Chapter–4

Laws of Motion

CBSE CLASS XI NOTES

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Newtons law's of motion

First law (law of Inertia)

Every body continues in its state of rest or of uniform motion in a straight line unless an external unbalance force acts on it.

Second law

The rate of change of momentum is directly proportional to the external unbalance force and takes place in the direction of force.

Third law

To every action there is an equal and opposite reaction.

Define momentum?

It is defined as the Quantity of motion possessed by a moving body and is measured as the product of mass and velocity

$$\vec{p} = m\vec{v}$$

* unit $kg\,ms^{-1}$
* Dimension MLT^{-1}

show that $F = ma$?

According to Newtons second law the rate of change of linear momentum of a body is directly proportional to the external force applied on it and change in momentum takes place in the direction of force.

Let $\vec{P}$ be the momentum of a body

$\vec{F}$ - external force acting on it for a time interval Δt

According to Newtons second law.

$$\vec{F} \propto \frac{\Delta \vec{P}}{\Delta t}$$

$$\propto \frac{\Delta (m\vec{v})}{\Delta t}$$

$$\vec{F} = k \frac{\Delta (m\vec{v})}{\Delta t} = k m \frac{\Delta \vec{v}}{\Delta t}$$

where 'k' is a constant of proportionality, k = 1

$$\vec{F} = m\frac{\Delta \vec{V}}{\Delta t} = m\vec{a}$$

$$\vec{F} = m\vec{a}$$

where $\vec{a}$ is the acceleration of the body.

$$F = ma$$

If m = 1 kg, a = $1 m/s^2$ then F = 1N.

* S.I unit of force is Newton [N]

Differentiate Impulse and Impulsive force.

Impulse of a force is the product of the force and the time during which the force acts on the body.

$$\text{Impulse} = \text{Force} \times \text{time}$$

$$I = F \times dt.$$

unit ⇒ Ns,
Dimension ⇒ MLT^{-1}

* The force acting for very short time is called an impulsive force. It is measured as the rate of change of momentum.

eg; Force exerted by a bat when it hits a ball.

S.T Impulse is equal to change in momentum.

consider an object on which impulsive force 'F' acts for a short interval of time dt.

* According to Newtons second law

$$F = \frac{dP}{dt}$$

$$F \times dt = dP$$

$$I = dP$$

Impulse = change in momentum of the body.

Application

1. while catching a ball, a cricket player lowers his hands to save himself from getting hurt.
2. car, buses, trucks etc are provided with a spring system to avoid Jerks.

which law of motion is the real law of motion? Deduce Newtons first law and third law from second law of motion.

a) Newton's second law is the real law of motion.

b) Deduction of first law from second law.

According to Newtons second law, the external force $\vec{F}$, applied on a body and the acceleration '$\vec{a}$' produced are related to each other

$$\vec{F} = m\vec{a}$$

If no external force acts on the body $\vec{F} = 0$

then $m\vec{a} = \vec{F} = 0$

or $a = 0$

we know that $v = u + at$

If $a = 0$, $v = u + 0$

$$v = u$$

It implies that if no external force acts on a body. Then the body initially at rest ($u=0$), will remain at rest ($v=0$).

* If it is moving with some velocity (u) it will move with same velocity.

* hence newtons first law can be deduced from second law.

* Deduction of the third law from the second law.

consider two bodies A and B. let them collide. During collision let the body A exert a force F_{AB} on B (action) for a time Δt. and the body B exerts a force on A, F_{BA} (reaction) for the same time Δt.

change in linear momentum = Force × time

change in linear momentum of B $= F_{AB} \cdot \Delta t$

change in linear momentum of A $= F_{BA} \cdot \Delta t$

total change $= F_{AB}\Delta t + F_{BA}\Delta t$

Here no external force acts on it, $F = 0$.

$\therefore a = 0$

$\therefore$ Total change in momentum $= 0$

$$F_{AB}\Delta t + F_{BA}\Delta t = 0$$

$$(F_{AB} + F_{BA})\Delta t = 0$$

$$F_{AB} + F_{BA} = 0$$

$$\therefore F_{AB} = -F_{BA}$$

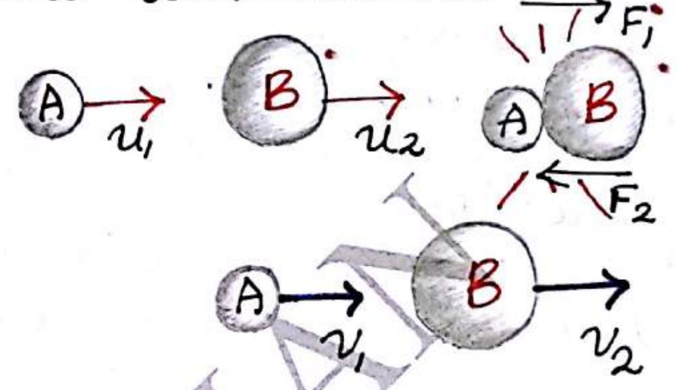
Hence action and reaction are equal and opposite. Hence Newtons third law of motion is contained in Newtons second law of motion.

* State and prove law of conservation of linear momentum based on third law.

law of conservation of linear momentum states that if no external force acts on a system, the linear momentum of the system remains constant.

Proof:-

consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 along a straight line in the same direction.



After collision let them move with velocities V_1 and V_2 in the same direction. let Δt be the time of contact during collision.

$$F_1 = \frac{m_1(V_1 - u_1)}{\Delta t}$$

$$F_2 = \frac{m_2(V_2 - u_2)}{\Delta t}$$

According to Newtons 3rd law of motion

$$F_2 = -F_1$$

$$\frac{m_2(V_2 - u_2)}{\Delta t} = \frac{-m_1(V_1 - u_1)}{\Delta t}$$

$$m_2(V_2 - u_2) = -m_1(V_1 - u_1)$$

$$m_2V_2 - m_2u_2 = -m_1V_1 + m_1u_1$$

$$m_1V_1 + m_2V_2 = m_1u_1 + m_2u_2$$

Final momentum after collision = Initial momentum before collision.

Short way

$P_A, P_B \rightarrow$ momentum before collision

$P'_A, P'_B \rightarrow$ mometum after collision

$F_1 \Delta t = P'_A - P_A$

$F_2 \Delta t = P'_B - P_B$

since $F_1 = -F_2$.

$P'_A - P_A = -(P'_B - P_B)$

$P'_A - P_A = -P'_B + P_B$

$\therefore P'_A + P'_B = P_A + P_B$

ie, final p = Initial p.

Recoil of gun

Let 'M' be the mass of the gun and 'm' be the mass of the bullet. Initially both the bullet and gun are at rest. On firing the gun bullet moves with a velocity v and gun moves back with a recoil velocity V

* According to law of conservation of momentum.

Total momentum before firing = Total momentum after firing.

$$MV + mv = 0.$$

$$MV = -mv$$

$$V = -\frac{mv}{M}$$

-ve sign shows V and v is opposite direction.

* Frames of Reference.

A system of co-ordinate axes which defines the position of a particle in two or three dimensional space is called a frame of reference.

- → Inertial frame of reference
- → Non-inertial frame of reference.

(i) Inertial frame of reference

A frame of reference relative to which Newtons law of Inertia is valid is called an inertial frame
eg; earth's frame of reference is approximately inertial for most terrestrial phenomenon.

(ii) Non Inertial frame of reference.

All those frames of reference in which Newton's law of inertia does not hold good are called non-inertial frames.

* All accelerated frames are non-inertial.

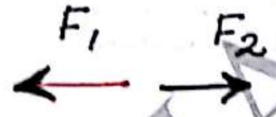
* Equilibrium of concurrent forces.

If a number of forces act at the same point they are called concurrent forces.

* condition to be in equilibrium.

concurrent forces are said to be in equilibrium when the resultant force is zero.

(1) A body acted upon by two forces will be in equilibrium if two forces are equal and opposite.

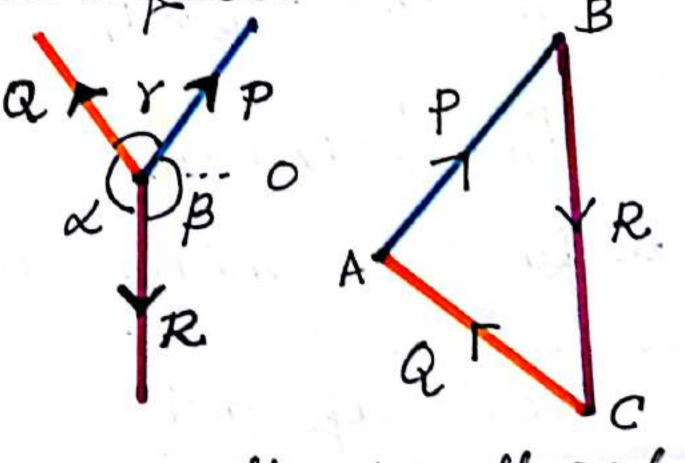


$F_1 = -F_2$

$F_1 + F_2 = 0$

(2) If three coplanar concurrent forces are acting at a point O. then 'O' will be in equilibrium if one the following conditions are satisfied.

(a)



the resultant of forces at 'O' should be zero.

$$\vec{P} + \vec{Q} + \vec{R} = 0$$

$$\vec{P} + \vec{Q} = -\vec{R}$$

(b) If 'P', 'Q' and 'R' represented by the sides of a Δgle. O will be in eq

...uilibruim.

i.e., $\frac{P}{AB} = \frac{Q}{CA} = \frac{R}{BC} = \text{constant}$

(c) Each force must be proportional to the sine of the angle between the other two forces. (Lami's theorem)

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = \text{const}$$

FRICTION

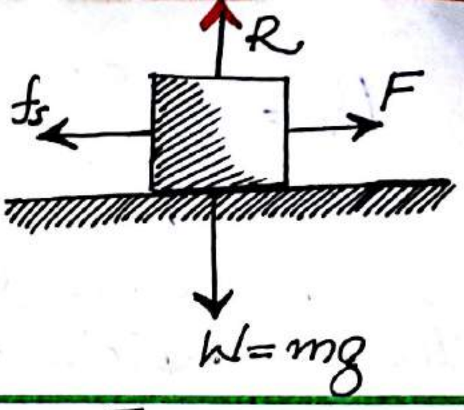
Friction is the opposing force that is set up between the surfaces of contact, when one body slides or rolls or tends to do so on the surface of another body.

Types

STATIC FRICTION (f_s)

It is the frictional force between two surfaces before there is relative motion between the surfaces in contact.

The maximum value of static friction is called as limiting static friction f_s^{max}



KINETIC FRICTION

Is the force of friction that is acting between two surfaces when one surface is in steady motion over the other surface.

* KINETIC FRICTION is less than LIMITING FRICTION f_s^{max}

Laws of Static friction

(1) The value of limiting static friction is independent of the shape or area of the surfaces in contact, when normal reaction between the surfaces remains same.

(2) LIMITING STATIC FRICTION f_s^{max} is directly proportional to normal reaction R.

$$f_s^{max} \propto R$$

$$f_s^{max} = \mu_s R$$

$$\mu_s = f_s^{max}/R$$

$$\mu_s = \frac{f_s^{max}}{R}$$

where μ_s is called co-efficient of static-friction.

KINETIC FRICTION

- → Sliding friction
- → Rolling friction

Sliding friction: It is the kinectic friction between two surfaces when a body slides over another body.

Rolling friction: It is the kinectic friction between the surfaces when a body rolls over another body.

Rolling friction < sliding friction.

Coefficient of Static friction (μ_s)

$$\mu_s = \frac{f_s^{max}}{R}$$

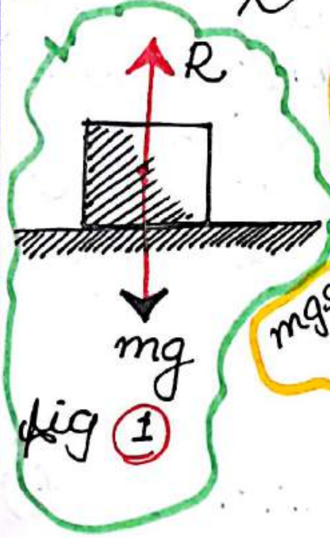
It is defined as the ratio between limiting static friction f_s^{max} and the normal reaction R between the surfaces.

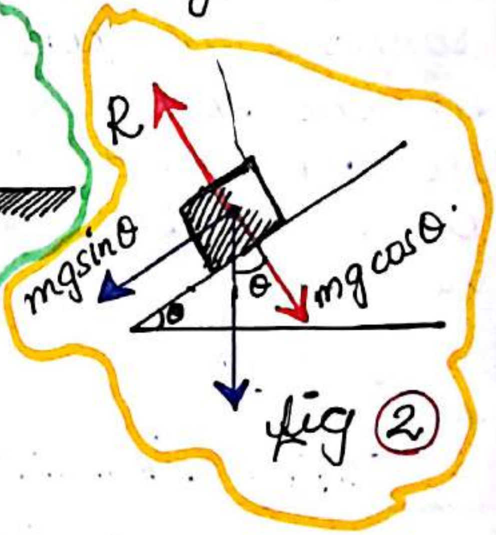
Co-efficient of Kinetic friction (μ_K)

$$\mu_K = \frac{f_K}{R}$$

* $\mu_K = \frac{f_K}{R} = \frac{f_K}{mg}$ fig ①

* $\mu_K = \frac{f_K}{R} = \frac{f_K}{mg\cos\theta}$. fig ②





$\therefore \mu_s = \frac{f_s^{max}}{R} = \frac{f_s^{max}}{mg\cos\theta}$.

* $f_s^{max} = \mu_s mg\cos\theta$

* $f_K = \mu_K mg\cos\theta$.

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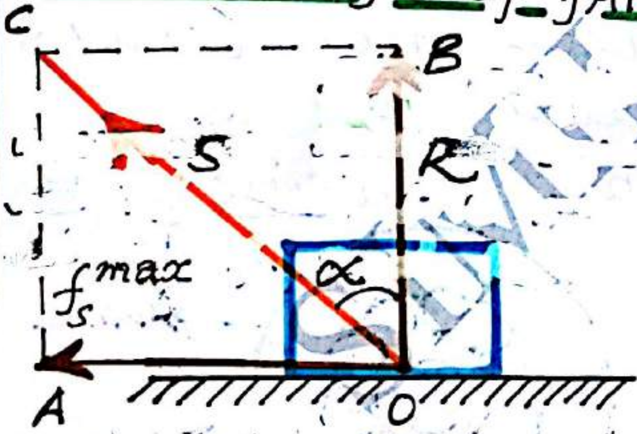
What are the laws of kinetic friction?

1. The kinetic friction has a constant value depending on the nature of the two surfaces in contact

2. The kinetic friction f_K is proportional to the normal reaction R. since $f_K < f_s^{max}$, $\mu_K < \mu_s$

3. The kinetic friction between the two surfaces is independent of the relative velocity between the surfaces.

* Define Angle of friction?



The angle of friction may be defined as the angle which the resultant of the limiting friction and normal reaction makes with normal reaction

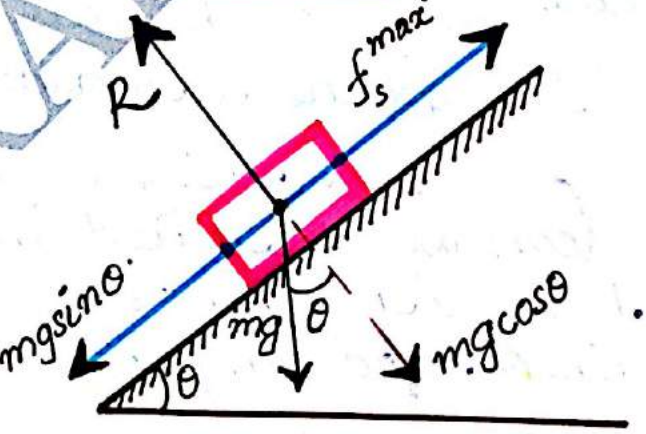
$$\tan\alpha = \frac{BC}{OB}$$

$$\tan\alpha = \frac{f_s^{max}}{R} = \mu_s$$

$$\tan\alpha = \mu_s$$

∴ Tangent of angle of friction is equal to co-efficient of limiting friction.

* Define Angle of Repose. Show that Angle of Repose is equal to Angle of friction?



The Angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide.

Let a body of mass 'm' be placed on a rough inclined plane. Increase the inclination gradually until the body is just on the point of sliding down. This angle is called angle of repose.

The forces acting on the body are

(a) Weight of the body mg acting vertically downwards.

(b) The reaction 'R' acting perpendicular to the plane upwards

(c) The limiting friction f_s^{max} acting parallel to the plane upwards.

The weight 'mg' can be resolved into two components $mg \sin\theta$ and $mg \cos\theta$

$$mg \sin\theta = f_s^{max}$$

$$mg \cos\theta = R$$

$$\frac{mg \sin\theta}{mg \cos\theta} = \frac{f_s^{max}}{R}$$

$$\tan\theta = \mu_s$$

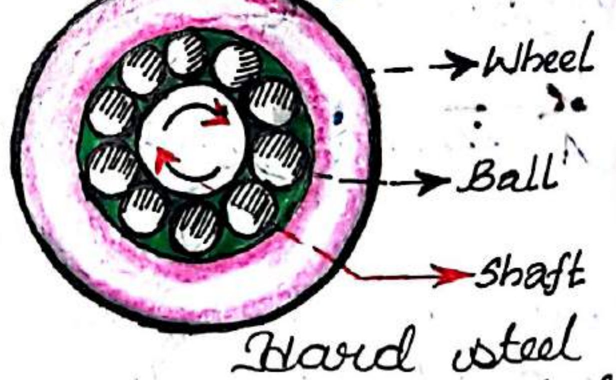
∴ The tangent of the angle of repose is equal to co-efficient of static friction.

Hence the Angle of repose is equal to the Angle of friction.

* Explain Rolling friction with an example?

Rolling friction between two surfaces in contact is the force of friction which comes into play when one body rolls over another. co-efficient of rolling friction μ_r - much smaller than sliding friction μ_K.

eg; Ball bearing.



Hard steel balls are placed between the moving parts like co-axial cylinders. The balls rotate as the cylinders turn relative to each other. This reduces friction.

* **What are the advantages and disadvantages of friction?**

Advantages

1. → helps in walking
2. Trains can run on rails
3. To hold a pen
4. Nails and screws join two surfaces due to force of friction.
5. climbing tree
6. fixing a nail on the wall

Disadvantages

1. Reduce efficiency of machine
2. causes wear and tear of moving parts of machine
3. Due to friction energy is wasted as heat.

* **What are the methods to reduce friction?**

1. By polishing the surfaces
2. By rolling friction: Friction between moving parts like coaxial cylinder can be reduced by the use of ball bearings.
3. By lubrication: Oil and greece are used as lubricants to reduce friction and to protect the moving parts from over heating.

* **Define Centrepetal Force?**

when a particle moves around a circle, there is centrepetal acceleration. The force acting on the particle that gives this acceleration is called centrepetal force.

centrepetal force = mass × centrepetal acceleration

$$F = m \times \frac{v^2}{r} \qquad a_c = \frac{v^2}{r}$$

$$\therefore F = \frac{mv^2}{r}.$$

$$F = mr\omega^2 = mv\omega$$

* **Give examples of centrepetal force?**

1. when a body is tied to one end of a string is whirled in a

horizontal circle, the necessary centrepetal force is supplied by the tension T in the string.

2. For the earth moving round the sun, the necessary centrepetal force is provided by the gravitational force of attraction between earth and sun.

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* **Find an expression for the maximum velocity of a car moving through a flat horizontal circular portion of a car road of radius r.**

Let a car of mass 'M' moves with a velocity V through a circular portion of radius r. The two forces acting on the car are.

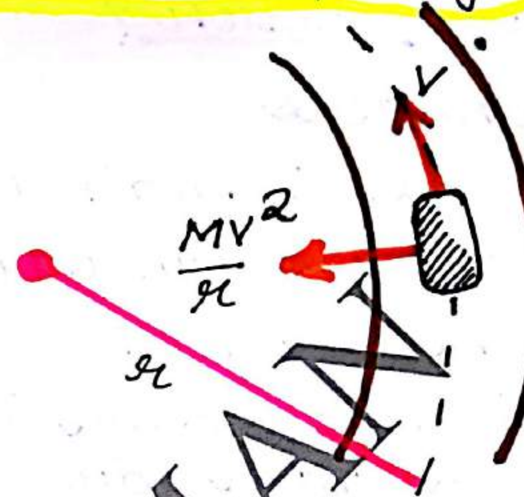
1. Weight Mg of the car acting vertically downward, which is balanced by the normal reaction R

$\therefore R = Mg$

2. centrepetal force $\frac{MV^2}{r}$ acting towards the centre of the circle.

This is provided by the frictional force

$F = \mu_s R = \mu_s Mg$



For safe running, the centrepetal force should be equal or less than the available frictional force

$$\frac{MV^2}{r} \leq F$$

$$\frac{MV^2}{r} \leq \mu Mg$$

$$V^2 \leq \mu r g$$

$$V^2 \leq \mu r g.$$

$$V_{max} = \sqrt{\mu r g}$$

This is the maximum velocity for the safe running of car.