MECHANICAL PROPERTIES OF SOLIDS

As we studied in the last chapter, a rigid body generally means a hard solid object having a definite shape and size. When suitable forces are applied on a body, it undergoes a change in length, volume or shape. Thus a wire fixed at its upper end pulled down by a weight at its lower end, undergoes a change in length. A solid body under compression undergoes a change in shape or change in volume etc.

Deforming force

If an external force is applied on a body, it may suffer a change in length or change in volume or change in shape. Such a body is said to be **deformed**. The applied force is called deforming force. Thus the force required to change the length, volume or shape of a body is called deforming force.

Elasticity

When the deforming forces are removed, the body shows a tendency to recover its original condition.

"The property of a material body by which it regains its original dimensions (length, volume, shape) on the removal of the deforming forces is called elasticity".

Bodies which can completely recover their original condition on the withdrawal of the deforming forces are called **perfectly elastic** (Quartz fibre is the nearest approach to perfectly elastic).

Plasticity

Some bodies do not show any tendency to recover their original conditions They are said to be plastic and this property is known as plasticity. (Paraffin wax, putty, kneaded flour etc are close to ideal plastics)

There is a maximum value for the deforming force, beyond which the body ceases to be elastic. Thismaximum value of the deforming force is called **elastic limit** of the body.

Stress & Strain

Stress (σ):-

When the deforming force is applied to a body, internal forces are set up in the body which opposes the deformation and would tend to restore the body back to its original condition. Restoring force setup inside the body per unit area is called **stress**.

$$\therefore \text{ stress} = \frac{\text{restoring force}}{\text{area}}$$

As the restoring force set up in the body is equal and opposite to the external deforming force (within elastic limits), the stress may be measured as the external force acting per unit area.

$$\text{i.e., } \quad \text{stress} = \frac{\text{external applied force}}{\text{area}} = \frac{F}{a}$$

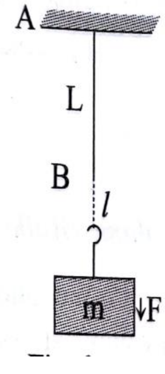
Unit: N/m^2 or pascal (Pa). Dimension $ML^{-1}T^{-2}$.

There are three ways in which a solid may change its dimensions when an external force acts on it.

Types of stress:-

1. **Tensile and compressive stress:-**

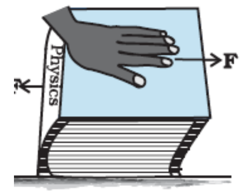
When a wire (cylinder) is stretched by a force, **applied normal to its cross sectional area**, the restoring force per unit area is called **tensile stress**.



If the body is **compressed** under the action of applied forces the restoring force per unit area is called **compressive stress**.

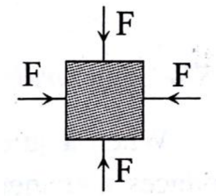
(Tensile or compressive stress can also be termed as **longitudinal stress or normal stress)**.

2.Shearing stress:-



If the deforming force applied tangential to a body (i.e. parallel to its cross sectional area), then the restoring force per unit area developed due to this tangential force is known **tangential or shearing stress**.

3. Volume (Bulk) stress:-



When a deforming force applied normally and uniformly all over the surface of a body, then the restoring force per unit area developed is called bulk stress.

Strain

When a deforming force acts on a body, it undergoes change in its dimensions and the body is said to be deformed or strained.

The ratio of the change in dimension of a body to the original dimension is called strain.

$$\text{strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$; It has **no unit** as it is the ratio of two similar quantities.

Types of Strain :-

There are three kinds of strains depending upon the nature of the deformation produced.

1. **Linear or longitudinal strain**

If the deforming force produces change in the length of the body, then the strain is known as linear or longitudinal strain.

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}}$$

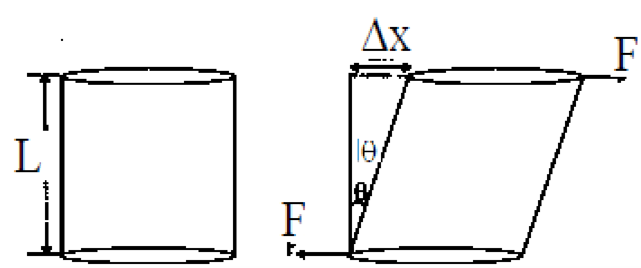
If ΔL is the increase in length of a wire of original length L, then Longitudinal strain $= \frac{\Delta L}{L}$

2. **Volume strain:-**

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

3. **Shearing strain**

Consider a cylinder of length L. Let a tangential force be applied so that it undergoes a relative displacement Δ x .

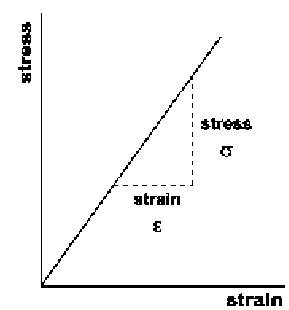


The strain so produced is knownas shearing strain and is defined as shearing strain = $\frac{\Delta X}{L}$

But $\frac{\Delta X}{L} = \tan\theta$; where θ is the angular displacement of the cylinder from the vertical (original position of cylinder).

Since θ is very small, $\tan\theta$ is nearly equal to the angle θ. So, $\theta = \frac{\Delta X}{L}$.

Hooke's law



Within elastic limit, stress is directly proportional to strain.

stress α strain

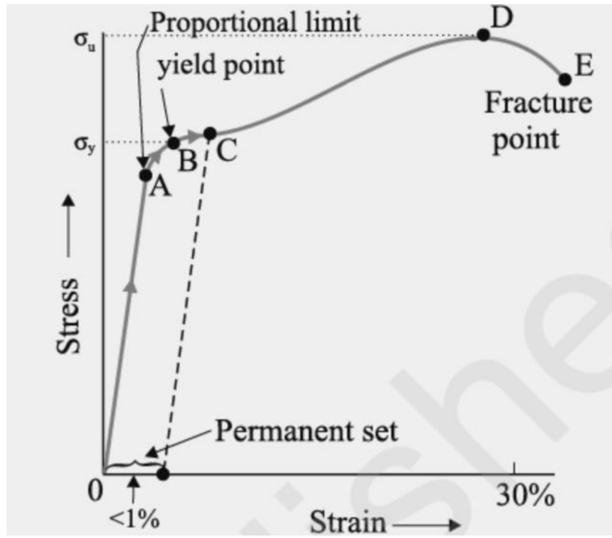
stress = K strain

Where 'K' is the proportionality constant and is known as **modulus of elasticity**.

<u>Stress - strain curve:-</u>

The relation between stress and strain for a given material under tensile stress can be found experimentally. Let a wire or cylinder is stretched by an applied force. The applied force is gradually increased and the change in length is noted. A graph is plotted between stress and the strain produced. A typical graph for a metal is shown in fig.

The stress - strain curves vary from material to material.



From the graph, we can that in the region between **<u>Region O to A,</u>** the curve is linear. In this region, Hooke's law is obeyed. The body regains its original dimension when the applied force is removed. In this region, the solid behaves as an elastic body.

<u>In the region from A to B,</u> stress and strain are not proportional, but the body returns to its original dimension when the load is removed. The point A is called **proportional limit**. The point B in the curve is known as **yield point (elastic limit)**. The corresponding stress is known as **yield strength** (S_y) of the material.

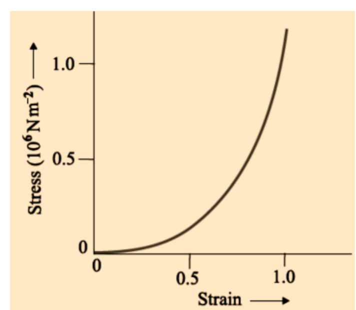
If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not equal to zero. The material is said to have a **permanent set**. The deformation is said to be **plastic deformation**.

The point D on the graph is the ultimate tensile strength (S_u) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. E is known as **fracture point**.

*Materials which break as soon as the stress is increased beyond the elastic limit are called **brittle** (Eg: cast iron)

*Materials which have a good plastic range are called **ductile** (Eg: Cu, Ag, Al etc) Such materials can be used for making springs, sheets etc.

<u>Stress-strain graph for elastomers:-</u>



***Materials like rubber and elastic tissue of aorta exhibit a different stress - strain relationship. By applying a relatively small stress, the length of rubber can be increases to several times its original length. On removal of the stress, it returns to its original length. Thus rubber has a large elastic region. But it has no well-defined plastic region. They don't obey Hooke's law as the stress - strain graph is not a straight line.

Stress - strain curve for the elastic tissue of Aorta or rubber is as shown.

Substances that can be elastically stretched to large values of strain are called **elastomers**.

<u>Elastic moduli (Modulus of elasticity)</u>

According to Hooke's law, within elastic limit,

stress α strain or $\frac{\text{Stress}}{\text{strain}} = K$; where, K is known as modulus of elasticity.

There are three types of moduli - Young's modulus, Bulk modulus and Shear modulus or Rigidity modulus
according to the deformation in length, volume or shape respectively. Unit of modulus of elasticity is N/m^2 or pascal (same as that of stress).

1. Young's modulus (Y):-

The ratio of tensile or longitudinal stress to the longitudinal strain is called Young's modulus.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$\text{longitudinal stress} = \frac{F}{A} \qquad \text{longitudinal strain} = \frac{\Delta L}{L} \qquad \therefore \quad Y = \frac{F/A}{\Delta L/L}$$

$$\text{OR} \quad Y = \frac{FL}{A \Delta L}$$

Now if $F = mg$. and r is the radius of cross section of wire, then area $A = \pi r^2$.

$$\text{Therefore,} \quad Y = \frac{mgL}{\pi r^2 \Delta L}$$

Note: It is noticed that for metals, Young's moduli are large. That is these materials require large force to produce mall change in length. It is also noticed that the force required to produce a strain in a steel wire is greater than that required to produce the same strain in aluminium, brass or copper wires of same cross-sectional area. It is for this reason that steel is preferred in heavy - duty machines and in structural designs. Wood, bone concrete and in glass have rather small Young's moduli.