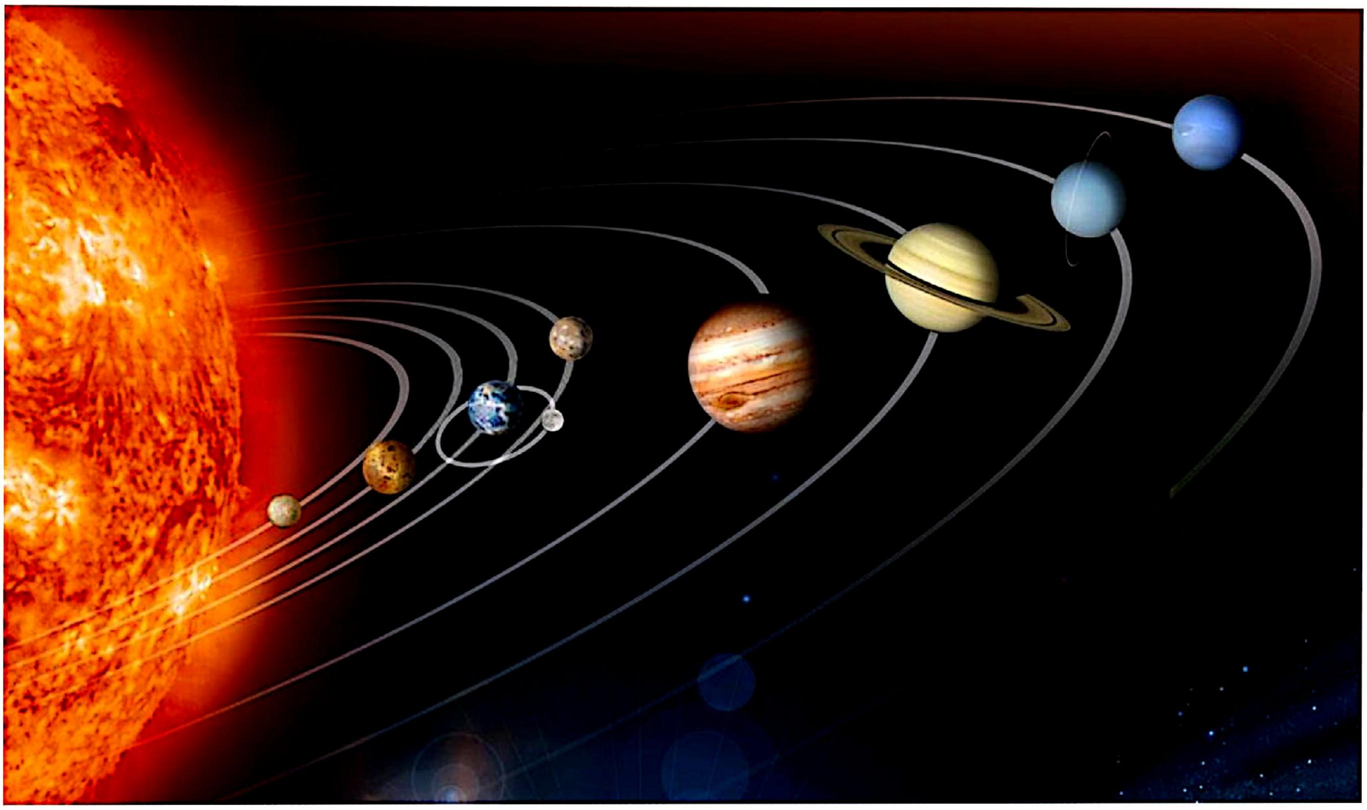
Chapter–8:

Gravitation



CBSE CLASS XI NOTES

\

Dr. SIMIL RAHMAN

Guidelines to NCERT Exercises

8.1. *Answer the following :*

(*a*) *You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means ?*

[Central Schools 08]

(*b*) *An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity ?*

(*c*) *If you compare the gravitational force on the Earth due to the Sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of Sun. Why ?*

Ans. (*a*) No. The shell does not shield other bodies lying outside it from exerting gravitational forces on a particle lying inside it.

(*b*) Yes. If the size of spaceship orbiting around the earth is large enough, the astronaut inside the spaceship can detect the variation in g.

(*c*) The tidal effect depends inversely upon the cube of the distance whereas the gravitational force varies inversely as the square of the distance. As the moon is closer to earth than the sun, so its tidal effect is greater than that of the sun. The ratio of these two effects is

$$\frac{T_m}{T_s} = \left(\frac{d_s}{d_m}\right)^3 = \left(\frac{1.5 \times 10^{11}}{3.8 \times 10^8}\right)^3 = 61.5 \times 10^6.$$

8.2. *Choose the correct alternative :*

(*i*) *Acceleration due to gravity increases/decreases with increasing altitude.*

(*ii*) *Acceleration due to gravity increases/decreases with increasing depth (assume the Earth to be a sphere of uniform density).*

(*iii*) *The effect of rotation on the effective value of acceleration due to gravity is greatest at the equator/poles.*

(*iv*) *Acceleration due to gravity is independent of mass of the Earth/mass of the body.*

(*v*) *The formula* $-GMm(1/r_2 - 1/r_1)$ *is more/less accurate than the formula* $mg(r_2 - r_1)$ *for the difference of potential energy between two points* r_2 *and* r_1 *distance away from the centre of the Earth.*

Ans. (*i*) At altitude h, $g_h = g\left(1 - \frac{2h}{R}\right)$.

Thus acceleration due to gravity *decreases* with increasing altitude.

(*ii*) At depth d, $g_d = g\left(1 - \frac{d}{R}\right)$

Thus acceleration due to gravity *decreases* with increasing depth.

(*iii*) At latitude λ, $g_\lambda = g\left(1 - \frac{R\omega^2 \cos^2 \lambda}{g}\right)$

For maximum variation (decrease) of g, $\cos \lambda = 1$ or $\lambda = 0°$.

Thus the effect of rotation on the effective value g is greatest at the *equator* (for which $\lambda = 0°$.)

(*iv*) As $g = \frac{GM}{R^2}$, where M is the mass of the earth.

Thus acceleration due to gravity is independent of mass m of the body.

(*v*) The formula $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is *more* accurate than the formula $mg(r_2 - r_1)$, because the value of g varies from place to place.

8.3. *Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?*

Ans. Let period of revolution of the earth

$$= T_e$$

∴ Period of revolution of planet,

$$T_p = \frac{1}{2} T_e$$ [∵ The planet goes around the sun twice as fast as earth]

Orbital size of the earth, $r_e = 1$ AU

Orbital size of the planet, $r_p = ?$

From Kepler's third law of planetary motion

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3}$$

$$\therefore \quad r_p = \left(\frac{T_p}{T_e}\right)^{2/3} r_e = \left(\frac{\frac{1}{2} T_e}{T_e}\right)^{2/3} \times 1 \text{ AU}$$

$$= (0.5)^{2/3} \text{ AU} = \mathbf{0.63\ AU}.$$

8.4. I_0, *one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is* 4.22×10^8 *m. Show that the mass of Jupiter is about one thousandth that of the sun.*

Ans. Orbital period of Jupiter's satellite,

$$T = 1.769 \text{ days} = 1.769 \times 24 \times 60 \times 60 \text{ s}$$

Orbital radius of Jupiter's satellite,

$$R = 4.22 \times 10^8 \text{ m}$$

Mass of Jupiter is given by

$$M_J = \frac{4\pi^2}{G} \cdot \frac{R^3}{T^2} = \frac{4\pi^2}{G} \cdot \frac{(4.22 \times 10^8)^3}{(1.769 \times 24 \times 60 \times 60)^2}$$

Orbital period of the earth around the sun,

$$T = 1 \text{ year} = 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Orbital radius of the earth,

$$R = 1.496 \times 10^{11} \text{ m}$$

Mass of the sun,

$$M_S = \frac{4\pi^2}{G} \cdot \frac{(1.496 \times 10^{11})^3}{(365.25 \times 24 \times 60 \times 60)^2}$$

$$\therefore \frac{M_J}{M_S} = \frac{(4.22 \times 10^8)^3}{(1.769 \times 24 \times 60 \times 60)^2} \times \frac{(365.25 \times 24 \times 60 \times 60)^2}{(1.496 \times 10^{11})^3}$$

$$= \frac{1}{1046} \approx \frac{1}{1000}.$$

Hence the mass of Jupiter is about one thousandth that of the sun.

8.5. *Let us assume that our galaxy consists of 2.5×10^{11} s stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the milky way to be 10^5 ly.*

Ans. Mass of galaxy,

$$M = 2.5 \times 10^{11} \text{ solar masses}$$

$$= 2.5 \times 10^{11} \times 2 \times 10^{30} \text{ kg} = 5 \times 10^{41} \text{ kg}$$

Orbital radius of the star,

$$R = 50{,}000 \text{ ly} = 50{,}000 \times 9.46 \times 10^{15} \text{ m}$$

$$= 4.73 \times 10^{20} \text{ m}$$

As $$M = \frac{4\pi^2}{G} \cdot \frac{R^3}{T^2}$$

$$\therefore \quad T^2 = \frac{4\pi^2}{G} \cdot \frac{R^3}{M} = \frac{4 \times 9.87 \times (4.73 \times 10^{20})^3}{6.67 \times 10^{-11} \times 5 \times 10^{41}}$$

$$= 1.25275 \times 10^{32}$$

$$\therefore \quad T = 1.12 \times 10^{16} \text{ s} = \frac{1.12 \times 10^{16}}{365.25 \times 24 \times 60 \times 60} \text{ years}$$

$$= \mathbf{3.54 \times 10^8 \text{ years.}}$$

8.6. *Choose the correct alternatives :*

(*a*) *If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.*

(*b*) *The energy required to rocket an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of the Earth's influence.*

Ans. (*a*) The total energy of an orbiting satellite is negative of its kinetic energy.

(*b*) The energy required to rocket an orbiting satellite out of Earth's gravitational influence is *less* than the energy required to project a stationary object at the same height out of Earth's influence.

8.7. *Does the escape speed of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched ? Explain your answer.*

Ans. Escape velocity, $v_e = \sqrt{\frac{2GM}{r}}$.

(*a*) Escape velocity is independent of the mass *m* of the body to be projected.

(*b*) As the gravitational potential, $V = GM / R$ depends slightly on the latitude of the point, so escape velocity also depends (slightly) on the latitude of the location from where the body is projected.

(*c*) Escape velocity is independent of the direction of projection.

(*d*) As the gravitational potential depends on the height of location, so escape velocity also depends on height of location.

8.8. *A comet orbits the sun in a highly elliptical obit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit ? Neglect any mass loss of the comet when it comes very close to the sun.*

Ans. (*a*) According to Kepler's second law of planetary motion, a planet moves faster when it is close to the sun and moves slower, when away from the sun. So its linear speed keeps on changing.

(*b*) As the planet moves under the influence of a purely radial force, its angular momentum remains constant.

(*c*) As the linear speed of the planet continuously changes, so its kinetic energy also keeps on changing.

(*d*) As the distance of the planet from the sun continuously changes, so its potential energy keeps on changing.

(*e*) Total energy of the planet always remains constant.

8.9. *Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.*

Ans. (*b*), (*c*) and (*d*) are affected in space.

In the following two exercises, choose the correct answer from among the given ones :

8.10. *The gravitation intensity at the centre C of the drumhead defined by a hemispherical shell has the direction indicated by the arrow [see Fig. 8.46]*

(i) a, (ii) b, (iii) c, (iv) zero.

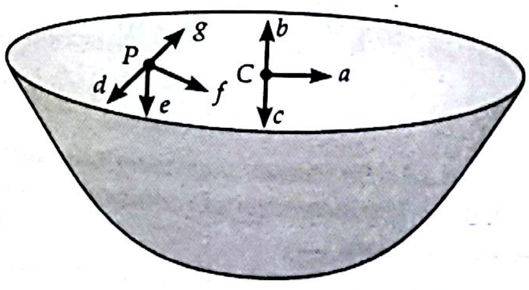


Fig. 8.46

Ans. Complete the hemisphere to sphere, as shown in Fig. 8.47. Gravitational potential V is a constant. So gravitational intensity $\left(E=-\frac{dV}{dr}\right)$ is zero at C (for the complete sphere). For the hemisphere, the net gravitational intensity will point along (c) at the centre C.

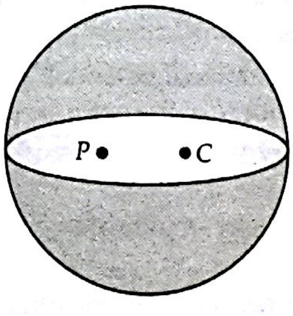


Fig. 8.47

8.11. *For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.*

Ans. By the similar reasoning as in the above problem, the direction of gravitational intensity at P will be along (e).

8.12. *A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero ? Mass of sun $= 2\times10^{30}$ kg, mass of the earth $= 6\times10^{24}$ kg. Neglect the effect of other planets etc. Orbital radius $= 1.5\times10^{11}$ m.*

Ans. Given $M_s = 2\times10^{30}$ kg,

$$M_e = 6\times10^{24}\text{ kg},\quad r = 1.5\times10^{11}\text{ m}$$

Let m be the mass of the rocket. Let at distance x from the earth, the gravitational force on the rocket be zero. Then at this distance,

Gravitational pull of the earth on the rocket
= Gravitational pull of the sun on the rocket

i.e., $$\frac{GM_e m}{x^2} = \frac{GM_s m}{(r-x)^2}$$

or $$\frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

or $$\frac{r-x}{x} = \sqrt{\frac{M_s}{M_e}} = \sqrt{\frac{2\times10^{30}}{6\times10^{24}}} = \frac{10^3}{\sqrt{3}} = 577.35$$

or $$r - x = 577.35\,x$$

or $$578.35\,x = r = 1.5\times10^{11}$$

or $$x = \frac{1.5\times10^{11}}{578.35} = \mathbf{2.59\times10^8\ m.}$$

8.13. *How would you 'weigh the sun', that is estimate its mass ? The mean orbital radius of the earth around the sun is 1.5×10^8 km.*

Ans. Refer to the solution of Example 4 on page 8.6.

8.14. *A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun ?*

Ans. According to Kepler's law of periods,

$$\left[\frac{T_S}{T_E}\right]^2 = \left[\frac{r_S}{r_E}\right]^3$$

But $$\frac{T_S}{T_E} = 29.5 \text{ and } r_E = 1.5\times10^8\text{ km}$$

$\therefore$ $$(29.5)^2 = \left(\frac{r_S}{1.5\times10^8}\right)^3$$

or $$r_S = 1.5\times10^8\times(29.5)^{2/3} = \mathbf{14.32\times10^8\ km.}$$

8.15. *A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?*

Ans. Here $mg = 63$ N, $h = R/2$

As $$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+\frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$g_h = \frac{4}{9}g$$

$\therefore$ $$mg_h = \frac{4}{9}mg = \frac{4}{9}\times63 = \mathbf{28\ N.}$$

8.16. *Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface ?*

Ans. Here $mg = 250$ N, $d = R/2$

Acceleration due to gravity at depth $d = R/2$, below the earth's surface will be

$$g_d = g\left(1-\frac{d}{R}\right) = g\left(1-\frac{R/2}{R}\right) = \frac{g}{2}$$

$\therefore$ New weight $= mg_d = \frac{mg}{2} = \frac{250}{2} = \mathbf{125\ N.}$

8.17. *A rocket is fired vertically with a speed of 5 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of earth $= 6.0\times10^{24}$ kg, mean radius of the earth $= 6.4\times10^6$ m, $G = 6.67\times10^{-11}$ Nm^2 kg^{-2}.*

Ans. Here $v = 5$ kms^{-1} $= 5000$ ms^{-1},

$$M = 6.0\times10^{24}\text{ kg},\quad R = 6.4\times10^6\text{ m},$$
$$G = 6.67\times10^{-11}\text{ Nm}^2\text{ kg}^{-2}.$$

Suppose the rocket goes upto a height h before returning to the earth. Clearly at this height, velocity of rocket will become zero. By the law of conservation of energy,

(K.E. + P.E.) at the earth's surface
= (K.E. + P.E.) at height h

or $$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{(R+h)}$$

$$v^2 - \frac{2GM}{R} = -\frac{2\,GM}{R+h}$$

or

$$\frac{2GM}{R+h} = \frac{2GM}{R} - v^2 = \frac{2GM - v^2 R}{R}$$

or

$$R + h = \frac{2GMR}{2GM - v^2 R}$$

or

$$= \frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 6.4 \times 10^6}{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} - (5000)^2 \times 6.4 \times 10^6}$$

$$= \frac{5.12 \times 10^{21}}{6.4 \times 10^{14}} = 8.0 \times 10^6 \text{ m}$$

$$h = 8.0 \times 10^6 - R = 8.0 \times 10^6 - 6.4 \times 10^6$$

$$= \mathbf{1.6 \times 10^6 \text{ m.}}$$

8.18. *The escape velocity of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth ? Ignore the presence of the sun and other planets.*

Ans. Escape velocity, $v_e = 11.2 \text{ kms}^{-1}$

Velocity of projection, $v = 3v_e$

Let m be the mass of projectile and v_0 be the velocity of the projectile after escaping gravitational pull.

By the law of conservation of energy,

$$\frac{1}{2} mv_0^2 = \frac{1}{2} mv^2 - \frac{1}{2} mv_e^2$$

or

$$v_0 = \sqrt{v^2 - v_e^2} = \sqrt{(3v_e)^2 - v_e^2} = \sqrt{8v_e^2}$$

$$= \sqrt{8 \times (11.2)^2} = 22.4\sqrt{2} = \mathbf{31.68 \text{ kms}^{-1}}.$$

8.19. *A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence ? Mass of the satellite = 200 kg, mass of the earth = 6.0 × 10^{24} kg, radius of the earth = 6.4 × 10^6 m, G = 6.67 × 10^{-11} Nm^2 kg^{-2}.*

Ans. Total energy of the satellite in the orbit is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2} mv^2 - \frac{GMm}{R+h}$$

$$= \frac{1}{2} m \frac{GM}{R+h} - \frac{GMm}{R+h} = -\frac{GMm}{2(R+h)}$$

$$\left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

Here $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, $M = 6.0 \times 10^{24}$ kg

$m = 200$ kg, $R = 6.4 \times 10^6$ m,

$h = 400$ km $= 400 \times 10^3$ m

$$\therefore \quad E = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2(6.4 \times 10^6 + 400 \times 10^3)}$$

$$= -\frac{6.67 \times 6.0}{6.8} \times 10^9 = -5.89 \times 10^9 \text{ J}$$

Energy required to rocket the satellite

$$= \mathbf{5.89 \times 10^9 \text{ J.}}$$

8.20. *Two stars each of 1 solar mass (= 2 × 10^{30} kg) are approaching each other for a head-on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. Use the known value of G.*

Ans. Mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $R = 10^7$ m

Initial P.E. of the two stars when they are 10^{12} m apart,

$$U_i = -\frac{GM \times M}{r} = -\frac{GM^2}{10^{12}} \qquad [r = 10^9 \text{ km} = 10^{12} \text{ m}]$$

When the stars are just to collide, distance between their centres

= twice the radius of each star $= 2R = 2 \times 10^7$ m

Final P.E. of the two stars when they just collide,

$$U_f = -\frac{GM \times M}{2R} = -\frac{GM^2}{2 \times 10^7}$$

Change in P.E. of the stars

$$= U_i - U_f = -\frac{GM^2}{10^{12}} - \left(-\frac{GM^2}{2 \times 10^7}\right)$$

$$= \frac{GM^2}{2 \times 10^7} - \frac{GM^2}{10^{12}} \simeq \frac{GM^2}{2 \times 10^7} \qquad \left[\because \frac{GM^2}{10^{12}} << \frac{GM^2}{2 \times 10^7}\right]$$

Let v be the velocity of each star just before collision. Then

Change in K.E. of the stars

= Final K.E. – Initial K.E.

$$= 2 \times \frac{1}{2} Mv^2 - 0 = Mv^2$$

By conservation of energy,

$$Mv^2 = \frac{Gm^2}{2 \times 10^7}$$

or

$$v = \sqrt{\frac{GM}{2 \times 10^7}} = \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{2 \times 10^7}}$$

$$= \mathbf{2.6 \times 10^6 \text{ ms}^{-1}}.$$

8.21. *Two heavy spheres each of mass 100 kg and radius 0.1 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres ?*

Take G = 6.67 × 10^{-11} Nm^2 kg^{-2}.

Ans. Let A and B be the positions of the two spheres and P be the midpoint of AB.

Gravitational field at P due to mass at A,

$$E_1 = \frac{GM}{r^2} = \frac{G \times 100}{(0.5)^2}, \text{ along } PA$$

Gravitational field at P due to mass at B,

$$E_2 = \frac{G \times 100}{(0.5)^2}, \text{ along } PB$$

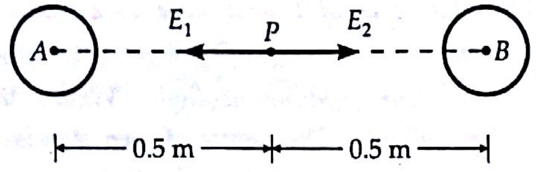


Fig. 8.48

As E_1 and E_2 have equal magnitudes but opposite directions, so the resultant gravitational field at P is zero.

As gravitational potential is a scalar quantity, so total potential at P is

$$V = V_A + V_B = -\frac{GM}{r} - \frac{GM}{r} = -\frac{2GM}{r}$$

$$= -\frac{2 \times 6.67 \times 10^{-11} \times 100}{0.5} = -\mathbf{2.668 \times 10^{-8}\ J\ kg^{-1}}.$$

8.22. *A geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of the satellite ? Mass of the earth = 6×10^{24} kg and radius = 6400 km.*

Ans. Here $M = 6 \times 10^{24}$ kg, $R = 6400$ km,

$h = 36{,}000$ km

$r = R + h = 6400 + 36{,}000 = 42{,}400 \text{ km} = 4.24 \times 10^7$ m

Gravitational potential,

$$V = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4.24 \times 10^7}$$

$$= -\mathbf{9.43 \times 10^6\ J\ kg^{-1}}.$$

8.23. *A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rev/s. Will an object placed on its equator remain stuck to its surface due to gravity ? Mass of the sun = 2×10^{30} kg.*

Ans. Here $M = 2.5 \times$ Mass of the sun

$= 2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ kg

$R = 12 \text{ km} = 12 \times 10^3$ m

Acceleration due to gravity on the surface of the star is

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30}}{(12 \times 10^3)^2} = 2.316 \times 10^{12}\ \text{ms}^{-2}$$

Now $\nu = 1.5$ rps, so $\omega = 2\pi\nu = 3\pi$ rad s^{-1}

Centripetal acceleration of the object $= R\omega^2$

$= 12 \times 10^3 \times (3 \times 3.14)^2 = 1.065 \times 10^6\ \text{ms}^{-2}$

As the acceleration due to gravity on the surface of the star is greater than the centripetal acceleration of the object, so the object will remain struck to its surface.

8.24. *A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system ? Mass of the spaceship 1000 kg, mass of the sun = 2×10^{30} kg, mass of Mars = 6.4×10^{23} kg, radius of Mars 3395 km, radius of the orbit of Mars = 2.28×10^8 km, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.*

Ans. Energy of the spaceship in the orbit is

$$E = \text{K.E.} + \text{P.E.} = \frac{1}{2} mv^2 - \frac{GMm}{(R+h)}$$

$$= \frac{1}{2} m \cdot \frac{GM}{R+h} - \frac{GMm}{R+h} \qquad \left[\because v = \sqrt{\frac{GM}{R+h}}\right]$$

$$= -\frac{GMm}{2(R+h)}$$

But $m = 1000$ kg, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2},

$M = 2 \times 10^{30}$ kg

$R = 2.28 \times 10^8 \text{ km} = 2.28 \times 10^8 \times 10^3$ m,

$h = 3395 \text{ km} = 3395 \times 10^3$ m

$$\therefore \quad E = -\frac{6.67 \times 10^{-11} \times 2 \times 10^{30} \times 1000}{2(2.28 \times 10^8 \times 10^3 + 3395 \times 10^3)}$$

$$= -2.9 \times 10^{11}\ \text{J}.$$

Energy needed to rocket the spaceship out of solar system = $\mathbf{2.9 \times 10^{11}}$ **J.**

8.25. *A rocket is fired vertically from the surface of Mars with a speed of 2 kms^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it ? Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.*

Ans. Total energy of the rocket

$$E = \frac{1}{2} mv^2 - \frac{GMm}{R}$$

Since 20% energy is lost, hence energy remained

$$= 80\% \text{ or } E = \frac{80}{100} E = \frac{4}{5} E = \frac{4}{5}\left[\frac{1}{2} mv^2 - \frac{GMm}{R}\right].$$

At the highest point, distant h from the surface of Mars, its total energy will be potential. Hence

$$\frac{4}{5}\left[\frac{1}{2} mv^2 - \frac{GMm}{R}\right] = -\frac{GMm}{R+h}$$

or $$\frac{2}{5} m\left(v^2 - \frac{2GM}{R}\right) = -\frac{GMm}{R+h}$$

or $$\frac{2}{5} \cdot \frac{v^2 R - 2GM}{R} = -\frac{GM}{R+h}$$

or $$R + h = -\frac{5RGM}{2(v^2 R - 2GM)}$$

But $R = 3395 \text{ km} = 3395 \times 10^3$ m,

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

$M = 6.4 \times 10^{23}$ kg, $v = 2 \text{ kms}^{-1} = 2 \times 10^3$ ms^{-1}

$\therefore \quad R + h$

$$= -\frac{5 \times 3395 \times 10^3 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{2[(2 \times 10^3)^2 \times 3395 \times 10^3 - 2 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{23}]}$$

$$= -\frac{7.25 \times 10^{20}}{2[1.36 \times 10^{13} - 8.54 \times 10^{13}]}$$

$$= \frac{7.25 \times 10^7}{2 \times 7.18} = 5.05 \times 10^6 \text{ m}$$

or $h = 5.05 \times 10^6 - 3395 \times 10^3$

$= 1655 \times 10^3 \text{ m} = $ **1655 km.**