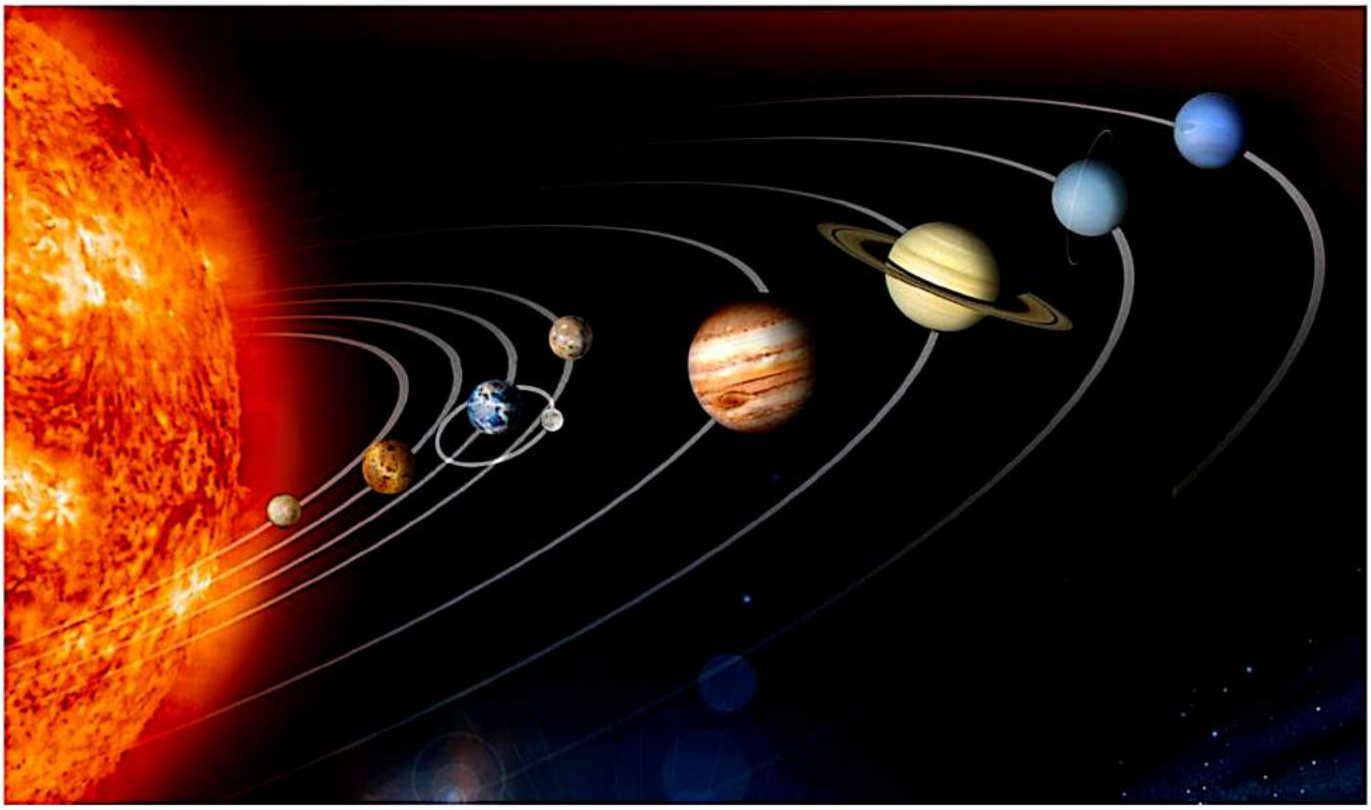
Chapter–8:

Gravitation



CBSE CLASS XI NOTES

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Gravitation

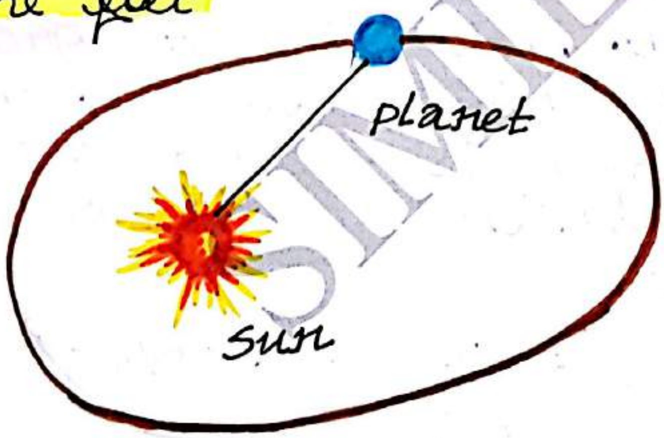
* Gravitation is the force of attraction between any two bodies in the universe

* Gravity :- Gravity is the force of attraction between the earth and any object lying on or near its surface.

* State keplers Laws of planetary motion.

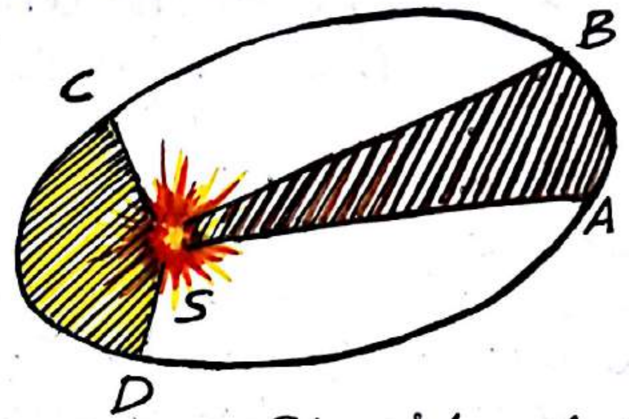
Keplers first law:-
(Law of orbits)

Every planet revolving around the sun is in an elliptical orbit with sun as one of the foci



Keplers second Law
(law of areas)

The radius vector sweeps out equal areas in equal intervals of time



The planet takes equal intervals of time to travel from A to B and C to D. so it has to travel faster to cover distance CD.

$$\text{areal velocity} = \frac{dA}{dt} = \text{constant}$$

Keplers third Law
[law of periods or the harmonic law]

The square of the time period of revolution of a planet around the sun is directly proportional to the cube of its semi major axis

$$T^2 \propto r^3$$

$$\frac{T^2}{r^3} = \text{constant} = K$$

$$k = \frac{4\pi^2}{GM} = 10^{-13}\ s^2/m^3$$

* State Newtons Law of Gravitation ?

every particle of matter in the universe attracts every other particle with a force of attraction which varies directly with the product of the masses and inversely with the square of the distance between them

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

where G – gravitational constant

$G - 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

* Define gravitational constant. Give its dimensional formula?

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

if $m_1 = m_2 = 1$ and $r = 1$

the $\boxed{F = G}$ (or) $\boxed{G = F}$

∴ Gravitational constant is numerically equal to the force of attraction between two unit masses seperated by unit distance.

Its Dimensional formula is $M^{-1}L^3T^{-2}$.

* Write the nature of gravitational force?

1. It is always attractive
2. It is independent of the medium in which it acts.
3. It is conservative
4. It holds good over wide range of distances.
5. It is a central force
6. It is an action-reaction pair.

* What is called acceleration due to gravity?

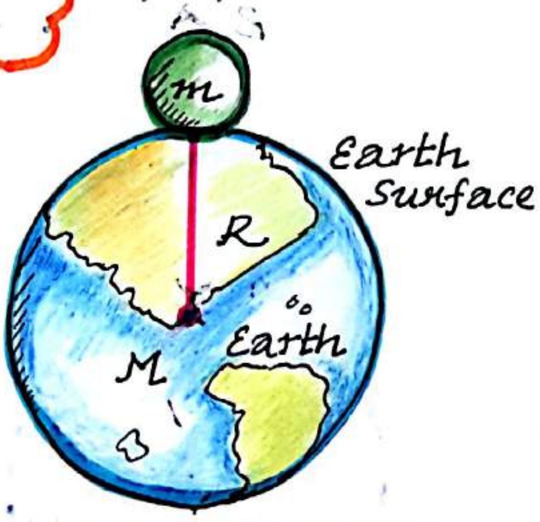
The uniform acceleration produced in a freely falling body due to gravitational pull of the earth is called acceleration due to gravity

$$\boxed{g = \frac{GM}{R^2}} = 9.8\ m/s^2$$

* Derive the expression for acceleration due to gravity 'g'?

consider earth as a sphere of mass 'M' and radius 'R' and a body of mass 'm' is lying on the surface of the earth. According to universal law of gravitation

$$F = \frac{GMm}{R^2} \quad \text{.... (1)}$$



According to Newtons IInd law

$$F = mg \quad \text{.... (2)}$$

(1) = (2)

$$mg = \frac{GMm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}}$$

It gives the value of acceleration due to gravity on the surface of the earth.

* Calculate mass and Density of the earth?

we know that acceleration due to gravity

$$\boxed{g = \frac{GM}{R^2}}$$

where

G - Gravitational constant

M & R - mass and radius of earth.

Mass

$$g = \frac{GM}{R^2}$$

$$\therefore \boxed{M = \frac{gR^2}{G}} = \frac{9.8 \times (6.38 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M = 6 \times 10^{24}\,kg$$

Density

earth is considered as a sphere

$\therefore$ volume of earth $= V = \frac{4}{3}\pi R^3$

$$\text{Density} = \frac{\text{Mass}}{\text{volume}} = \frac{M}{V}$$

$$\frac{\frac{gR^2}{G}}{\frac{4}{3}\pi R^3}$$

$$D = \frac{gR^2}{G} \times \frac{3}{4\pi R^3} = \frac{3g}{4\pi RG}$$

$$\boxed{D = \frac{3g}{4\pi RG}}$$

* Discuss the variation of acceleration due to gravity with

(a) shape of the earth

(b) Altitude

(c) Depth.

(a) Variation of 'g' due to the shape of earth.

$$g = \frac{GM}{R^2}$$

$$\boxed{g \propto \frac{1}{R^2}}$$

acceleration due to gravity is inversely proportional to square of radius of earth. Greater the radius, smaller the acceleration due to gravity.

Equatorial radius is greater than polar radius.

Therefore, 'g' is more at poles.

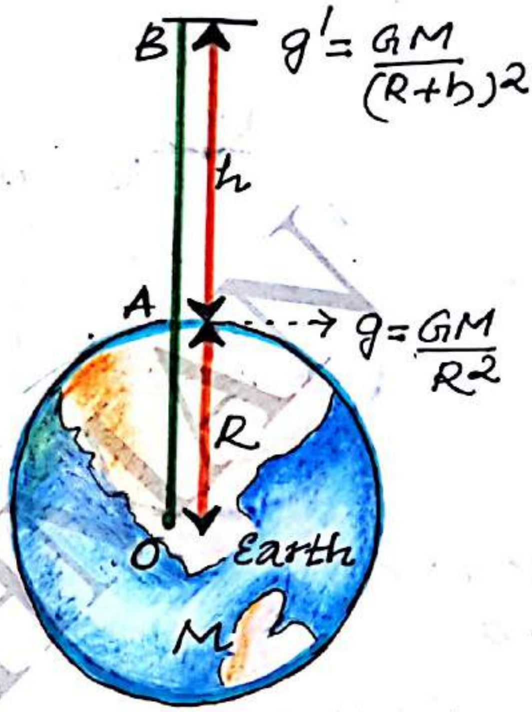
$$\boxed{\begin{array}{c} R_{equator} > R_{poles} \\ g_{equator} < g_{pole} \end{array}}$$

(b) Variation of 'g' with altitude.

consider that earth is a sphere of radius 'R' and mass 'M'.

Let 'g' be the value of acceleration due to gravity at a point on the surface of the earth

$$\boxed{g = \frac{GM}{R^2}} \quad \dots \text{(1)}$$



Let 'g'' be the acceleration due to gravity at a heigh 'h' from the earths surface

$$\boxed{g' = \frac{GM}{(R+h)^2}} \quad \dots \text{(2)}$$

(2) ÷ (1)

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{R^2(1+\frac{h}{R})^2} = \frac{1}{(1+\frac{h}{R})^2}$$

$$\boxed{\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}}$$ expanding by binomial theorem

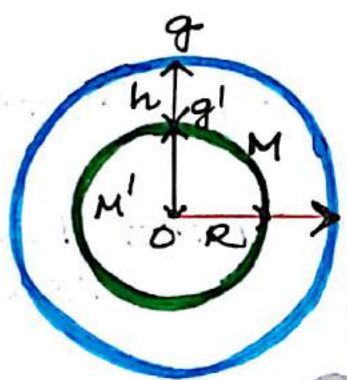
$$\boxed{\frac{g'}{g} = 1 - \frac{2h}{R}} \qquad \boxed{\therefore g' = g\left(1 - \frac{2h}{R}\right)}$$

gravity decreases

Variation of 'g' with depth.

Consider that earth is a homogeneous sphere of radius 'R' and mass 'M' with centre 'O'. If 'g' is the acceleration due to gravity on the surface of the earth

$$\boxed{g = \frac{GM}{R^2}}$$



$$g = \frac{GV\rho}{R^2}$$

$$g = G\frac{\frac{4}{3}\pi R^3\rho}{R^2}$$

$$g = G\frac{4}{3}\pi R\rho$$

$$\boxed{g = \frac{4}{3}G\pi R\rho} \quad \ldots\ (1)$$

where 'ρ' is the mean density of the earth.

Let g' be the acceleration due to gravity at a depth 'h' below the surface of the earth.

$$g' = \frac{GM'}{(R-h)^2} = \frac{GV'\rho}{(R-h)^2}$$

$$g' = G\frac{\frac{4}{3}\pi(R-h)^3\rho}{(R-h)^2}$$

$$g' = \frac{4}{3}G\pi(R-h)\rho \quad \ldots\ (2)$$

(2) ÷ (1)

$$\frac{g'}{g} = \frac{\frac{4}{3}G\pi(R-h)\rho}{\frac{4}{3}G\pi R\rho}$$

$$\frac{g'}{g} = \frac{R-h}{R} = 1 - \frac{h}{R}$$

$$\therefore \boxed{g' = g\left(1 - \frac{h}{R}\right)}$$

Therefore, the value of acceleration due to gravity decreases with depth.

* Acceleration due to gravity at the centre of earth $h = R$

$$\therefore g' = g\left(1 - \frac{R}{R}\right) = 0$$

$$g_{centre} = 0$$

* Acceleration due to gravity is maximum at the surface of earth.

Satellites

A satellite is a body which is constantly revolving in an orbit around a planet

- Natural satellites (eg; Moon)
- Artificial Satellites (eg; Sputnik-I)

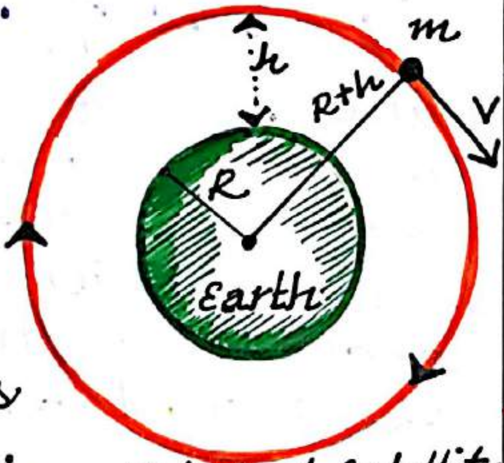
Derive an expression for Orbital velocity and time period of a satellite?

The velocity with which a satellite

moves in its closed orbit is called orbital velocity

consider a satellite of mass 'm' in a closed orbit of Radius $R+h$ with orbital velocity V.

Let 'M' be the mass of the earth and 'R' its radius.



orbit of Satellites

when satellite is in stable orbit, the centripetal force is provided by the gravitational force.

$$\frac{mV^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$V^2 = \frac{GM}{(R+h)}$$

$$\boxed{V = \sqrt{\frac{GM}{R+h}} \quad \therefore V = \sqrt{\frac{gR^2}{R+h}}} \quad \ldots (1)$$

$$GM = gR^2$$

If the orbit is very close to the earth 'h' is negligible to 'R'.

$\therefore$ (1) becomes

$$\boxed{V = \sqrt{\frac{GM}{R}} = \sqrt{\frac{gR^2}{R}} \quad V = \sqrt{gR}}$$

* The velocity corresponding to minimum orbit is called first cosmic velocity

first cosmic velocity $V = \sqrt{\frac{GM}{R}}$

$$V = \sqrt{gR}$$

$$V = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \text{ km/s}$$

TIME PERIOD OF A SATELLITE

Time period of a satellite is the time taken by the satellite to revolve once round the earth.

$$\text{Time period} = \frac{\text{circumference}}{\text{orbital velocity}}$$

$$T = \frac{2\pi(R+h)}{V} = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$T = 2\pi(R+h)\sqrt{\frac{R+h}{GM}}$$

$$\boxed{T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

For minimum orbit 'h' is negligible to 'R'.

$$\therefore \boxed{T = 2\pi\sqrt{\frac{R^3}{GM}}} = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

GeoStationary Satellite

* state the conditions which should be satisfied so that a satellite appears stationary?

Geostationary satellites revolves around the earth from west to east in an orbit (parking orbit) in the equatorial plane of earth at a height of 36000km above the surface of the earth.

The orbit in which geostationary satellites are revolving is called parking orbit or geo stationary orbit.

conditions

1. The period of revolution ~~of~~ around the earth should be same as that of earth (exactly 24 hours)
2. They should rotate in the same direction as the rotation of the earth (west to east)
3. It should rotate in an orbit in equatorial plane.

* GRAVITATIONAL FIELD

It is the space around a body where gravitational influence is felt.

* Define Intensity of gravitational field?

Intensity of gravitational field at a point can be defined as the force experienced by a unit mass placed at that point.

$$I = \frac{F}{m} = \frac{GMm}{R^2 \cdot m} = \frac{GM}{R^2}$$

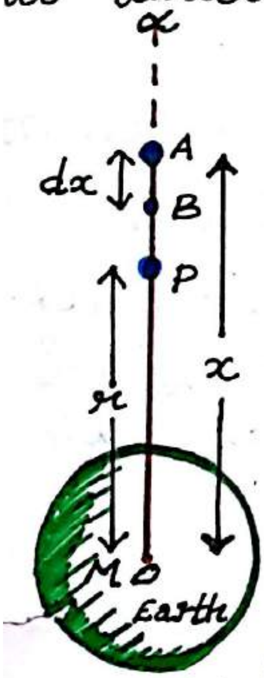
$$\boxed{I = \frac{GM}{R^2}} \quad \therefore GM = gR^2$$

$$\boxed{I = g}$$

Therefore intensity of gravitational field at a point is equal to acceleration due to gravity at that point.

* **Define gravitational potential. Derive the expression for the potential at a point due to a point mass.?**

gravitational potential can be defined as the work done in bringing unit mass from infinity to that point.



consider P be a point at a distance 'r' from earths mass M

consider a point A at a distance x from M.

Gravitational potential = work done in bringing a unit mass from infinity to the point A.

work done in moving the unit mass through infinitesimally small distance 'dx' is

$$dw = F\,dx$$

$$dw = \frac{GM}{x^2}\,.\,dx.$$

Total work done to move from ∞ to P

$$\int dw = \int_{\infty}^{r} \frac{GM}{x^2}\,dx = GM\int_{\infty}^{r} x^{-2}\,dx$$

$$W = GM\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r} = -GM\left[x^{-1}\right]_{\infty}^{r}$$

$$W = -GM\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GM}{r}$$

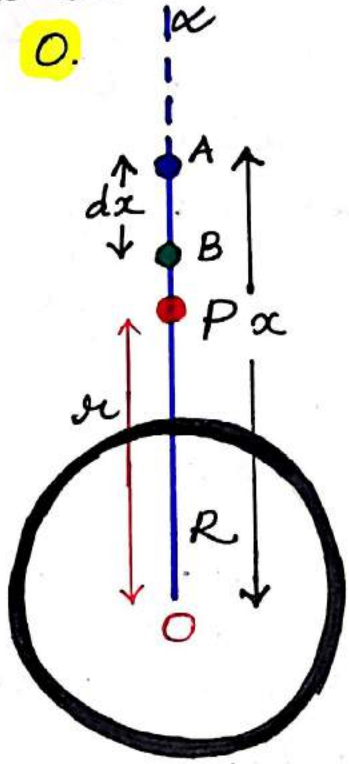
potential $\boxed{V = -\frac{GM}{r}}$

Define gravitational potential energy. Derive the expression for gravitational potential energy?

Gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against gravitational force.

consider that earth is a uniform sphere of radius R and mass M.

Let us calculate gravitational potential energy of the body 'm' lies at the point 'P' at a distance 'r' from O.



Gravitational potential energy at point P = W
= work done in bringing the body from infinity to P.

Work done in moving the body through infinitesimally small distance 'dx' is

$$dW = F\,dx = \frac{GMm\,dx}{x^2}$$

Total work done to move from ∞ to P

$$\int dW = \int_{\infty}^{r} \frac{GMm}{x^2}\,dx$$

$$= GMm \int_{\infty}^{r} x^{-2}\,dx$$

$$W = GMm\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$W = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

This work done is stored as gravitational potential energy (U)

$$U = -\frac{GMm}{r}$$

Discussion of Result

1) If the body is moved from a point at a distance r_1 to a point at distance r_2 ($r_1 > r_2$) than change in gravitational P.E

$$\Delta U = -GMm\left[\frac{1}{x}\right]_{r_1}^{r_2}$$

$$\Delta U = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

2) If the body is moved from the surface of earth $r_1 = R$ to a point at a distance 'h' above the surface of earth ($r_2 = R+h$), then the change in potential energy =

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\Delta U = GMm\left[\frac{R+h-R}{R(R+h)}\right]$$

$$\Delta U = GMm\left[\frac{h}{R(R+h)}\right]$$

if $h << R$ neglect h

$$\Delta U = \frac{GMmh}{R^2}$$

$$GM = gR^2 \Rightarrow \Delta U = \frac{gR^2mh}{R^2}$$

$$\therefore \Delta U = mgh$$

$$\Delta U = P.E$$

Derive the expression for the energy of an orbiting Satellite

when a satellite of mass 'm' revolves around a planet of mass M in an orbit, it possess both potential and kinectic energy.

consider the orbit is at a height 'h' from the surface of the earth.

$$r = R+h$$

potential energy of satellite in its orbit.

$$U = -\frac{GMm}{R+h} \quad \cdots \to (1)$$

As satellite is revolving in its orbit around the planet

centripetal force = gravitational force.

$$\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$mv^2 = \frac{GMm}{R+h} \quad \cdots \to (2)$$

kinectic energy of satellite $= \frac{1}{2}mv^2 \quad \cdots\cdots (3)$

$$K.E = \frac{1}{2}\frac{GMm}{R+h} \quad \cdots\cdots (4)$$

$$\therefore \text{Total Energy} = P.E + K.E$$

$$= -\frac{GMm}{R+h} + \frac{1}{2}\frac{GMm}{R+h}$$

$$T.E = E = -\frac{GMm}{2(R+h)}$$

Binding energy of satellite.

* Total energy is negative. It makes the system to bind.

What is meant by escape velocity? Derive the expression for escape velocity?

The minimum velocity with which a body must be projected so that it may escape from the gravitational field of the earth is called escape velocity.

Let 'M' be the mass of the earth and 'R' its radius. Let 'V_e' be the escape velocity.

K.E of the body near the surface of earth $= \frac{1}{2}mv^2$

P.E of the body on the surface of earth

$= -\frac{GMm}{R}$ [when it moves from α to R]

$P.E = \frac{GMm}{R}$ [when the body moves from R to α]

At the point where body escapes

$K.E = P.E$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$v^2 = \frac{2GM}{R}$$

$$\boxed{V = \sqrt{\frac{2Gm}{R}}} \quad Gm = gR^2$$

$$\therefore V = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

$$\boxed{V = \sqrt{2gR}}$$

escape velocity from earth = 11.2 km/s