LAWS OF MOTION

SIMIL PHYSICS reference text

INTRODUCTION

We have studied in the previous chapters, the motion of bodies with uniform acceleration without taking into account the external factors that produced the acceleration.

For centuries the problem of motion and its causes were a central theme of natural philosophy. The Greek philosopher Aristotle stated that a body will move with uniform velocity so long as a constant force acts on it. It was only in the sixteenth century Galileo contradicted the statement. From experiments on inclined planes he inferred that a body in motion will continue to move with constant velocity if no external force acts on it. Later Isaac Newton (1642–1727 A. D.) studied the problem of motion in detail and formulated them in three laws, named after him.

NEWTON'S LAWS OF MOTION

First Law (Law of Inertia)

Every body continues in its state of rest or of uniform motion in a straight line unless compelled by an external unbalanced force to change that state.

Second Law

The rate of change of momentum is directly proportional to the external unbalanced force and takes place in the direction of the force.

Third Law

To every action there is an equal and opposite reaction.

Discussion of the First Law

Inertia and Force

It is a matter of common experience that a body at rest remains at rest, unless a force acts on it. But we do not observe any body moving with uniform velocity in the absence of a force. For example, if the engine of a car running on a level road is switched off it comes to rest in a short distance. This is due to the frictional force acting on the car. If we could remove frictional force entirely, the car will move with uniform velocity even if the engine is switched off.

According to the first law, a body by itself cannot change its state of rest or its uniform velocity. This is the property of all material bodies and is called **inertia**. *It is the property of a body by which it tends to resist changes in its state of rest or uniform motion in a straight line.*

It is a matter of common experience that if a person jumps out of a moving vehicle, he falls forward. This happens because at the moment he jumps out he has a velocity equal to that of the vehicle. But on reaching the ground his feet are brought to rest while the upper part of the body still continues to move in the direction of motion due to inertia.

For the same reason if a cyclist suddenly stops, he falls over the handle.

First law also helps us to define force. According to this law, a force is required to change the state of rest or uniform motion of a body along a straight line.

Hence **force** *is that which changes or tends to change the state of rest or uniform motion of a body along a straight line.*

Discussion of the Second Law

From the first law it is clear that force changes the state of rest of a body or changes its velocity. Thus force produces acceleration. The second law gives us a relation between force and acceleration.

Linear momentum

The effect of force on a body is completely understood only if we know its mass and velocity. *The product of mass and its linear velocity is called linear momentum of the body.* It is a vector quantity. If m is the mass of a body and v its velocity, the momentum,

$$\vec{P} = m\vec{v}.$$

Unit: kg ms^{-1}; *Dimensions:* MLT^{-1}

Measurement of force–To show that F = ma

Consider a body of mass m moving with a velocity v. Its momentum $P = mv$. A force F acts on the body for a time Δt. Let the momentum change by an amount $\Delta P = \Delta(mv)$. According to Newton's second law, $F \propto$ rate of change of momentum

$$\vec{F} \propto \frac{\Delta\vec{P}}{\Delta t} \propto \frac{\Delta(m\vec{v})}{\Delta t}$$

$\therefore\ \vec{F} = k \times \dfrac{\Delta(m\vec{v})}{\Delta t}$, where k is the constant of proportionality.

Since the mass of a body generally remains constant, $\vec{F} = k \times m \times (\Delta\vec{v}/\Delta t) = k \times m\vec{a}$; where a is the acceleration of the body.

Unit of force is so chosen as to make $k = 1$. If $F = 1$, $m = 1$ and $a = 1$, then $k = 1$

Thus, $\vec{F} = \text{m}\vec{a}$

Unit force: It is that force which produces unit acceleration on unit mass.

The S. I. unit of force is **newton** (N). *It is that force which produces an acceleration of 1 ms^{-2} on a mass of 1 kg.*

Dimensions of force : MLT^{-2}

Note: The equation $\vec{F} = m\vec{a}$, is a vector equation. We can consider a rectangular co-ordinate system with unit vectors i, j and k in the X, Y and Z directions.

Then

$$F = F_x i + F_y j + F_z k \text{ and } a = a_x i + a_y j + a_z k.$$

Newton's second law can be written in the component form,

$$F_x = ma_x;\ F_y = ma_y;\ F_z = ma_z.$$

The x, y and z components of force and acceleration have to satisfy the above equaions individually.

Two forces acting on a body

Let two forces $\vec{F_1}$ and $\vec{F_2}$ acting independently on a mass m produce accelerations $\vec{a_1}$ and $\vec{a_2}$ respectively. If they act simultaneously on the body, the combined effect of the forces on the body is obtained by taking the vector sum of the two forces.

$$\vec{F} = \vec{F_1} + \vec{F_2}$$

Since $\vec{F_1} = m\vec{a_1}$ and $\vec{F_2} = m\vec{a_2}$; $\vec{F} = m(\vec{a_1} + \vec{a_2}) = m\vec{a}$; where a is the vector sum of a_1 and a_2

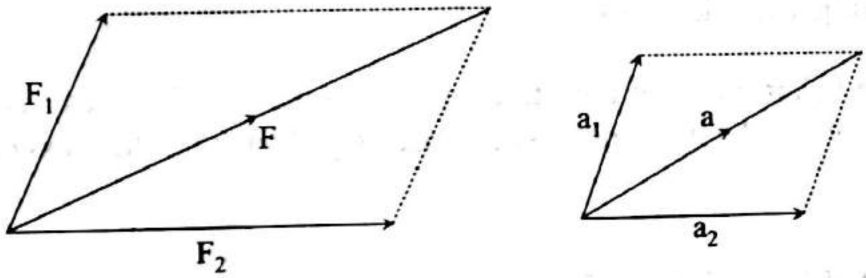


Fig. 5

Mass and weight

Mass is the basic property of matter. It is a measure of the quantity of matter that the object contains. It does not depend on pressure, temperature or the location of the object in space. Mass is a scalar quantity. Mass is expressed in kg.

The weight of a body is the force of gravity acting on the body. If m is the mass of a body and g the acceleration due to gravity at a place, the weight of the body is given by $W = mg$. Hence the weight of a body changes from place to place. Weight is a vector quantity. Since the weight of a body is a force, it is measured in newton.

Gravitational unit of force is kilogram weight (kg wt). $1\, kgwt = g$ newton.

Inertial Mass and Gravitational Mass

Inertial mass

From Newton's second law of motion, the acceleration produced by a certain force F is given by $a = F/m$, where m is the mass of the body. Thus the acceleration on a body is inversely proportional to the mass of the body. For a given force, heavier a body, less will be the acceleration. Thus mass is a measure of the resistance offered by the body to the changes in its velocity when a force is applied. It is a measure of the inertia of the body.

If two bodies arc accelerated by the same force so that the corresponding accelerations are a_1 and a_2 respectively, the ratio of their inertial mass is

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

The ratio of the inertial masses of two bodies is equal to inverse ratio of accelerations produced on the bodies by the same force.

Gravitational mass

The gravitational force on a body is proportional to its mass. From the equation, $W = mg$, the mass of the body $m = W/g$.

The ratio of the weight of the body to the acceleration due to gravity is called gravitational mass.

Though inertial and gravitational masses are defined differently, the two masses are equivalent.

Measurement of gravitational mass

A common balance can be used to compare the masses of bodies. The gravitational pull on a given body and a standard mass are made equal and thus the mass of the body may be determined.

Examples

IV.1. What force acting on a mass of 15 kg for one minute can change its velocity from $10\,ms^{-1}$ to $50\,ms^{-1}$? [NCERT]

$$u = 10\,ms^{-1};\ v = 50\,ms^{-1};\ t = 60\,s;\ m = 15\,kg;\ a = ?;\ F = ?$$
$$v = u + at;\ 50 = 10 + a \times 60;\quad 60a = 40;$$
$$a = 40/60 = (2/3)\,ms^{-1};$$
$$F = ma = 15 \times (2/3) = \mathbf{10\,N}$$

IV.2. A bullet of mass 0.01 kg moving with a velocity $100\,ms^{-1}$ strikes a wooden plank of thickness 0.5 m and emerges with a velocity $30\,ms^{-1}$. Find the resistance offered by the plank, assuming it to be uniform.

$$m = 0.01\,kg;\ u = 100\,ms^{-1};\ S = 0.5\,m;\ v = 30\,ms^{-1};\ a = ?;\ F = ?$$
$$v^2 - u^2 = 2aS;\ 30^2 - 100^2 = 2 \times a \times 0.5$$
$$a = (30^2 - 100^2) = -9100\,ms^{-2};$$
$$F = ma = 0.01 \times 9100 = \mathbf{91\,N}$$

IV.3. A bullet of mass 20 grams travelling with a velocity of $16\,ms^{-1}$ penetrates a sand bag and is brought to rest in 0.05 second. Find the depth of penetration and the average retarding force of the sand.

$$v = 16\,ms^{-1};\ u = 0;\ t = 0.05\,s;\ S = ?;\ a = ?;\ F = ?$$
$$v = u + at;\ 0 = 16 + a \times 0.05;$$
$$0.05a = -16;\ a = -320\,ms^{-2}$$
$$F = ma = 0.02 \times -320 = -6.4\,N;$$

(Negative sign shows that force is opposite to the direction of motion)

$$S = \left[\frac{u+v}{2}\right]t = \left(\frac{16+0}{2}\right)0.05 = \mathbf{0.4\,m}$$

IV.4. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body. [NCERT]

Resultant force, $F = \sqrt{8^2 + 6^2} = \sqrt{100} = 10\,N$

It acts in a direction making an angle θ with 8 N force such that,

$$\tan\theta = 6/8 = 0.75;\ \theta = 37^\circ$$
$$F = ma;\quad a = F/m = 10/5 = \mathbf{2\,ms^{-2}}$$

The acceleration of the body is $2\,ms^{-2}$ at an angle 37° with the force 8 N.

IV.5. A stream of water flowing horizontally with a speed of $15\,ms^{-1}$ rushes out of a tube of cross-sectional area $10^{-2}\,m^2$ and hits a vertical wall nearby. What is the force exerted on the wall by the impact of water assuming that it does not rebound? [NCERT]

$$\text{Velocity of flow} = 15\,\text{ms}^{-1}$$

$$\text{Area of cross section} = a = 10^{-2}\,\text{m}^2$$

$$\text{Volume of water flowing out per second} = av = 10^{-2} \times 15\,\text{m}^3\text{s}^{-1}$$

$$\text{Mass of water flowing out per second} = 10^{-2} \times 15 \times 1000 = 150\,\text{kgs}^{-1}$$

$$m = 150\,\text{kg};\ u = 15\,\text{ms}^{-1};\ v = 0;\ t = 1\,s;\ F = ?$$

Force exerted on water, $F = ma = m(v-u)/t = 150(0-15)/1 = -2250\,N$

$\therefore$ Force exerted on the wall $= \mathbf{2250\,N}$

IV.6. A particle of mass 0.4 kg moving with a constant speed of $10\,\text{ms}^{-1}$ to the north is subjected to a constant force of 8 N directed towards south for 30 s. Take the instant the force is applied to be $t = 0$ and the position of the particle at that time to be $x = 0$. Predict the position at $t = -5$ s; 25 s; 100 s [NCERT]

Let the north direction be positive

(a) At $t = -5$ s the force was not acting and it was moving with constant speed $v = 10\,\text{ms}^{-1}$

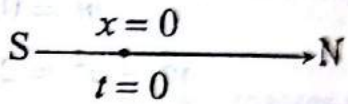


Fig. 7

$$x_t = x_0 + vt = 0 + 10 \times -5 = -50\,\text{m}$$

(b) At the instant $t = 25$ s

During the time the force acts in the opposite direction and the particle experiences a retardation.

Retardation, $a = F/m = 8/0.4 = 20\,\text{ms}^{-2}$

$$x_t = x_0 + ut + (1/2)at^2 = 0 + 10\times25 + (1/2)\times -20\times25^2 = 250 - 6250 = \mathbf{-6000\,m}$$

(c) At the instant, $t = 100$ s

For the first 30 seconds it experiences retardation and thereafter it moves with uniform velocity.

Position and velocity after 30 seconds

$$x(t) = x(0) + ut + (1/2)at^2 = 0 + ut + (1/2)at^2$$
$$= 10 \times 30 + (1/2) \times (-20)30^2 = 300 - 9000 = -8700\,\text{m}$$
$$v(t) = u + at = 10 - 20 \times 30 = -590\,\text{ms}^{-1}$$

Position after 100 seconds $= -8700 - 590 \times 70 = -8700 - 41300 = \mathbf{-50\,000\,m}$

Motion of a body in a lift

Consider a body of mass m inside a lift. The force acting on the body are:-

(i) the weight mg of the body acting vertically downwards and

(ii) the reaction R of the lift acting vertically upwards.

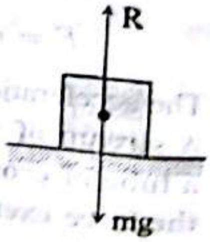


Fig. 8

1. *When the lift is stationary or moving up or down with uniform velocity, the unbalanced force,*

$$F = R - mg = ma$$

$$\therefore\ R - mg = 0 \quad (\because\ a = 0)\ \therefore\ R = mg$$

2. *When the lift moves up with acceleration a,*

   the unbalanced force, $F = R - mg = ma.$

$$\therefore\ R = mg + ma = m(g + a)$$

3. *When the lift moves down with acceleration a,*

   the unbalanced force, $F = mg - R = ma.$

$$R = mg - ma = m(g - a)$$

4. *When the lift falls down freely i.e.*, when $a = g$, then,

$$R = mg - ma = m(g - a) = m(g - g) = 0$$

There is no reaction and the body is apparently weightless.

If a body is suspended from a spring balance attached to the ceiling of a lift which is moving up or down with acceleration a, there is an apparent change in the reading.

$$\text{Apparent weight } = m(g \pm a) \quad \text{newton} = m\frac{(g \pm a)}{g}\ \text{kg. wt.}$$

Examples

IV.7. A man weighing 70 kg stands in a weighing scale in a lift which is moving (a) upwards with uniform speed of $10\,ms^{-1}$ (b) downwards with uniform acceleration of $5\,ms^{-2}$ (c) upwards with uniform acceleration of $5\,ms^{-2}$. What would be the reading of the scale in each case? [NCERT]

(a) *When the lift moves with constant speed* $10\,ms^{-1}$,

$$R = mg = 70 \times 9.8 = 686\,N = \mathbf{70\,kgwt}$$

(b) When the lift moves downwards with acceleration,

$$mg - R = ma;\ R = mg - ma = m(g - a)$$

$$R = 70(9.8 - 5) = 336\,N = \mathbf{34.29\,kgwt}$$

(c) When the lift moves up with uniform acceleration,

$$R - mg = ma; \quad R = m(g + a) = 70(9.8 + 5)$$

$$= 70 \times 14.8 = 1036\,N = \mathbf{105.7\,kgwt}$$

Note: If the lift falls freely under gravity $a = g$ and $R = 0$. The reading of the weighing machine is zero.

IV.8. A helicopter of mass 1000 kg rises with a vertical acceleration of $15\,ms^{-2}$. The crew and the passengers weigh 300 kg. Give the magnitude and direction of the (a) force on the floor by the crew and the passengers, (b) action of the rotor of the helicopter on the surrounding air and (c) force on the helicopter due to surrounding air ($g = 10\,ms^{-2}$) [NCERT]

(a) When the helicopter is rising up with acceleration, the reaction on the floor,

$$R = m(g + a) = 300(10 + 15) = 300 \times 25 = \mathbf{7500\ N}$$

downwards

(b) Total mass of the helicopter and the crew = 1300 kg. The action of the rotor of the helicopter on the surrounding air, $= m(g + a) = 1300(10 + 15) = \mathbf{32500\,N}$ downwards

(c) Force on the helicopter due to surrounding air, *i.e.*, Reaction, $R =$ **32500 Ndownwards;**

IV.9. A monkey of mass 40 kg climbs on a rope which can withstand a maximum tension of 600 N. In which of the following cases will the rope break? The monkey (a) climbs up with an acceleration of $6\,ms^{-2}$, (b) climbs down with acceleration of $4\,ms^{-2}$, (c) climbs up with a uniform speed of $5\,ms^{-1}$ and (d) falls down the rope nearly freely under gravity [$g = 10\,ms^{-2}$]. Ignore the mass of the rope. [NCERT]

Mass of monkey, $m = 40$ kg

Maximum tension the rope can withstand, $T = 600$ N

In each case the actual tension in the rope will be equal to the apparent weight of monkey (R).

The rope will break when R exceeds T

(a) When the monkey climbs up with an acceleration of $6\,ms^{-2}$; $R = m(g + a) = 40(10 + 6) = 640$ N. This is greater than T. Hence the rope will **break**

(b) When the monkey climbs down with an acceleration of $4\,ms^{-2}$ $R = 40(10 - 4) = 540$ N This is less then T. Hence the **rope will not break.**

(c) When the monkey climbs with uniform speed $v = 5\,ms^{-1}$; $a = 0$

$R = mg = 40 \times 10 = 400$ N which is less then T. The rope will not break

(d) When the monkey falls down nearly freely $a = g$; $R = m(g - g) = \mathbf{0}$ So, the rope will not break, e.g. (i) Force exerted on a ball when it is hit with a bat. (ii) Force exerted on a bullet when it is fired from a gun.

Since the time of action of the impulsive force is very small we measure the effect of the force called *impulse*.

Impulse (I)

Impulse of a force is the product of the force and the time during which the force acts on the body.

$$\text{Impulse} = \text{Force} \times \text{time};\ I = F \times dt.$$

The impulse is a measure of the total effect of the force.

To show that the impulse is equal to the change in momentum

Consider an impulsive force acting on a body for a short interval of time dt. According to Newton's second law,

$F = (dp/dt)$ $\therefore$ Impulse, $I = F \times dt = dp$, *the change in momentum of the body.*

Unit of impulse: Ns; Dimensions: MLT^{-1}, same as that of momentum.

Examples

IV.10. Two objects of masses 3 kg and 4 kg are connected by a weightless string which passes over a frictionless pulley. The objects are initially held at a height of 5 m from the ground with equal length of the string on either side and then released. Find (a) the acceleration of the system (b) distance between masses 2 seconds after start (c) time taken by the 4 kg mass to reach the ground. ($g = 9.8\ ms^{-2}$)

(a)

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = \frac{(4-3)}{(4+3)} \times 9.8 = \frac{9.8}{7} = \mathbf{1.4\ ms^{-2}}$$

(b) Distance travelled by either mass

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 1.4 \times 2^2 = 2.8\ m$$

Distance between masses = 2 × 2.8 = **5.6 m**

(c)

$u = 0;\quad a = 1.4\ ms^{-2};\quad S = 5\ m;\quad t = ?$

$$S = ut + \frac{1}{2}at^2;\quad 5 = (1/2) \times 1.4 \times t^2 \quad \therefore t = \mathbf{2.67}\ s$$

IV.11. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. [NCERT]

$u = 12\ ms^{-1};\quad v = -12\ ms^{-1};\quad m = 0.15\ kg;\ I = ?$

$I = (mv - mu) = m(v - u) = 0.15(-12 - 12) = \mathbf{-3.6\ Ns}$

IV.12. Two masses of 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless vertical pulley. Find the acceleration of the masses, and the tension of the string when the masses are released. $g = 10\ \text{ms}^{-2}$ **[NCERT]**

$$a = \frac{(m_1 - m_2)}{m_1 + m_2} g = \frac{(12-8) \times 10}{(12+8)} = \mathbf{2\ ms^{-2}}$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 12 \times 8 \times 10}{(8+12)} = \mathbf{96\ N}$$

IV.13. A horizontal force of 500 N pulls two masses 10 kg and 20 kg (lying on a frictionless table) connected by a light string. What is the tension in the string? Does the answer depend on which mass end the pull is applied? **[NCERT]**

$$m_1 = 10\,\text{kg};\ m_2 = 20\,\text{kg}$$

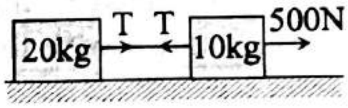


Fig. 11

(a) Let $F = 500\,\text{N}$ be the force applied on 10 kg mass; T be the tension in the string and a the acceleration of the system.

For the motion of $m_1 = 10\,\text{kg}$,

$$F - T = m_1 a \tag{1}$$

For motion of $m_2 = 20\,\text{kg}$,

$$T = m_2 a \tag{2}$$

$$\therefore \quad F = (m_1 + m_2)a;\ 500 = (10+20)a = 30a$$

$$a = (500/30) = (50/3)\ \text{ms}^{-2}$$

$$T = m_2 a = 20 \times (50/3) = \mathbf{333.3\,N}$$

If the pull is applied on the mass 20 kg

Let T be the tension. Since acceleration is the same,

$$T = m_1 a = 10 \times (50/3) = \mathbf{166.7\,N}$$

IV.14. The position-time graph of a particle of mass 4 kg moving in one dimension is given in the figure. What is the force on the particle for (a) $t < 0$, $t > 4$ s, $0 < t < 4$ s? (b) Impulse at $t = 0$ and $t = 4$ s **[NCERT]**

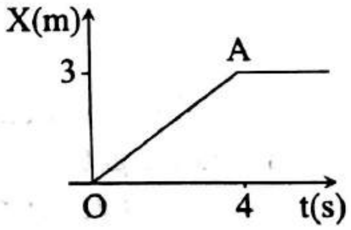


Fig. 12

(a) For $t < 0$ and $t > 4$ s the particle is at rest. Hence no force acts on it. For, $0 < t < 4$ s *i.e.*, along OA, the particle moves with uniform velocity. Hence no force acts on it.

(b) Just before $t = 0$, the body is at rest, *i.e.*, $u = 0$ and just after $t = 0$, the body moves with a uniform velocity $v = (3/4)\ \text{ms}^{-1}$

$$\therefore \quad \text{At } t = 0,\ \text{Impulse} = \text{change in momentum of the body}$$

$$= m[v - u] = 4 \times (3/4) = \mathbf{3\,kgms^{-1}}$$

Just before $t = 4$ s; the body is moving with a uniform velocity (3/4) and just after $t = 4$ s, the body comes to rest. *i.e.*, $u = (3/4)\text{ms}^{-1}$; $v = 0$ $\therefore$ At $t = 4$ s, Impulse $= 4[0 - (3/4)] = \mathbf{-3\,kg\,ms^{-1}}$

IV.15. A jet of water issuing horizontally at the rate of 10 kg/s strikes a vertical wall with a velocity $10\,\text{ms}^{-1}$ and rebounds with half the original velocity. What is the force exerted by the jet on the wall?

$$m = 10\,\text{kg};\quad t = 1\,\text{s};\quad u = 10\,\text{ms}^{-1};\ v = -5\,\text{ms}^{-1};\ F = ?$$

$$F = m\left[\frac{v-u}{t}\right] = 10\left[\frac{-5-10}{1}\right] = -150\,\text{N}.$$

This is the force exerted by the wall on the jet.

$\therefore$ Force exerted by the jet on the wall $=$ **150 N**

IV.16. A billiard ball of mass m moving with a velocity u strikes a rigid wall and gets reflected without any loss of speed as shown in the figure. Calculate the impulse imparted to the ball if it (a) strikes normally on the wall and (b) strikes at an angle θ with the normal to the wall. **[NCERT]**

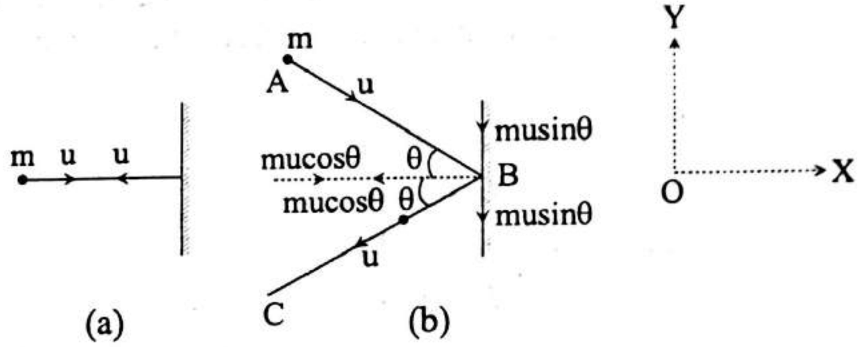


Fig. 13

(a)

Initial momentum of the ball $=$ mu

Final momentum of the ball $=$ $-$mu

Impulse imparted on the ball $=$ change on momentum

$= -mu - mu = -2mu$, normal to the wall

(b) Initial momentum of the ball $= mu$, along AB. Resolving this into two components we get,

(i) $mu\cos\theta$, normal to the wall along the positive direction of the X-axis.

(ii) $mu\sin\theta$, along the wall (negative direction of Y-axis.)

Final momentum of the ball $= mu$, along BC. Resolving this into two components we get,

(i) $mu\cos\theta$, normal to the wall along the negative direction of X-axis.

(ii) $mu\sin\theta$, along the negative direction of Y-axis.

Change in momentum along the X-axis $= -mu\cos\theta - mu\cos\theta$

$= -2mu\cos\theta$

Change in momentum along the Y axis $= -mu\sin\theta - (-mu\sin\theta) = 0$

$\therefore$ Impulse imparted on the ball $= -2mu\cos\theta$, normal to the wall along the negative direction of the X-axis.

IV.17. A batsman deflects a ball of mass 0.15 kg by an angle of 45° without changing its initial speed of 108 km/hr. What is the impulse imparted to the ball?[NCERT]

$u = 108\,\text{km/hr} = 30\,\text{ms}^{-1};\ \theta = 45/2 = 22.5°$

Impulse imparted to the ball

$= -2mu\cos\theta = -2 \times 0.15 \times 30 \times \cos 22.5$

$= \mathbf{-8.315\,kg\,ms^{-1}}$

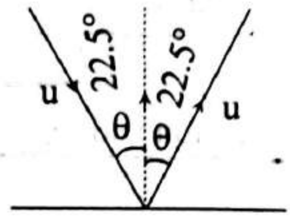


Fig. 14

Impulse imparted to the ball is $8.315\,\text{kg ms}^{-1}$ directed along the bisector of the initial and final directions.

Discussion of third law

According to Newton's third law, action and reaction are equal and opposite. Still they do not cancel each other as they act on different bodies.

Illustration

1. A book lying on a table exerts a force on the table which is equal to the weight of the book. This is action. The table in turn exerts an equal reaction on the book which supports it.

2. While walking, a person presses against the ground in the backward direction (action) by his feet. The ground pushes the person forward with an equal force (reaction).

Note: *Newton's second law is the real law of motion; the first and the third laws can be obtained from the second law.*

(a) Deduction of the first law from the second law

According to Newton's second law, $F = ma$. If no unbalanced force acts on the body, $F = 0$. $\therefore ma = 0$, Since $m \neq 0$, $a = 0$.

This means that if there is no unbalanced force, there is no acceleration for the body. The body will be either at rest or will be moving with a uniform velociy. This is Newton's first law of motion.

(b) Deduction of the third law from the second law

Consider an isolated system of two bodies A and B. Let them collide. During collision, let the body A exert a force F_1 on B (action) for a time Δt and the body B exert a force F_2 on A (reaction) for the same time Δt

Change in linear momentum = Force × time

Change in linear momentum of B = $F_1 \times \Delta t$

Change in linear momentum of A = $F_2 \times \Delta t$

Total change in linear momentum = $F_1\Delta t + F_2\Delta t$

$F_1\Delta t + F_2\Delta t$

As no external force acts on the system, according to Newton's second law, $a = 0$ and hence the total change in linear momentum of the system must be zero. Then,

$$F_1 \Delta t + F_2 \Delta t = 0$$

$$F_1 = -F_2$$

Hence action and reaction are equal and opposite.

Law of conservation of linear momentum

If no external force acts on a system of several particles, the vector sum of linear momenta of the system remains constant.

Here the total momentum of the system is the vector sum of the momenta of individual particles constituting the system. The law implies that in an isolated system although the momentum of individual particles may change due to interactions within the system, the total momentum vector will remain the same.

The law can be proved on the basis of (a) Newton's second law and (b) Newton's third law.

(a) Proof of the law based on Newton's second law

Consider an isolated system consisting of n particles of masses $m_1, m_2, m_3 \ldots m_n$ moving with velocities $v_1, v_2, v_3 \ldots v_n$. The total linear momentum is given by,

$$P = m_1 v_1 + m_2 v_2 + m_3 v_3 \cdots + m_n v_n$$

If an external force F acts on the system, then, $F = \dfrac{dP}{dt}$.

If $F = 0$, then $\dfrac{dP}{dt} = 0 \therefore \quad P = \text{constant}$

Thus the total momentum of the system remains constant in the absence of external force.

(b) Proof of the law based on Newton's third law

Consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 along a straight line in the same direction. After collision, let them move with velocities v_1 and v_2 in the same direction. Let Δt be the time of contact during collision.

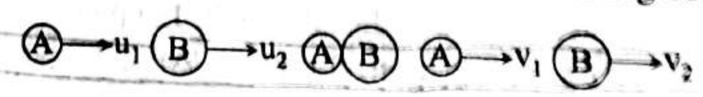


Fig. 15

Force exerted by A on B, *i.e., action*, $F_1 = m_2 \dfrac{(v_2 - u_2)}{\Delta t}$

Force exerted by B on A, *i.e., reaction*, $F_2 = m_1 \dfrac{(v_1 - u_1)}{\Delta t}$

By third law of motion, $F_2 = -F_1$

$$m_2\frac{(v_2 - u_2)}{\Delta t} = -m_1\frac{(v_1 - u_1)}{\Delta t}$$

$$m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$$

$$m_2v_2 + m_1v_1 = m_1u_1 + m_2u_2$$

Thus the total momentum after collision is equal to the total momentum before collision. In other words, the total momentum remains constant.

Examples

IV.18. A body of mass 10 kg moving with a velocity $20\,\mathrm{ms^{-1}}$ along a straight line collides with another body of mass 8 kg moving in the same direction with a velocity $5\,\mathrm{ms^{-1}}$. After collision if the velocity of the former is reduced to $16\,\mathrm{ms^{-1}}$, calculate the final velocity of the latter.

$m_1 = 10\,\mathrm{kg}$; $m_2 = 8\,\mathrm{kg}$; $u_1 = +20\,\mathrm{ms^{-1}}$; $u_2 = +5\,\mathrm{ms^{-1}}$; $v_1 = +16\,\mathrm{ms^{-1}}$; $v_2 = ?$

By law of conservation of momentum,
Momentum after collision = Momentum before collision

$$10 \times 16 + 8 \times v_2 = 10 \times 20 + 8 \times 5$$

$$160 + 8v_2 = 200 + 40 = 240;\ 8v_2 = 80;\ v_2 = \mathbf{10\,ms^{-1}}$$

IV.19. An explosion blows a rock into three parts. Two pieces go off at right angles to each other; a 100 kg piece at $12\,\mathrm{ms^{-1}}$ and 200 kg piece at $8\,\mathrm{ms^{-1}}$. The third piece flies off with a velocity $25\,\mathrm{ms^{-1}}$. Calculate the mass of the third piece and its direction of motion.

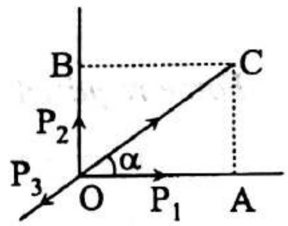


Fig. 16

Since the momentum before explosion is zero, by law of conservation of momentum, the vector sum of momenta of all pieces after explosion must be zero.

$$P_1 = 100 \times 12 = 1200\,\mathrm{kg\,ms^{-1}};\ P_2 = 200 \times 8 = 1600\,\mathrm{kg\,ms^{-1}}$$

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0; \quad \vec{P}_3 = -(\vec{P}_1 + \vec{P}_2)$$

$$P_3 = \sqrt{P_1^2 + P_2^2} = \sqrt{1200^2 + 1600^2} = 2000\,\mathrm{kg\,ms^{-1}}$$

$$\text{Mass of the third piece} = \frac{2000}{25} = 80\,\mathrm{kg}$$

$$\mathrm{Tan}\,\alpha = 1600/1200 = 1.33 \therefore \alpha = 53^\circ$$

Third mass moves in the direction making an angle $180^\circ - 53^\circ = 127^\circ$ with the direction of P_1.

Recoil of a gun

When a shot is fired from a gun, the shot acquires a large forward momentum due to explosion of the gun powder. Consequently the gun gets an equal backward momentum and the gun is pushed backward with a velocity V. This backward motion of the gun is called recoil of the gun. The velocity V of the gun just after the firing is called the *recoil velocity* of the gun; the velocity v of the bullet just after the firing is called the *muzzle velocity* of the bullet.

Let M be the mass of the gun and m that of the shot. Before firing both are at rest. After firing let v be the velocity of the shot and V that of the gun. By law of conservation of linear momentum,

$$\text{momentum after firing} = \text{momentum before firing}$$

$$mv + MV = 0$$

$$\text{Recoil velocity,} \quad V = -mv/M$$

The negative sign indicates that the gun is recoiling.

Examples

IV.20. A bullet of mass 10 g is fired from a gun of mass 8 kg with a velocity 160 ms^{-1}. Find the velocity of recoil of the gun. Find also the force required to stop the gun in a distance of 25 cm

Momentum of the gun = momentum of shot

$$MV = mv; \quad V = \frac{m}{M}v = 0.01 \times \frac{160}{8} = \mathbf{0.2\,ms^{-1}}$$

Force required to stop the gun:

$$u = +0.2\,\text{ms}^{-1}; \quad S = 0.25\,\text{m}; \quad v = 0; \quad a = ?$$

$$v^2 = u^2 + 2aS; \quad 0 = 0.2^2 + 2 \times a \times 0.25; \quad a = -\frac{0.04}{0.5} = -0.08\,\text{ms}^{-2}$$

Force required to stop the gun, $F = Ma = 8 \times 0.08 = \mathbf{0.64\,N}$

IV.21. A machine gun has a mass of 10 kg. It fires 30 g bullets at the rate of 6 bullets per second each with a speed of 400 ms^{-1}. What force must be applied to the gun to keep it in position?

$$\text{Total mass of the bullet fired per second} = 6 \times 0.03 = 0.18\,\text{kg}$$

$$\text{velocity of bullet} = 400\,\text{ms}^{-1}$$

$$\text{Force} = \text{rate of change of momentum} = \frac{\text{change in momentum}}{\text{Time}}$$

$$= \frac{0.18 \times 400}{1} = \mathbf{72\,N}$$

VARIABLE MASS PROBLEM

So far we have considered the application of Newton's laws to bodies of fixed mass. But there are cases where the mass varies with time. A typical example is the rocket propulsion.

Rocket Propulsion

Consider a rocket that is continuously losing mass due to ejection of combustion gases.

Let M be the initial mass of the rocket and V its initial velocity, which is generally zero. At any time t let m be its mass. At time $(t + dt)$, let the mass of the rocket be $(m - dm)$; where dm is the mass of the gas ejected in time dt. Let v and $(v + dv)$ be the velocity of the rocket at time t and $(t + dt)$ respectively. Let v_g be the velocity of the ejected gas with respect to the earth in a direction opposite to that of the rocket.

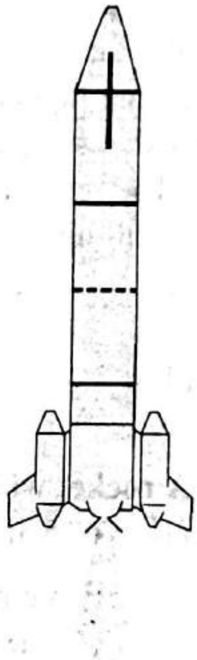


Fig. 17

Applying the law of conservation of linear momentum at time t and $t + dt$ we have

$$mv = (m - dm)(v + dv) + dm(-v_g) \qquad (1)$$

Expanding

$$mv = mv + m \times dv - dm \times v - dm \cdot dv - dm \times v_g \qquad (2)$$

As dm and dv are small quantities their product is negligible.

$$\therefore \qquad m \times dv = dm(v + v_g) \qquad (3)$$

If v_r is the relative velocity of exhaust gases with respect to rocket and knowing that the rocket and exhaust gases are moving in opposite directions we have

$$v + v_g = -v_r$$

$$m \times dv = -v_r dm; \quad dv = -v_r \frac{dm}{m}$$

Integrating both sides within proper limits

$$\int_V^v dv = \int_M^m -v_r \frac{dm}{m}$$

$$v - V = -v_r[\log m]_M^m = -v_r[\log m - \log M] = v_r \log \frac{M}{m}$$

If the initial velocity $V = 0$ at $t = 0$ we have

$$\boldsymbol{v = v_r \log_e(M/m)}$$

If the fuel consumption has a time rate α, the velocity after time t is given by

$$\boldsymbol{v = v_r \log_e[M/(M - \alpha t)]}$$

The thrust on the rocket $F = v_r(dm/dt)$

Multistage rockets

With a single stage rocket, the maximum velocity attained is of the order of $4.6\,\text{kms}^{-1}$. For obtaining higher velocities the rocket is designed in stages. When the fuel of the first stage is exhausted, the rocket casing is detached and dropped off, and the second stage is ignited. The removal of the surplus mass contained in the first stage considerably helps in attaining still higher velocity.

Examples

IV.22. A rocket with a lift off mass 20,000 kg is blasted upward with an initial acceleration of $5\,\text{ms}^{-2}$. Calculate the initial thrust of the blast. **[NCERT]**

$$\text{Force required to support the mass} = mg = 20000 \times 9.8 = 196000\,\text{N}$$

$$\text{Force to produce acceleration} = ma = 20000 \times 5 = 100000\,\text{N}$$

$$\text{Total force} = mg + ma = 196000 + 100000 = \mathbf{2.96 \times 10^5\,N}$$

IV.23. Fuel is consumed at the rate of $100\,\text{kgs}^{-1}$ in a rocket. The exhaust gases are ejected at a speed of $4.5 \times 10^4\,\text{ms}^{-1}$ with respect to the rocket. What is the thrust experienced by the rocket? Also calculate the velocity of the rocket at the instant when the mass is reduced to 1/10th of its initial mass. Take the initial velocity of the rocket to be zero.

$$dM/dt = 100\,\text{kgs}^{-1}; \quad u = 4.5 \times 10^4\,\text{ms}^{-1}$$

$$\text{Thrust} \quad F = u \times (dM/dt) = 4.5 \times 10^4 \times 100 = \mathbf{4.5 \times 10^6\,N}$$

$$\text{Velocity of rocket, } v = u \log_e(M_0/M) = 2.303 \times 4.5 \times 10^4 \times \log 10$$

$$= \mathbf{1.036 \times 10^5\,ms^{-1}}$$

Frame of reference

A system of co-ordinate axes which defines the position of a particle or event in two or three dimensional space is called a frame of reference. The essential requirement of a frame of reference is that it should be rigid.

The simplest frame of reference is the familiar Cartesian system of coordinates. The description of an event essentially depends on the reference frame from which the event is observed. A given event will be described differently by observers in different frames of reference.

Inertial frame of reference

A frame of reference relative to which Newton's law of inertia is valid is called an inertial frame. For practical purpose an inertial frame is that frame in which a body moves with constant velocity only if there is no net force acting on it. Newton's first law of motion is an affirmation of the existence of inertial frame. All frames of reference moving with constant velocity with respect to an inertial frame are also inertial frames of reference. An ideal inertial frame does not exist. The earth's frame of reference is approximately inertial for most terrestrial phenomena.

Noninertial frame of reference

All those frames of reference in which Newton's law of inertia does not hold good are called noninertial frames. All accelerated frames are noninertial.

Consider a body of mass m moving with acceleration a in an inertial frame S. In that frame force $F = ma$. The force acting on it in a reference frame S' moving with acceleration α with respect to S is

$$F' = ma - m\alpha.$$

The frame S' is a noninertial frame. Putting $-m\alpha = F_p$ we have

$$F' = F + F_p \qquad \text{If } F = 0, \qquad F' = F_p = -m\alpha$$

ie,, even when no force actually acts on the particle in the inertial frame S, a force F_p appears to be acting on it in the S' frame. Therefore $F_p = -m\alpha$ does not actually exist but appears to come into play consequent of the acceleration of the frame S' with respect to S. Therefore it is called a fictitious or pseudo force. The second law for general non inertial frame takes the form

$$F + F_p = ma.$$

Examples of Pseudo forces

1. If a lift is moving up or down with constant speed its acceleration is zero. Hence the weight of a man measured inside the lift is same as that outside.

   If the lift is moving up with acceleration a the weight of the man measured is $m(g + a)$. which is greater than his actual weight. An extra fictitious weight is added to his weight.

2. Suppose a stone is rotated by a string in a horizontal circle of radius r with uniform speed v. For an inertial observer the tension T in the string provides the necessary centrepetal acceleration and according to the second law

   $$T = mv^2/r.$$

   Now we imagine a frame rotating with the stone. Relative to this frame the stone is at rest even though it is subjected to a force T. The rotating frame is noninertial. Here we can write the second law as

   $$T - \frac{mv^2}{r} = m \times a,$$

   where a is the acceleration of the stone relative to the rotating stone's frame. The term $-mv^2/r$ is the pseudo force appearing with rotating frame. This pseudo force is acting outward along the radius. It is called *centrifugal force*.

3. Consider a rotating table on which a small pebble is lodged against a raised rim at the outer edge. An observer on the earth watching the pebble would say that the pebble is in uniform circular motion. The pebble relative to this observer is not in equilibrium at all.

Another observer on the table would observe that the pebble is at rest. If one pulls it a bit towards the center it moves back as if under a force directed radially outward. He would say that the pebble is in equilibrium under the action of this outward force (centrifugal force) and the radially inward force exerted by the rim. Thus we may say that centrifugal force is not a real force but it arises due to acceleration of the rotating frame.

4. If a particle is moving in a rotating table along a straight line it appears to move along a curved path to an observer on the table. To him the curvature appears to arise from a force acting on particle at right angles to its velocity. This pseudo force is called Coriolis force.

Equilibrium of concurrent forces

The forces acting at a point are called concurrent forces. These forces are said to be in equilibrium when the resultant force is zero.

1. *A body acted upon by two forces* will be in equilibrium if the two forces are equal and opposite.

2. *Three coplanar concurrent forces*

Consider three coplanar forces P, Q and R acting at O. Then O will be in equilibrium if one of the following laws is satisfied.

1. The vector sum of $\vec{P}$ and $\vec{Q}$ must be equal and opposite to $\vec{R}$ *i.e.*, $\vec{P}+\vec{Q}=-\vec{R}$
2. If P, Q and R can be represented by the sides of a triangle taken in order, O will be in equilibrium. (The law of triangle of forces).

$$\frac{P}{LM}=\frac{Q}{NL}=\frac{R}{MN}=\text{constant}$$

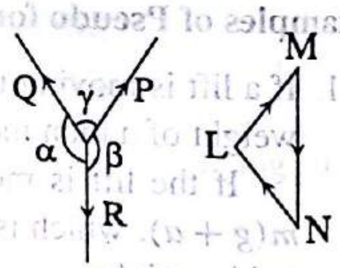


Fig. 18

3. Each force must be proportional to the sine of the angle between the other two. (Lami's theorem)

$$\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}=\text{constant}$$

General condition of equilibrium of a point acted upon by a number of coplanar forces

(a) The sum of the resolved components of all forces in the X-direction must be zero. *i.e.*, $\Sigma\vec{F}_x=0$

(b) The sum of the resolved components of all forces in the Y-direction must be zero. *i.e.*, $\Sigma\vec{F}_y=0$

Examples

IV.24. A mass of 6 kg is suspended by a rope of length 2 m from a ceiling. A force 50 N in the horizontal direction is applied at the midpoint of the rope. What is the angle the rope makes with the vertical in equilibrium? ($g = 10$ ms^{-2}). Neglect the mass of the rope. **[NCERT]**

Let T_1 and T_2 be the tensions of the segments of the rope and F the horizontal force; θ be the angle the rope makes with the vertical.

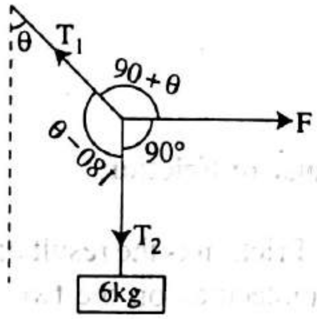


Fig. 19

$T_2 = W = mg = 6 \times 10 = 60$ N; $F = 50$ N; $\theta = ?$

Using Lami's theorem,

$$\frac{W}{\sin(9\theta + \theta)} = \frac{F}{\sin(180 - \theta)}$$

$$\text{Tan}\,\theta = \frac{F}{W} = \frac{50}{60} \quad \therefore \theta = 39.8°$$

IV.25. *A circular roller of mass 5 kg and radius of 0.3 m hangs by a string AC of length 0.5 m and rests against a smooth vertical wall at B as shown in the figure. Find the tension T in the string and the force N exerted against the wall at B.

Here, $\sin\alpha = \frac{0.3}{0.5} = 0.6$ and

$$\cos\alpha = \frac{0.4}{0.5} = 0.8$$

Resolving the forces in the horizontal and vertical directions, we get,

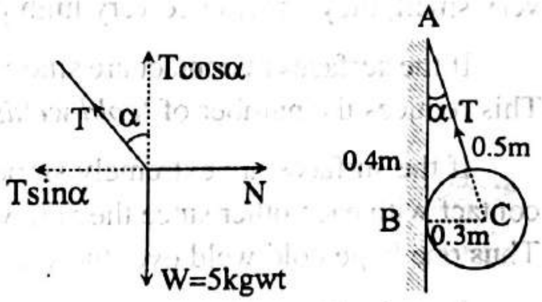


Fig. 20

$$W = T\cos\alpha; \quad T = W/\cos\alpha = \frac{5}{0.8} = \mathbf{6.25\ kg\ wt}$$

$$N = T\sin\alpha = 6.25 \times 0.6 = \mathbf{3.75\ kg\ wt}$$

FRICTION

When two perfectly smooth surfaces are in contact, the only force acting between them is the reaction which acts normal to the surfaces. The smallest applied force is now sufficient to make one body slide over the other. But, no such perfectly smooth bodies actually exist.

Ordinarily when one body slides over another, there is some opposition to the relative motion. This is referred to as *friction*.

Friction is the force which opposes relative motion between two surfaces in contact with each other.

The frictional force always acts tangential to the surface of contact and it is due to surface irregularities at molecular scale.

*Cause of Friction

Friction is the result of molecular interaction. It is due to the force of attraction between the molecules on the two surfaces in contact. This force of attraction is called *adhesive force*.

When two bodies are placed in contact, the actual microscopic area of contact is much less than apparent macroscopic area of contact. Even the most finely polished surface is not plane on the atomic scale. At these microscopic areas of contact, the molecules on opposite surfaces in contact are within the range of intermolecular force of attraction which is about 10^{-8} m. As a result, adhesive force between the molecules at the microscopic contact point is very strong and these contact points actually become *cold 'welded'*. If the surfaces are rough, the normal force acts only at point of actual contact. Since these areas of contact are very small, they experience very high pressure and form *cold welds*.

If the surfaces in contact are smooth, the point like areas of contact decrease in number. This reduces the number of '*cold welds*'.

If the surfaces are extremely smooth, the molecules of the two surfaces are directly in contact with each other since they are within the range of intermolecular force of attraction. Thus one large cold weld over the whole area of contact is formed.

In order to produce relative motion between the two surfaces in contact, the '*cold welds*' have to be teared off. The force of friction is, therefore, the force required to break these *welds.*

Because of the above reasons, the smoothening or polishing of two surfaces more and more, reduces the friction initially, but at later stages increases the friction as the distance between the molecules on two surfaces goes below the range of intermolecular force of attraction.

Types of friction

Static friction (f_s)

It is the frictional force between two surfaces before there is relative motion between the surfaces in contact. Its magnitude is always equal to the external force which tends to cause relative motion.

Limiting static friction (f_s^{max})

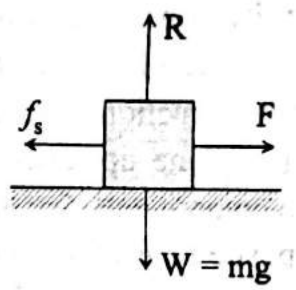


Fig. 21

As the external force which tries to produce relative motion between the surfaces in contact increases, the frictional force also increases, till relative motion just starts. This maximum value of the frictional force before the body just slides over the surface of another body is called *limiting* static *friction* f_s^{max}.

Laws of static friction

1. The magnitude of the limiting static friction is independent of the area, when the normal reaction between the surfaces remains same.
2. The limiting static friction f_s^{max} is directly proportional to normal reaction R.
   i.e., $f_s^{max} = \mu_S \times R$; where μ_S is called *coefficient of static friction.*

Kinetic friction

Kinetic friction between two surfaces in contact is the force of friction which calls into play when there is relative motion between the surfaces.

Kinetic friction f_k is less than limiting friction f_s^{max}

Kinetic friction is of two types. (a) Sliding friction. (b) Rolling friction.

Sliding friction is the kinetic friction between two surfaces when a body slides over another body. The rolling friction is the kinetic friction between the surfaces when a body rolls over another body. Rolling friction is less than sliding friction.

Coefficient of friction (μ)

(i) Coefficient of static friction (μ_s)

Coefficient of static friction between two surfaces is defined as the ratio of the limiting static friction (f_s^{max}) to the normal reaction (R) between the surfaces.

$$\mu_s = f_s^{max}/R$$

(ii) Coefficient of kinetic friction (μ_k)

It is defined as the ratio of the kinetic friction (f_k) to the normal reaction (R) between the surfaces

$$\mu_k = f_k/R$$

Note: Normal reaction (R)

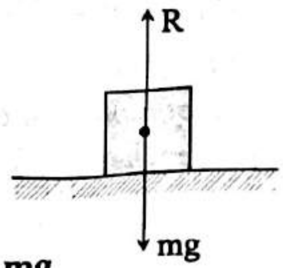


Fig. 22

Ex: (i) Consider a body of mass m on a horizontal surface. The weight mg of the body acts vertically downwards. The normal reaction R by the surface on the body acts vertically upwards perpendicular to the surface.

$$\therefore R = mg \;\; \therefore \mu_s = f_s^{max}/R = f_s^{max}/mg \;\therefore \mathbf{f_s^{max} = \mu_s mg}$$

Similarly, $\mu_k = f_k/R = f_k/mg \;\; \therefore \mathbf{f_k = \mu_k mg}$

Ex: (ii) Consider a body of mass m on an inclined plane of angle θ. The weight mg of the body acts vertically downwards. The normal reaction R of the plane on the body acts perpendicular to the plane upwards. Resolving the weight of the body into two components, we get, (i) $mg\cos\theta$ perpendicular to the plane opposite to R and (ii) $mg\sin\theta$ along the plane downwards. So, the normal reaction, $\mathbf{R = mg\cos\theta}$.

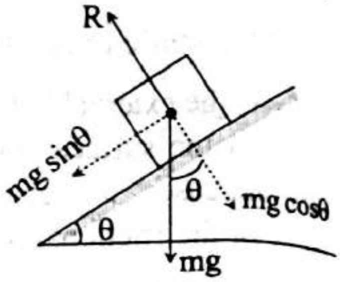


Fig. 23

$$\therefore \mu_s = f_s^{max}/R = f_s^{max}/mg\ \cos\theta \therefore f_s^{max} = \mu_s \mathbf{mg\cos\theta}$$

Similarly, $\mu_k = f_k/R = f_k/mg\ \cos\theta \therefore \mathbf{f_k = \mu_k mg\cos\theta}$

Laws of kinetic friction

1. The kinetic friction has a constant value depending on the nature of the two surfaces in contact.
2. The kinetic friction f_k is proportional to the normal reaction R. *i.e.*, $f_k = \mu_k \times R$, where the constant of proportionality μ_k is called coefficient of kinetic friction.
   Since $f_k < f_s^{max}$, $\mu_k < \mu_s$
3. The kinetic friction between the two surfaces is independent of the relative velocity between the surfaces.

Angle of friction (Limiting angle of friction)

The resultant of the normal reaction R and the limiting friction f_s^{max} is called the total reaction S.

The angle of friction (λ) *is the angle between the directions of normal reaction R and the resultant reaction* (*S*).

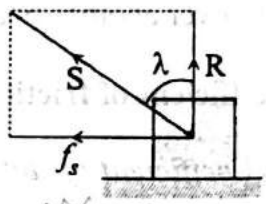


Fig. 24

From the figure 20,

$$\tan\lambda = \frac{f_s^{max}}{R} = \mu_s$$

Hence the tangent of the angle of friction gives the coefficient of friction.

Angle of repose (α)

Let a body of mass m be placed on a rough inclined plane. The inclination is gradually increased until the body is just on the point of sliding down. Let α be the inclination at that instant. This angle α is called angle of repose.

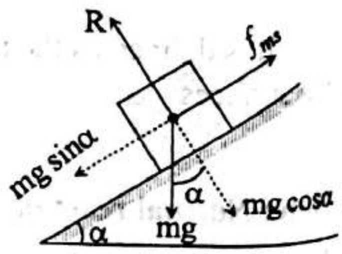


Fig. 25

Angle of repose of an inclined plane with respect to the surface of a body in contact with it is the angle of inclination of the plane when the body just starts sliding down the plane under its own weight.

The forces acting on the body are:-

1. the weight of the body mg acting vertically downwards
2. the reaction R acting perpendicular to the plane upwards and

3. the limiting friction f_s^{max} acting parallel to the plane upwards.

The weight mg of the body can be resolved in directions parallel to the plane and perpendicular to it. The components are '$mg \sin\alpha$' and '$mg \cos\alpha$'. Since the body is in equilibrium,

$$f_s^{max} = mg \sin\alpha \quad \cdots (1) \text{ and } R = mg\cos\alpha \quad (2)$$

Dividing equation (1) by equation (2)

$$f_s^{max}/R = \frac{mg \sin\alpha}{mg\cos\alpha} = \tan\alpha \quad \therefore \quad \mu_s = \tan\alpha$$

The tangent of the angle of repose is equal to coefficient of static friction.

Hence, the angle of repose is equal to the angle of friction.

Rolling friction

Rolling friction between two surfaces in contact is the force of friction which calls into play when a body rolls over another. The corresponding coefficient is denoted by μ_r which is much smaller than kinetic sliding friction μ_k for the same two surfaces.

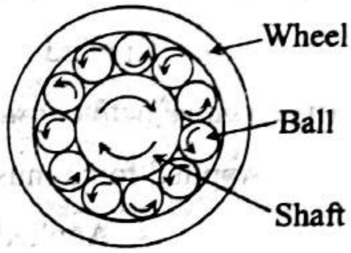


Fig. 26

It is our common experience that it is easier to roll a body than to slide it along the ground. This is also the principle on which ball bearings work. Hard steel balls are placed between the moving parts like coaxial cylinders. The balls rotate as the cylinders turn relative to each other. This considerably reduces friction.

Disadvantages of friction

Due to friction the whole energy given to a machine is not converted to useful work; a portion of energy is wasted to overcome friction. This reduces the efficiency of the machine. The energy used in overcoming friction appears in the form of heat. Moreover, friction causes wear and tear of the moving parts of the machine due to rubbing of various parts against one another.

Advantages of friction

It is due to the existence of friction that we can walk on the ground, trains can run on rails, buses can ply on the road etc. In the absence of friction we cannot hold a pen or stop our vehicles. Nails and screws join two surfaces due to the force of friction. Without friction it is impossible to climb a tree or fix a nail on the wall. In fact life becomes impossible if there is no friction in nature. So friction is a necessary evil.

Lubrication

A lubricant is a substance which forms a thin layer between two surfaces in contact. It fills the depressions present on the surfaces of contact and hence friction is reduced. Generally oil and grease are used as lubricants to reduce friction as well as to protect the moving parts from overheating. A modern lubricant is a mixture of mineral oil, vegeta oil and colloidal thin oil. For heavy machinery thick oil or grease is required because t oil would be squeezed out from between the surfaces in contact by great pressure. In very heavy machinery solid lubricant graphite is used. For light machinery like watches, sewing machines etc, thin oil is used.

Flow of compressed pure dry air acts as lubricant. It reduces friction between the moving parts by providing an elastic cushion. It also prevents overheating and settling of dust on moving parts.

Motion on a plane surface

1. Horizontal surface

(i) *To move with uniform velocity*

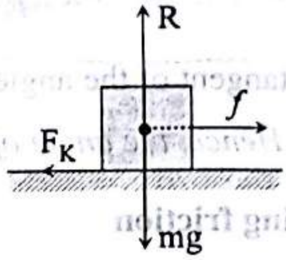


Fig. 27

To maintain constant velocity the force to be applied is equal to the frictional force.

$$\text{Applied force } f = F_k = \mu_k R = \mu_k mg$$

(ii) *To move with constant acceleration*

Applied force must be sufficient to overcome friction and also produce acceleration

$$\text{Applied force } f = F_k + ma = \mu_k mg + ma = m(\mu_k g + a)$$

2. Inclined surface

(i) *Motion up the plane*

$$\text{Total opposing force } = mg\sin\theta + F_k$$

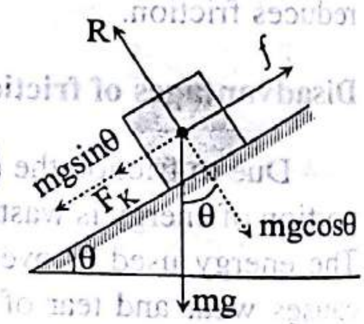


Fig. 28

Force required to produce an acceleration a is given by

$$f = mg\sin\theta + F_k + ma$$
$$= mg\sin\theta + \mu_k mg\cos\theta + ma$$

For the body to move up with constant velocity the force to be applied

$$f = mg\sin\theta + \mu mg\cos\theta$$

(ii) *Motion down the plane*

In this case $mg\sin\theta$ acts downwards and $\mu mg\cos\theta (F_k)$ acts upwards. If $mg\sin\theta$ is greater than $\mu mg\cos\theta$ the body slides down with acceleration

$$a = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = g(\sin\theta - \mu\cos\theta)$$

Examples

IV.26. Two blocks of masses $m = 1\,\text{kg}$ and $M = 2\,\text{kg}$ are in contact on a frictionless table. A horizontal force $F = 3\,\text{N}$ is applied to m. Find the force of contact between the blocks. Will the force of contact remain the same if F is applied to M?

As the blocks are rigid, both the masses move together with the same acceleration,

$$a = \frac{F}{M+m} = \frac{3}{1+2} = 1\ \text{ms}^{-2}$$

Force on the mass $M = Ma = 2 \times 1 = 2\,N$

If the force is applied on M, the force on the mass $m = 1 \times 1 = 1\,\text{N}$

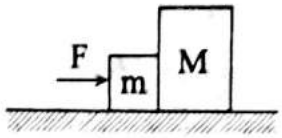


Fig. 29

IV.27. A body of mass m moves up a rough inclined plane at an angle θ with the horizontal. The body is pulled parallel to the plane by a force F. Find the acceleration.

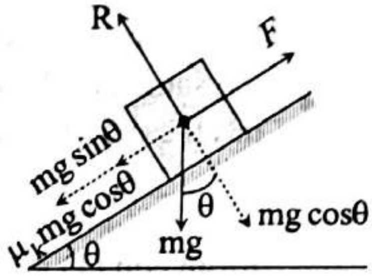


Fig. 30

The downward force $= mg\sin\theta + \mu_k mg\cos\theta$

The unbalanced force acting upwards

$$= F - mg\sin\theta - \mu_k mg\cos\theta = ma$$

$$\therefore \mathbf{a} = (\mathbf{F}/\mathbf{m}) - (\sin\theta + \mu_k\cos\theta)\mathbf{g}$$

IV.28. A block of mass 50 kg is placed on a rough inclined plane. The inclination is gradually increased to 30° when the block starts sliding down. Find the magnitude of the limiting friction. **[NCERT]**

Downward force due to gravity $= mg\sin\theta = 50 \times 9.8 \times \sin 30 = 245\,\text{N}$

Limiting frictional force, $F = \mathbf{245\,N}$

IV.29. A horizontal force of 20 N pushes a block weighing 1 kg against a vertical wall. The coefficient of static friction between the wall and the block is 0.6. Assuming the block is not moving initially (a) will the block start sliding down? (b) what minimum horizontal force will keep the block pressed against the wall?

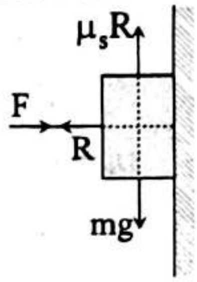


Fig. 31

Normal reaction between the surfaces, $R = 20\,\text{N}$

Maximum frictional force between the block and the wall $= \mu_s R = 0.6 \times 20 = 12\,\text{N}$ (a) Since the weight of the body 9.8 N is less than frictional force 12 N, the block will remain in equilibrium

(b) Minimum frictional force required to keep the body pressed against the wall

$$= \text{weight of the body} = 9.8\,\text{N}$$

$$\therefore \quad \mu_s \times R = 9.8;\ R = 9.8/0.6 = 16.33\,\text{N}$$

$\therefore$ Minimum horizontal force required $= \mathbf{16.33\,N}$

IV.30. If the force required just to move a body up an inclined plane is double the force required to prevent the body from sliding down, find the angle of inclination of the plane in terms of the coefficient of friction μ.

Force required just to move the body up the plane,

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$

Force required just to prevent it from moving down,

$$F_2 = mg\sin\theta - \mu mg\cos\theta.$$

But, $F_1 = 2F_2$, *i.e.*, $mg\sin\theta + \mu mg\cos\theta = 2(mg\sin\theta - \mu mg\cos\theta)$;

$\therefore \quad \tan\theta = 3\mu$

IV.31. *A body A lies on a smooth table and another body B is placed over A. The coefficient of friction between A and B is μ. What acceleration given to A will cause slipping to occur between A and B?

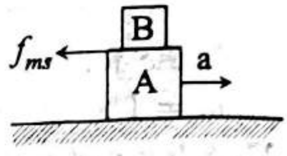


Fig. 32

The limiting frictional force for B, $f_s^{\max} = \mu R = \mu m_B g$. (1)

If the force of friction between A and B is less than $f_s^{\max}$ the body B will move with the same acceleration a of A and so $f = m_B a$. When slipping is about to occur f has the maximum value $f_s^{\max}$ and has maximum acceleration $a_{\max}$

$$\therefore \quad f_s^{\max} = m_B a_{\max} \quad (2)$$

From eqn (1) and (2), $a_{\max} = \mu g$

IV.32. The rear side of a truck is open and a box of 50 kg mass is placed 5 m away from the rear end. The coefficient of friction between the box and the surface below it is 0.15. On a straight road the truck starts from rest and accelerates at $2\,\text{ms}^{-2}$. At what distance from the starting point does the box fall off the truck? ($g = 10\,\text{ms}^{-2}$) **[NCERT]**

Net backward force on the box $= ma - \mu mg$

$= 50 \times 2 - 0.15 \times 50 \times 10 = 25\,\text{N}$

Net backward acceleration of the box, $a = \dfrac{F}{m} = \dfrac{25}{50} = 0.5\,\text{ms}^{-2}$

Time taken by the box to travel 5 m

$u = 0; \quad a = 0.5\,\text{ms}^{-2}; \quad S = 5\,\text{m}; \quad t = ?$

$S = ut + (1/2)at^2; \quad 5 = (1/2) \times 0.5 \times t^2; \quad t = \sqrt{20}\,\text{s}$

Distance travelled by the truck in this time

$u = 0; \quad a = 2\,\text{ms}^{-2}; \quad t = \sqrt{20}\,\text{s}; \quad S = ?$

$S = ut + (1/2)at^2 = (1/2) \times 2 \times 20 = \mathbf{20\,m}$

IV.33. A body of mass 50 kg is kept on a horizontal surface of coefficient of static friction 0.5. Find the least horizontal force required to start motion. If now the body is slightly disturbed and the same force continues to act, find the acceleration of the body. (Coefficient of kinetic friction = 0.48)

$m = 50\,\text{kg}; \quad \mu_s = 0.5; \quad F_s = \mu_s \times R = \mu_s \times mg$

$= 0.5 \times 50 \times 9.8 = 245\,\text{N}$

Force required to start motion $=$ **245 N**

As the body moves friction decreases. Hence the applied force is greater than the frictional force and the body moves with acceleration.

$\mu_k = 0.48; \quad F_k = \mu_k \times R = \mu_k \times mg$

$= 0.48 \times 50 \times 9.8 = 235.2\,\text{N}$

Force applied $= 245$ N

Unbalanced horizontal force, $F = 245 - 235.2 = 9.8$ N

$$\text{Acceleration, } a = F/m = \frac{9.8}{50} = \mathbf{0.196\,ms^{-2}}$$

CENTRIPETAL FORCE

When a particle moves round a circle, there is a centripetal acceleration. If v is the speed of the particle and r the radius of the path,

Centripetal acceleration $= v^2/r = r\omega^2$, where ω is the angular velocity.

The force acting on the body that gives this acceleration is called *centripetal force.*

$$\text{Centripetal force} = \text{mass} \times \text{centripetal acceleration}$$

$$i.e., \quad F = mv^2/r = mr\omega^2 = mv\omega$$

It is this force that keeps the body in a circular path by continuously changing its direction. On removing this force the body will fly off along the tangent with constant velocity.

When a body is moving round the circle with a uniform speed, no work is done as there is no displacement in the direction of the force.

When a body executes non-uniform circular motion, work is done by the tangential force.

Examples of centripetal force

1. When a body tied to one end of a string is whirled in a horizontal circle, the necessary centripetal force is supplied by the tension T in the string. The weight of the body does not have any component towards the centre of the circular path. So, $T = mv^2/r$.
2. For the earth moving round the sun, centripetal force is provided by the gravitational force of attraction between the earth and the sun.

Motion of a particle in a vertical circle

A particle of mass m is tied to the end of a light inextensible string and whirled in a vertical circle. Let r be the radius of the circle. Unlike the particle moving in a horizontal circle, the particle moving in a vertical circle is affected by gravity. The particle will not have uniform speed and the tension will change from point to point.

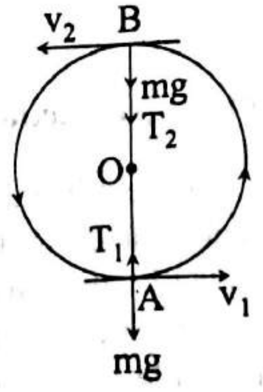


Fig. 33

When the particle is at the lowest point A (Fig. 29), let v_1 be the velocity. If T_1 is the tension in the string at that instant,

$$T_1 - mg = \frac{mv_1^2}{r}\,;\ T_1 = mg + \frac{mv_1^2}{r} \tag{1}$$

When the particle is at the highest point B, let v_2 be the velocity of the particle and T_2 the tension in the string. Then,

$$T_2 + mg = \frac{mv_2^2}{r} \quad \therefore\ T_2 = \frac{mv_2^2}{r} - mg \tag{2}$$

Special Cases

1. The difference between the tensions at the lowest and highest points.

$$T_1 - T_2 = mg + \frac{mv_1^2}{r} - \left(\frac{mv_2^2}{r} - mg\right) \tag{3}$$

$$= 2\,mg + \frac{m}{r}(v_1^2 - v_2^2) \tag{4}$$

Applying the law of conservation of energy for points A and B.

KE of particle at A = (KE + PE) of the particle at B

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mg \times 2r$$

$$v_1^2 = v_2^2 + 4rg\,;\ v_1^2 - v_2^2 = 4rg \tag{5}$$

Substituting in eqn (4)

$$T_1 - T_2 = 2mg + \frac{m}{r} \times 4rg = 2mg + 4mg = \mathbf{6mg}$$

2. Expression for the critical speed at the highest point

If v_2 is the minimum speed at the highest point so that the string does not slacken, then $T_2 = 0$.

From eqn (2), $mg = mv_2^2/r\,;\ v_2^2 = rg\,;\quad v_2 = \sqrt{\mathbf{rg}}$

3. Minimum speed at the bottom of a loop required for looping a full circle

Let v_1 be the speed required at the bottom of the loop so that the body may just reach the highest point. At the highest point the speed should be,

$$v_2 = \sqrt{rg}$$

From eqn (5) $\quad v_1^2 - v_2^2 = 4rg$

$$v_1^2 = v_2^2 + 4rg = rg + 4rg = 5rg$$

$$v_1 = \sqrt{\mathbf{5rg}}$$

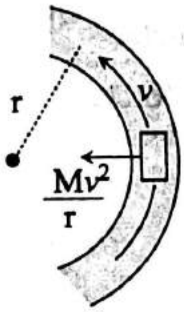
Centrifugal reaction

The centripetal force produces a centrifugal reaction. When we whirl a stone tied to the end of a string, the centripetal force necessary to keep the stone in the circular path is provided by the tension in the string. The hand in turn will experience an equal reaction force outward along the radius. It must be noted that the centripetal force acts on the rotating body while the centrifugal reaction force acts on the body at the centre. Since these two forces act on different bodies they do not cancel each other.

Examples in circular motion

(a) Car on a circular level road

Let a car of mass M move with constant speed v on a flat horizontal circular portion of a road of radius r.



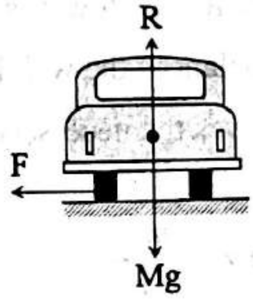


Fig. 34

The forces acting on the car are:-

(1) The weight Mg of the car acting vertically down, which is balanced by the normal reaction R of the ground acting vertically upwards, such that $R = Mg$

(2) Centripetal force (Mv^2/r) acting on the car, directed towards the centre of the circle. This force is provided by the force of friction between the car tyre and the road.

If F is the force of friction and μ the coefficient of friction,

$$F = \mu R = \mu Mg$$

For safe turning, the centripetal force should be equal or less than the available force of friction.

$$\frac{Mv^2}{r} \leq F \quad \text{or} \quad \frac{Mv^2}{r} \leq \mu_s Mg$$

$$v^2 \leq \mu rg; \quad v \leq \sqrt{\mu rg} \quad \therefore v_{\max} = \sqrt{\mu rg}$$

This is the maximum speed limit with which a car can have safe turning. If the speed of the car exceeds this value, the force due to friction will not be sufficient to provide the centripetal force and it will skid off the road.

(b) Banking of roads

The maximum speed limit a vehicle can have on a curved level road depends on μ the coefficient of friction between the tyres and the road. The force of friction is not a reliable source for providing the required centripetal force for the vehicle. The friction causes wear and tear of the tyres. Moreover, if the speed exceeds the limit, it may cause accident by skidding. To avoid wear and tear of the tyres and the accident by skidding, the roadbed is banked at curves.

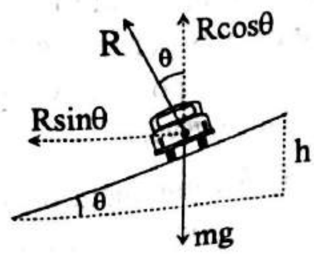


Fig. 35

At curves, the surface of he road is so laid that it slopes with the outer edge raised above the inner edge. This is called *banking of roads*. The angle between the surface of the road and the horizontal is called *banking angle*.

Let r be the radius of curvature of the road and θ the angle of banking and mg the weight of the vehicle. The reaction R of the road on the vehicle is perpendicular to the surface of the road. Resolving the reaction R into two components, we get, (i) $R\cos\theta$, vertically upwards and (ii) $R\sin\theta$ towards the centre of curvature of the curve.

$R\cos\theta$ component balances the weight of the vehicle.

$R\sin\theta$ component gives the necessary centripetal force.

$$\therefore R\sin\theta = \frac{mv^2}{r} \quad (1)$$

$$\text{and} \quad R\cos\theta = mg \quad (2)$$

$$\text{Eqns. (1)/(2)}, \quad \tan\theta = \frac{v^2}{rg}$$

Roads are usually banked for the average speed of vehicles passing over them. The speed limit at which the curve can be negotiated without wear and tear of the tyre is given by,

$$v = \sqrt{rg\tan\theta}$$

If the force of friction is also taken into account it can be shown that the safe limit of speed in this case is

$$v_{max} = \sqrt{\frac{rg[\tan\theta + \mu]}{1 - \mu\tan\theta}}$$

(c) Banking of rails

If the railway track is curved and rails are laid in a horizontal plane, the centripetal force is supplied by the flange pressure of the outer rails on the outer wheels. This causes the wear and tear of the rails. To avoid this and the derailment at high speed, the track is banked *i.e.*, the outer rail is laid raised a bit above the inner rail.

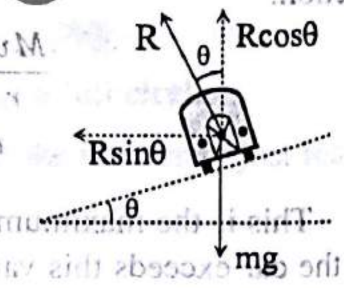


Fig.36

Let r be the radius of curvature of the track and θ the angle of banking, *i.e.*, the angular height of the outer rail over the inner.

Let R be the reaction at the wheels. The reaction is inclined at angle θ to the vertical.

The weight mg of the train acts vertically downwards; the normal reaction R of the rail on the train acts perpendicular to the line joining the rails upwards. Resolving the reaction R into two components we get, (i) $R\cos\theta$ vertically upwards and (ii) $R\sin\theta$ towards the centre of the curve.

$R\cos\theta$ component balances the weight mg of the train and $R\sin\theta$ component gives the necessary centripetal force.

$$R\sin\theta = \frac{mv^2}{r} \quad ; \quad R\cos\theta = mg \quad \therefore \tan\theta = v^2/rg$$

If l is the distance between the rails and h the height of the outer rail over the inner,

$$\tan\theta = h/l.$$

The optimum speed to avoid flange thrust is given by,

$$v = \sqrt{rg\tan\theta}$$

Fig. 37

(d) Motion of a cyclist along curved path

When a cyclist negotiates a curve, centripetal force has to be provided. For this, he leans towards the centre of curvature of the path so that the resultant reaction force R at the wheel is inclined to the vertical.

Let θ be the angle of inclination of the resultant reaction from the vertical. The horizontal component of this reaction provides the necessary centripetal force. The vertical component balances the weight of the cyclist and the cycle.

Let m be the mass of the cycle and the cyclist, v the velocity and r the radius of curvature of the path.

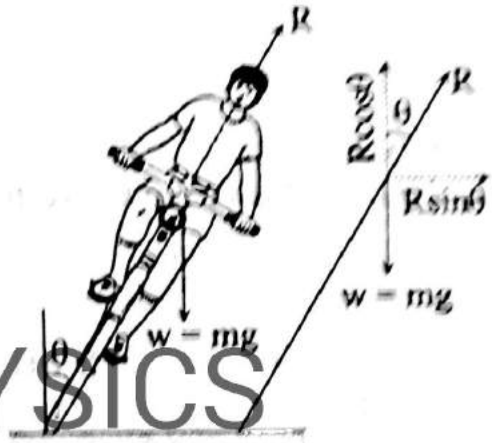


Fig. 38

Horizontal component of the reaction,

$$R\sin\theta = \frac{mv^2}{r} \qquad (1)$$

Vertical component of the reaction,

$$R\cos\theta = mg \qquad (2)$$

Dividing (1) by (2), $\tan\theta = \dfrac{v^2}{rg}$

Examples

IV.34. A light string 1 m long can just support a weight of 10 kg.wt. A body of mass 2 kg is whirled by the string in a horizontal circle. Find the greatest number of revolutions per second that can be made without breaking the string.

$$F = 10\text{ kg.wt} = 10 \times 9.8\ N,\ r = 1\ m;\ m = 2\text{ kg};\ n = ?$$

$$F = mr\omega^2 = mr(2\pi n)^2 = 4\pi^2 n^2 mr$$

$$n = \sqrt{\frac{F}{4\pi^2 mr}} = \sqrt{\frac{10 \times 9.8}{4\pi^2 \times 2 \times 1}} = \mathbf{1.115\ r.p.s}$$

IV.35. One end of a string of length 1 m is tied to a stone of mass 0.4 kg and the other end to a small pivot on a smooth vertical board. What is the minimum speed of the stone required at the lowest point so that the string does not slacken at any point during its motion along the vertical circle?

Minimum speed required at the topmost point for the string not to slacken is given by,

$$\frac{mv_2^2}{r} = mg; \quad v_2 = \sqrt{rg}$$

Let v_1 be the minimum speed at the lowest point so that the velocity at the topmost point is $v_2 = \sqrt{rg}$. By law of conservation of energy,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mg \times 2r$$

$$v_1^2 = v_2^2 + 4rg = rg + 4rg = 5rg$$

$$v_1 = \sqrt{5rg} = \sqrt{5 \times 1 \times 9.8} = \mathbf{7\ ms^{-1}}$$

IV.36. An aircraft executes a horizontal loop at a speed of $720\,\text{kmh}^{-1}$ with its wings banked at $15°$. What is the radius of the loop? **[NCERT]**

$$v = 720\ \text{km/h} = 200\ \text{ms}^{-1}; \quad \tan\theta = v^2/rg;$$

$$\therefore r = v^2/g\tan\theta = \frac{200 \times 200}{9.8 \times \tan 15} = \mathbf{15.2 \times 10^3\ m}$$

IV.37. A cyclist negotiates a curve of radius $100\,\text{m}$ with a velocity $10\,\text{ms}^{-1}$. How much should he lean from the vertical in doing so?

$$\tan\theta = \frac{v^2}{rg} = \frac{10^2}{100 \times 9.8} = 0.102 \quad \therefore \theta = \mathbf{5°49'}$$

IV.38. A meter gauge train is travelling at 36 km/h on a curved track of 600 m radius. How much the outer rail must be raised to avoid lateral pressure?

$$v = 36/\text{km/h} = 10\ \text{ms}^{-1}; \quad r = 600\ \text{m}; \quad l = 1\ \text{m}$$

$$\tan\theta = \frac{v^2}{rg}; \quad i.e., \quad \frac{h}{l} = \frac{v^2}{rg}$$

$$h = \frac{lv^2}{rg} = \frac{1 \times 10^2}{600 \times 9.8} = \mathbf{0.017\ m}$$

IV.39. A disc revolves in a horizontal plane at a steady rate of 3 r.p.s. A coin will remain on the disc if it is kept at a distance of 2 cm from the axis of rotation. What is the coefficient of friction between the coin and the disc? $g = 9.8\,\text{ms}^{-2}$.

Let m be the mass of the coin. Its angular velocity ω is the same as that of the disc. The centripetal force is provided by the friction F between the coin and the disc.

$\therefore F = mr\omega^2$, where r is the radius of the path of the coin.

But $F = \mu R = \mu mg$, where μ is coefficient of friction,

$\therefore \mu mg = mr\omega^2$

$$\mu = \frac{r\omega^2}{g} = \frac{r(2\pi n)^2}{g} = \frac{0.02 \times (2\pi \times 3)^2}{9.8} = \mathbf{0.724}$$

IV.40. A long playing record revolves with a speed of $33\frac{1}{3}$ r.p.m and has radius 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre. If the coefficient of friction between the coins and the record is 0.15, which of the two coins will revolve with the record? **[NCERT]**

For the coin to revolve with the record, the force of friction must be more than the necessary centripetal force

$$\text{ie} \quad mr\omega^2 \le \mu mg$$

$$r \le \frac{\mu g}{\omega^2} \le \frac{\mu g}{(2\pi n)^2} \le \frac{\mu g}{4\pi^2 n^2}$$

$$\mu = 0.15 \quad ; \quad n = \frac{33\frac{1}{3}}{60} = \frac{5}{9}$$

$$r \le \frac{0.15 \times 9.8 \times 81}{4\pi^2 \times 25} = \mathbf{0.12\ m = 12\ cm}$$

Hence the nearer coin which is 4 cm away from centre will stay on the record.

IV.41. A motor cyclist is making a vertical loop inside a 'death well'. What is the minimum speed required to perform a vertical loop if the radius of the chamber is 25 m? [NCERT]

At the uppermost point, $R + mg = mv^2/r$, where R is the normal reaction (downwards) on the motor cyclist by the ceiling of the chamber. The minimum possible speed at the uppermost point corresponds to $R = 0$

$$mg = \frac{mv^2}{r}; \; v = \sqrt{rg} = \sqrt{25 \times 9.8} = \mathbf{15.7\ ms^{-1}}$$

IV.42. A 70 kg man stands in contact against the wall of a cylindrical drum of radius 3 m rotating about its vertical axis. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall, when the floor is suddenly removed? [NCERT]

The horizontal reaction R of the wall on the man provides the needed centripetal force.

$$\therefore R = mr\omega^2$$

The frictional force F acting vertically upwards opposes the weight mg.
The man remains stuck to the wall after the floor is removed if,

$$mg < F; \quad F = \mu R$$

$$mg < \mu\, mr\omega^2$$

$$\therefore g < \mu\, r\omega^2$$

The minimum angular speed of rotation of the cylinder is when, $g = \mu\, r\omega^2$

$$\therefore \quad \omega_{min} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{9.8}{0.15 \times 3}} = \mathbf{4.7}\text{ rad s}^{-1}$$

IV.43. A circular race track of radius 500 m is banked at an angle of 20°. If the coefficient of friction between the tyres and the road is 0.15, calculate the (a) optimum speed of the car to avoid wear and tear and (b) the maximum permissible speed to avoid slipping. [NCERT]

(a) On banked road, the horizontal component of the normal reaction and frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the component of the normal reaction is enough to provide the centripetal force. The optimum speed is given by the relation,

$$\tan\theta = \frac{v^2}{rg};\quad v = \sqrt{rg\tan\theta}$$

$$\therefore v = \sqrt{500 \times 9.8 \times \tan 20} = \mathbf{42.23\ ms^{-1}}$$

(b) To calculate the maximum permissible speed,

$$v_{max} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

$$= \sqrt{500 \times 9.8\left(\frac{0.15 + \tan 20}{1 - 0.15 \times \tan 20}\right)} = \mathbf{51.6\ ms^{-1}}$$

IV.44. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0.1. Will the cycle slip while taking the turn?[NCERT]

The condition for cyclist not to slip is, $v^2 \leq \mu rg$

$$v^2 = 5 \times 5 = 25;\ \mu rg = 0.1 \times 3 \times 9.8 = 2.94$$

The condition is not obeyed. Hence the cyclist will slip while taking the turn.

IMPORTANT POINTS

Newton's laws of motion

1. *In the absence of net external force a body will either remain at rest or move with uniform velocity*
2. *Momentum* $p = mv$

   Rate of change of momentum is directly proportional to applied force.
3. *Action and reaction are equal and opposite. These two act on different bodies*

$$\text{Impulse } I = F \times \Delta t = mv - mu = \text{change in momentum}$$

Motion of a body in a lift

1. *Lift at rest or moving with uniform velocity*

$$\text{Reaction,} \quad R = mg$$

2. *Lift moving up with uniform acceleration* $R = m(g + a)$
3. *Lift moving down with uniform acceleration* $R = m(g - a)$

Law of conservation of momentum

The total momentum of an isolated system of particles is conserved

Friction. *Opposes relative motion between two surfaces in contact. Static friction f_s opposes impending relative motion; Kinetic friction f_k opposes actual relative motion*

$$f_s^{\max} = \mu_s R; \qquad f_k = \mu_k R$$

Inertial frame. *A reference frame relative to which Newton's law of inertia is valid.*

Non-inertial frame. *Frames of reference which have acceleration or rotation with respect to an inertial frame.*

Circular motion.

$$\text{Centripetal force} = \frac{mv^2}{r}$$

Car on a circular level road, *maximum permissible speed* $v_{\max} = \sqrt{\mu_s Rg}$

Car on a banked road, $v_{\max} = \left[\dfrac{rg(\mu + \tan\theta)}{1 - \mu/\tan\theta}\right]^{1/2}$,

If there is no friction, $v_{\max} = \sqrt{Rg \tan\theta}$

Rocket propulsion.

$$v = v_0 + v_r \log_e(M/m) = v_r \log_e(M/m), \quad if\ v_0 = 0$$