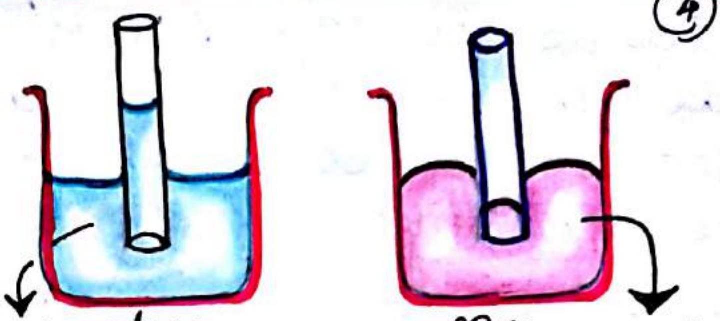
excess pressure
$\boxed{P = \frac{2T}{R}}$

where P – excess pressure = pressure difference
T – surface tension
R – Radius of the bubble.

Capillarity

A tube of very fine bore is called a capillary tube

The rise or fall of a liquid in a tube of very fine bore is called capillarity

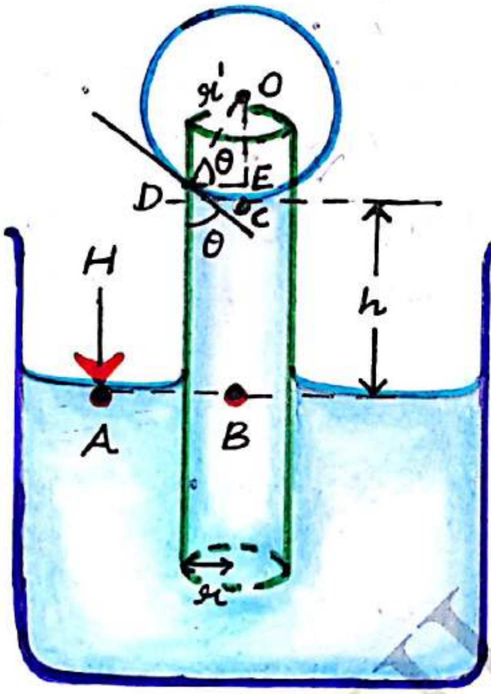


Write some applications of capillarity?

1. The tip of the nib of a pen is split to provide capillary action
2. Blotting paper absorbs ink by capillary action.
3. Water reaches every branch of plant by capillary action.
4. A sponge retains water on being dipped in water is due to capillary action.

*Give the expression for capilary ascent (Ascent formula)

Consider a capillary tube of radius 'r' open at both ends and dipped vertically in a liquid of density P and surface tension 'T'. The meniscus inside the liquid is concave. Let 'r' be the radius of concave meniscus, θ - angle of contact. and 'h' height of liquid column raised in the tube from the level of liquid outside.



→ The pressure just below the concave side of the liquid meniscus is in the tube will be less than that above it by an amount

$P = 2T/r'$

→ pressure just above the meniscus in the tube = H (atm pressure)

∴ pressure at C

$= H - 2T/r'$

pressure at A = pressure at B

i.e., $H = H - \frac{2T}{r'} + h\rho g$

$$\frac{2T}{r'} = h\rho g$$

$$\therefore h = \frac{2T}{r'\rho g} \quad \text{(1)}$$

Δ ODE, $OD = r'$, $DE = r$,

$$\cos\theta = \frac{DE}{DO} = \frac{r}{r'}$$

$$r' = \frac{r}{\cos\theta}$$

$$\text{(1)} \Rightarrow h = \frac{2T\cos\theta}{r\rho g}$$

→ This expression is called ascent formula

→ In case of water and water-like liquids which wet the tube

$\theta \simeq 0 \quad \therefore \cos\theta = 1$

$$\therefore h = \frac{2T}{r\rho g}$$

* How does temperature affect surface tension?

surface tension decreases when tempe

* pressure of impurity also changes the value of surface tension.

* **Explain the cleansing action of detergents?**

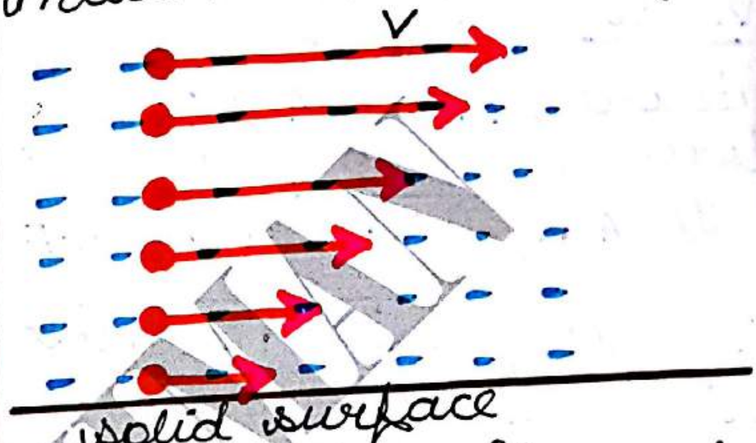
The dirty clothes having greasy stains cannot be cleaned by washing them in water. Because water cannot wet greasy dirt. But when detergent is added to water, it reduces surface tension, wets the grease. Detergent molecule attract water at one end and grease at the other end. When clothes are rinsed in water, the greasy dirt is wasted away by running water.

* **What is called viscosity?**

Viscosity of a fluid is the property of the fluid by virtue of which the fluid opposes the relative motion between its successive layers.

* **On what factors viscous force depend upon? give the expression for viscous force. Define co-efficient of viscosity?**

Consider a liquid flowing over a horizontal solid surface in the form of parallel layers. The layer at the top possesses maximum velocity.



Due to viscosity a force F acts in opposite direction to destroy the relative motion.

This viscous force F depends upon the following factors

1. It is directly proportional to the area of the layers in contact.

$$F \propto A$$

2. It is directly proportional to velocity gradient between layers.

$$F \propto \frac{dv}{dx}$$

$$\therefore F \propto A\frac{dv}{dx}$$

$$F = \eta A \frac{dv}{dx}$$

where η is a constant called coefficient of viscosity. η depends on nature of liquid.

*** Define coefficient of viscosity ?**

$$\text{viscous force} = F = \eta A \frac{dv}{dx}$$

If $A = 1$, $\frac{dv}{dx} = 1$ then $\eta = F$

$\therefore$ coefficient of viscosity of a liquid may be defined as the tangential viscous force, which maintains unit velocity gradient between two parallel layers, each of unit area. It's SI unit is Nsm^{-2}

Dimensions : $ML^{-1}T^{-1}$

*** What is called as stokes law ?**

The viscous force 'F' experienced by a spherical ball of radius 'a' moving through a viscous medium of coefficient of viscosity 'η' with a velocity 'v' is

$$\boxed{F = 6\pi a \eta v}$$

This is called stokes law

$F \propto V$

*** Define terminal velocity. Derive the expression for terminal velocity ?**

Terminal velocity is the maximum constant velocity acquired by a body while falling freely through a viscous medium.

consider a small and heavy ball of radius 'a' and density 'ρ' falling through a liquid of density 'σ' and coefficient of viscosity 'η'. The different forces acting on the ball are

(a) Weight of the ball acts downward.

$$W = mg = V\rho g = \frac{4}{3}\pi a^3 \rho g$$

(b) Upthrust (Buoyant force) of the displaced liquid. (acting upward)

upthrust = weight of the liquid displaced.

$U = mg = V\sigma g = \frac{4}{3}\pi a^3 \sigma g$

$\therefore U = \frac{4}{3}\pi a^3 \sigma g$

(c) Viscous force F acting upward

$F = 6\pi a \eta v$

where 'v' is the velocity of the ball. When the ball falls through the liquid, its velocity increases gradually. This gravitational force is balanced by upthrust and viscous force.

$\therefore$ The ball continues to move with uniform velocity. This velocity is called terminal velocity. At terminal velocity,

$\boxed{\text{Weight of the ball} = \text{upthrust} + \text{viscous force}}$

$$\frac{4}{3}\pi a^3 \rho g = \frac{4}{3}\pi a^3 \sigma g + 6\pi a \eta v$$

$$\therefore 6\pi a \eta v = \frac{4}{3}\pi a^3 \rho g - \frac{4}{3}\pi a^3 \sigma g$$

$$\therefore 6\pi a \eta v = \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

$$\therefore v = \frac{\frac{4}{3}\pi a^3 (\rho - \sigma) g}{6\pi a \eta}$$

$$v = \frac{2}{9} \frac{a^2 (\rho - \sigma) g}{\eta}$$

$$\boxed{v = \frac{2}{9} \frac{a^2 (\rho - \sigma) g}{\eta}}$$

If 'σ' is negligible compared with 'ρ'

$$v = \frac{2}{9} \frac{a^2 \rho g}{\eta}$$

$$\boxed{\therefore v \propto a^2}$$

$a \to$ radius

Fluid Flow

* Differentiate streamlined and turbulent flow.

Streamline flow	Turbulent flow
1, It is steady flow and highly ordered.	1, Unsteady flow and highly disordered.
2, velocity less than critical value	2, velocity greater than critical value

3, velocity at each point remains constant	3, velocity at any point varies with time.
4, Reynolds number lies between 0 to 2000	4, Reynolds number greater than 3000

* Define stream line flow?

A stream line flow of a liquid is a steady flow in which each layer of liquid follows the same path and has same velocity as that of its predecessor.

* Define stream line?

streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point.

* Define critical velocity (V_c)

critical velocity is the velocity at which streamline flow is changed into turbulent flow and vice versa.

If velocity $V > V_c$ flow is turbulent.

If $V < V_c$ flow is streamline.

* What is called Reynolds number R?

critical velocity $V_c = \frac{R\eta}{\rho D}$

∴ Reynolds number $\boxed{R = \frac{V_c \rho D}{\eta}}$

If R = 0 to 2000 – stream line

If $R > 3000$ – Turbulent

if R = 2000 to 3000 – flow may change from streamline to turbulent & vice versa.

* Derive the equation of continuity?

consider a liquid is flowing through a pipe AB of varying cross section

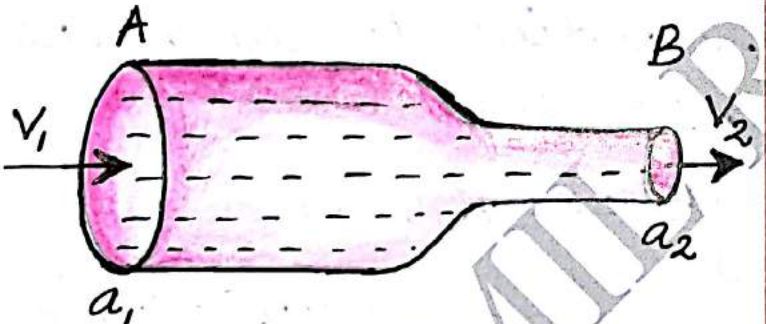
Let 'a_1' and 'a_2' be the cross sectional areas of the pipe at A and B respectively. Liquid enters with normal velocity V_1 and leaves with velocity V_2.

Quantity of liquid entering A per second = Quantity of liquid leaving B per second

$$Q_1 = Q_2$$

$$a_1 v_1 \rho = a_2 v_2 \rho$$

$$\boxed{a_1 v_1 = a_2 v_2}$$



$av =$ constant. This equation is called as equation of continuity

$$\boxed{V \propto \frac{1}{a}}$$

velocity is inversly proportional to cross sectional area.

* State and prove Bernoulli's theorem.

Statement: It states that, in a streamline flow of a fluid, the total energy (pressure energy, potential energy and kinetic energy) of a small amount of the fluid flowing without any friction remains constant throught its flow. (7)

pressure energy + potential energy + kinectic energy = const

$$\boxed{\text{Pressure energy} + P.E + K.E = \text{const}}$$

$$\boxed{\frac{P}{\rho} + gh + \frac{1}{2}V^2 = \text{const}}$$

proof

consider a liquid of density ρ flowing through a pipe of varying area of cross section.

Let A and B be the two sections of pipe perpendicular to the direction of flow.

At A

$V_1 \rightarrow$ volume of liquid entering the pipe

$P_1 \rightarrow$ pressure

a_1 → area of cross section

v_1 → velocity of liquid

h_1 → height from reference level.

At B

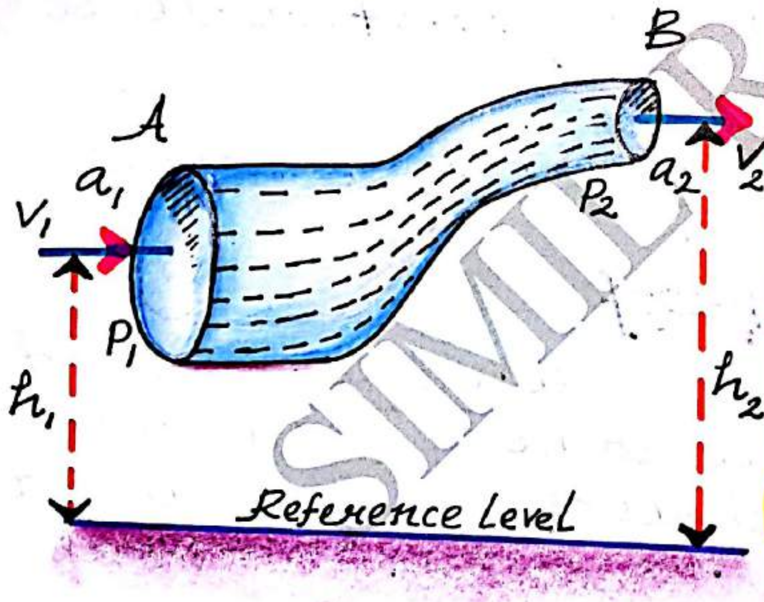
V_2 – volume of liquid leaving the pipe at B

P_2 → pressure

a_2 → area of cross section of pipe

V_2 → velocity of liquid leaving the pipe.

h_2 → height from reference level.



* Work done on unit mass of liquid at A

$$= \frac{P_1 V_1}{m} = \frac{P_1}{\rho}$$

* Work done by unit mass of liquid at B

$$= \frac{P_2 V_2}{m} = \frac{P_2}{\rho}$$

Net work done on unit mass of liquid flowing from A to B

$$= \frac{P_1}{\rho} - \frac{P_2}{\rho}$$

* Gain in potential energy when unit mass of liquid flows from A to B

$$= gh_2 - gh_1$$

* Gain in kinetic energy when unit mass of liquid flows from A to B $= \frac{1}{2}V_2^2 - \frac{1}{2}V_1^2$

* Gain in P.E & K.E on unit mass

$$= (gh_2 - gh_1) + (\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2)$$

According to work-energy theorem

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} = (gh_2 - gh_1) + (\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2)$$

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + gh_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + gh_2$$

Thus Bernoulli's theorem is proved.

Note:

$$\frac{P}{\rho} + \frac{1}{2}V^2 + gh = \text{constant}$$

$\frac{P}{\rho g}$ → pressure head

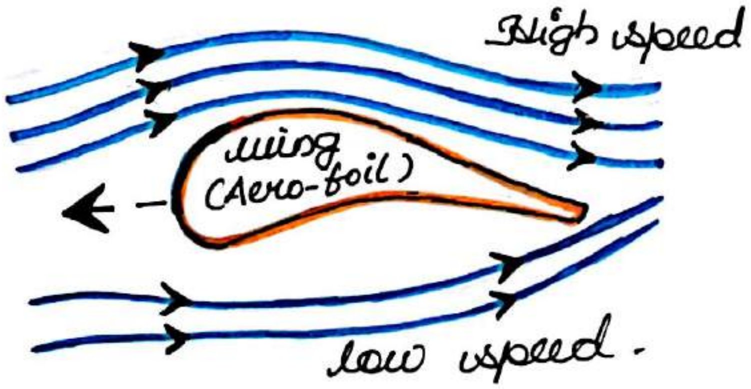
$\frac{V^2}{2g}$ → velocity head.

h → gravitational head.

Applications

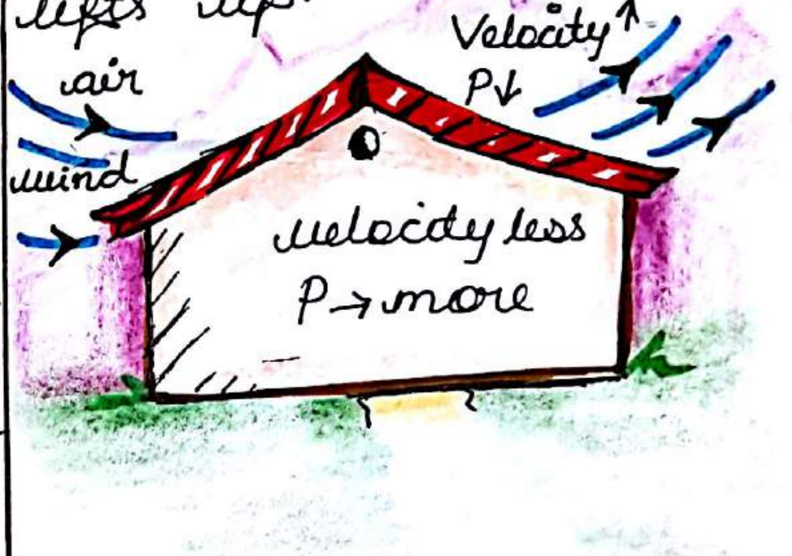
1, **lift of an aircraft**

The wings of an aeroplane are so designed that their upper surfaces are more curved than the lower surfaces. when the aircraft is moving, the air moves faster at the upper surface than the lower surface. As a result the pressure of air on the upper surface of the wing is less than the pressure at lower surface.



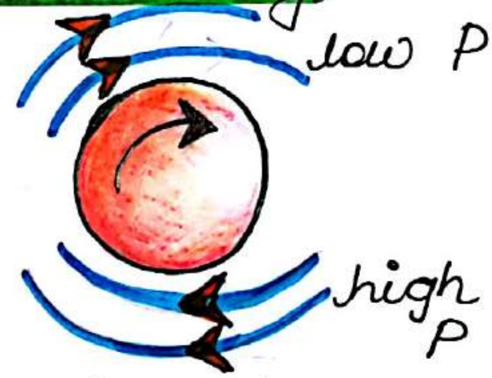
2, **Blowing of roofs of houses during storm**

During storm velocity of wind or air above the roof is greater. It reduces the pressure (above). so pressure at a point just below the roof is more. so roof lifts up.



3, **Atomiser / sprayer**

4, **magnus effect – swing bowling**



5, **cylindrical shape of bullet.**