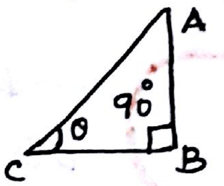
Motion in a Straight line

Simil Rahman. E
Physics Teacher.

Simil

Trignometry

consider right angle Δgle ABC



$$\sin\theta = \frac{opp}{hyp} = \frac{AB}{AC}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{BC}{AC}$$

$$\tan\theta = \frac{opp}{adj} = \frac{AC}{BC}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{1}{\tan\theta} = \cot\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(90-\theta) = +\cos\theta$$

$$\cos(90-\theta) = +\sin\theta$$

$$\sin(90+\theta) = \cos\theta$$

$$\cos(90+\theta) = -\sin\theta$$

$$\sin(180-\theta) = \sin\theta$$

$$\cos(180-\theta) = -\cos\theta$$

angle	0	30	45	60	90
sin	0	$1/2$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
Tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	α

Rest

If the position of an object does not change w.r.t time it is said to be at rest.

motion

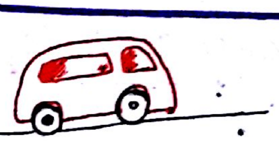
If the position of the object changes w.r.t time it is said to be in motion.

point object

If the distance travelled by the object is very large compared to the size of the object, it can be considered as point object.

Motion
- one dimensional
- two dimensional
- Three dimensional motion.

ONE DIMENSIONAL MOTION



motion restricted to a straight line.

Eg; Moving bus on straight line

* only one co-ordinate is required

2D motion

motion is restricted to a plane.

Eg; Boat on a lake

* 2D required.

3D motion

motion in a space

Eg; flight in space

* : Three co-ordinates required.

Differentiate between distance & displacement.

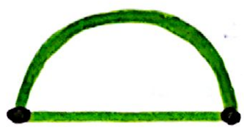
Distance	Displacement
(1) Length of the path covered by a particle	(1) shortest distance b/w Initial and final point.
(2) scalar Quantity	(2) vector Quantity.
(3) can never be '0' or -ve	(3) can be '0' or -ve
(4) never decreases with time	(4) May decrease.

calculate the distance and displacement in the following diagram.

(1) 

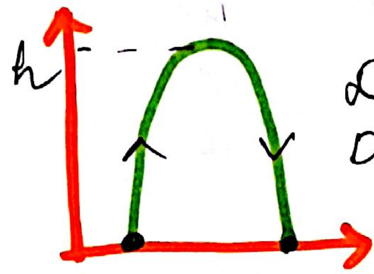
Distance $= 2\pi r$

Displacement $= 0$



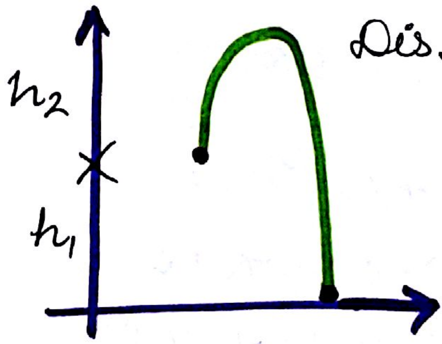
Distance $= \pi r$

Displacement $= 2r$



Distance $= 2h$

Displacement $= 0$



Distance $= h_2 + h_2 + h_1$
$= 2h_2 + h_1$

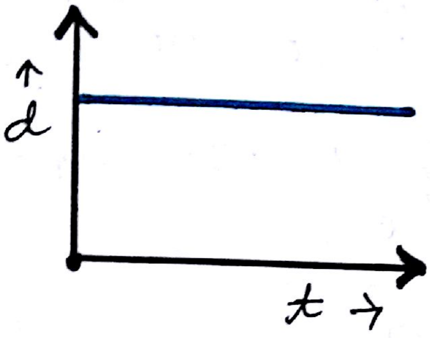
Displacement
$= h_f - h_i$
$= 0 - h_1$
$= \underline{-h_1}$

uniform motion

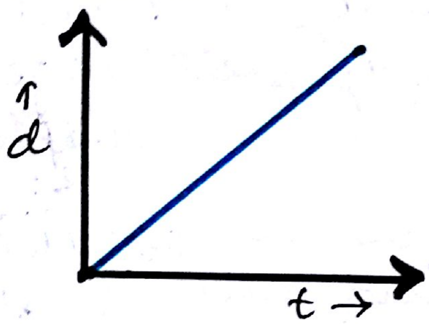
If a body covers equal distance in equal intervals of time, the body is said to be in uniform motion.

Position time graphs

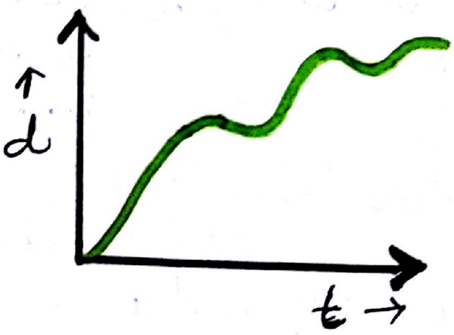
(1) Body at rest



(2) Body is Uniform motion



(3) Non uniform Motion



Rectilinear motion

motion along a straight line is called rectilinear motion.

Define speed and give its SI unit.

The time rate of change of position is called speed.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

S.I unit is m/s.

Average speed

Average speed is the ratio of total distance travelled by total time taken

$$\text{Avg speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

* SI unit is m/s.

Velocity

Is the speed of an object moving in a definite direction

or

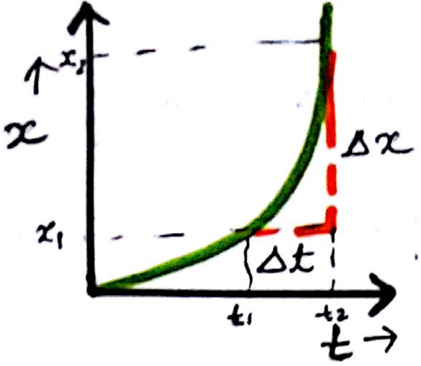
It is the time rate of displacement.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

SI unit is m/s.

Average Velocity

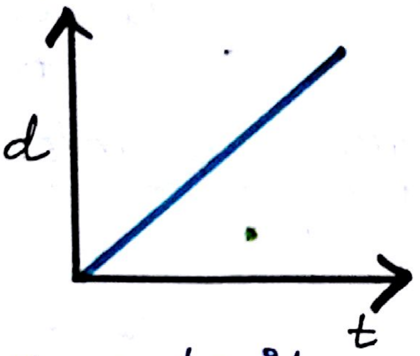
Average velocity can be defined as the change in position or displacement (Δx) divided by the time interval (Δt)



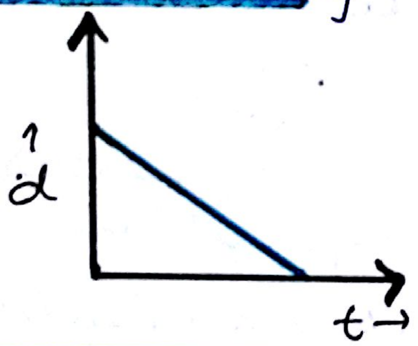
$$\text{Avg } V = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

position time graphs.

(a) Body moving with positive velocity.



(b) -ve velocity



Define Instataneous velocity.

The velocity of a particle at any instant of motion is called Instantaneous velocity.
It is also defined as the limit of average velocity as Δt becomes infinitely small

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Differentiate speed and velocity

speed	velocity
(1) Rate of change of position	(1) Rate of change of displacement
(2) It is a scalar Quantity	(2) vector Quantity
(3) It is only +ve	(3) can be +ve or -ve

Acceleration

It is the time rate of change of velocity

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{Time}}$$

$$a = \frac{v - u}{t}$$

* S.I unit is m/s^2
* Dimensional formula is LT^{-2}

Average Acceleration

It is the ratio of change in velocity to the time Interval.

$$\bar{a} = \frac{\text{change in velocity}}{\text{Time Interval}}$$

$$\bar{a} = \frac{\Delta V}{\Delta t}$$

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous Acceleration

It is the acceleration of an object at a given Instant of time

Instantaneous acceleration

$$= \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dv}{dt}$$

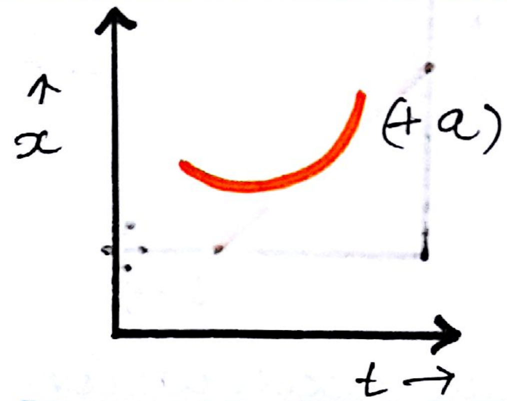
$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

uniform acceleration

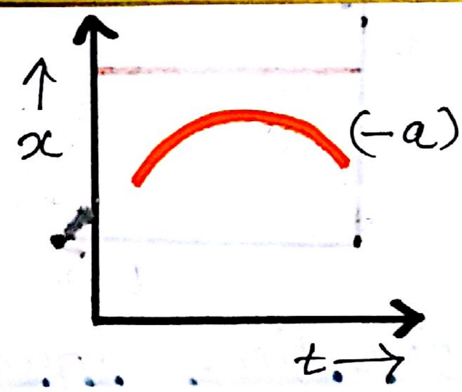
If the velocity changes in equal amount in equal intervals of time, the particle is said to move with uniform acceleration.

Position time graphs.

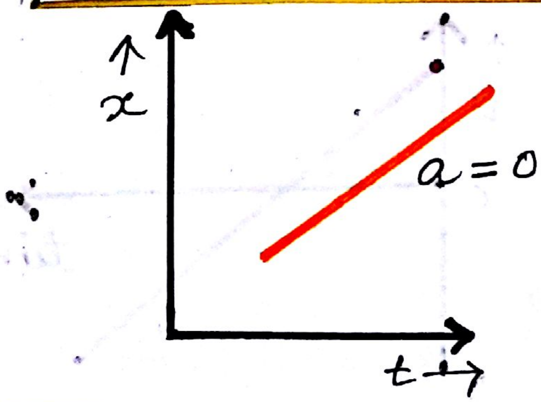
(i) +ve acceleration



(ii) -ve acceleration

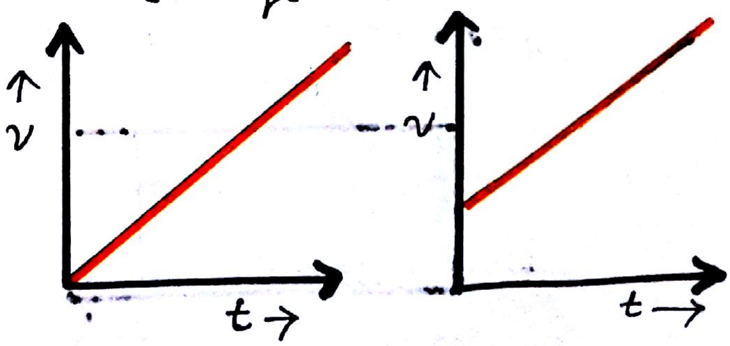


(iii) zero acceleration

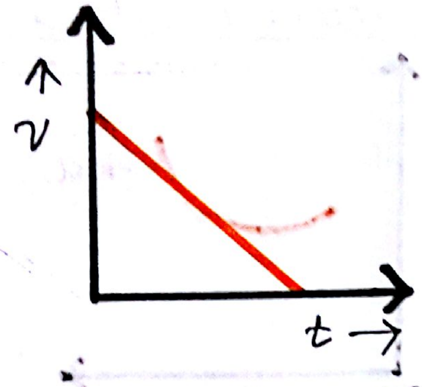


Velocity - time graphs (V-t)

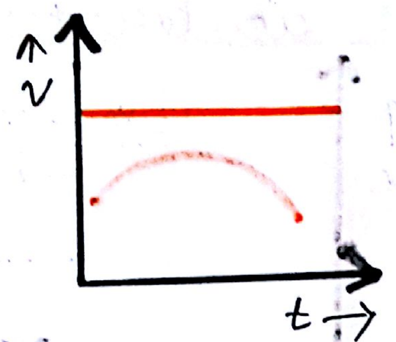
(a) positive acceleration -(uniform)



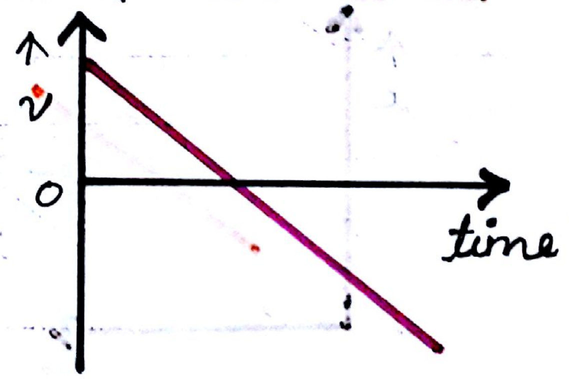
(b) constant negative acceleration



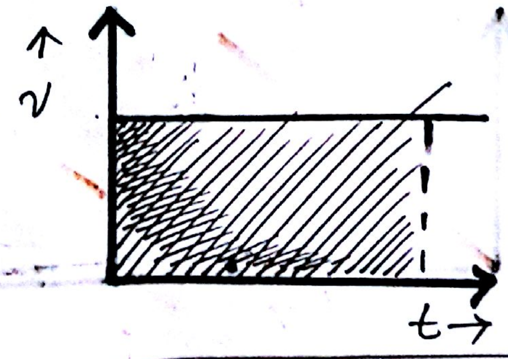
(c) zero acceleration.



* velocity time graph of a projectile.

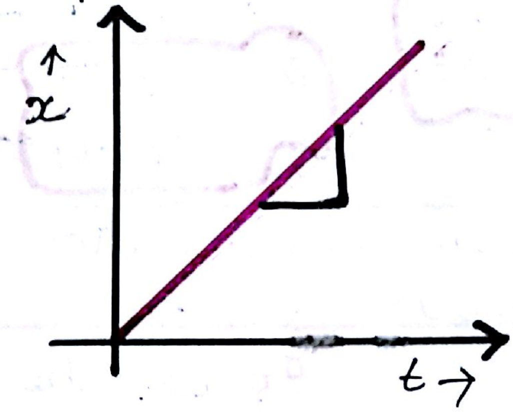


* which physical quantity can be calculated from area under v-t graph.



area $= v \times t$
$=$ displacement

* write the physical Quantity that can be calculated by the slope in position time graph?



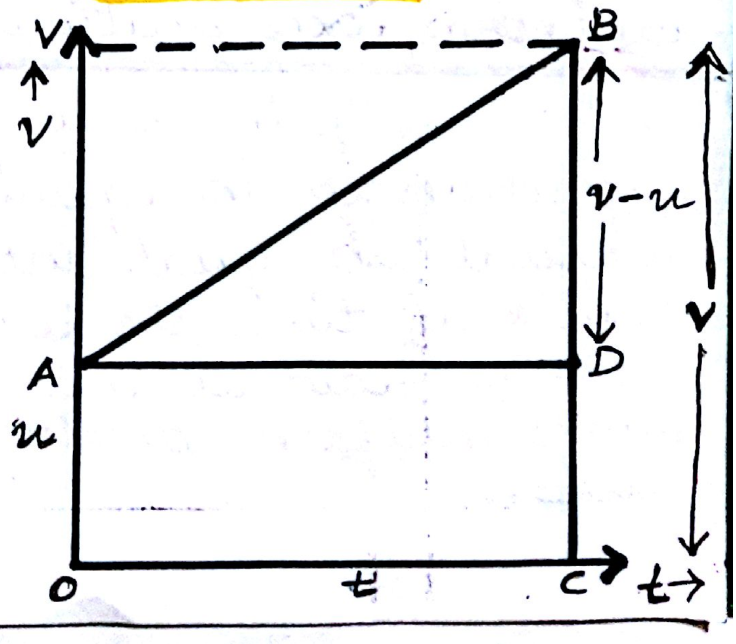
slope $= \dfrac{x}{t} =$ speed.

* Derive equations of motion by graphical method.

(1) $v = u + at$

(2) $s = ut + \frac{1}{2}at^2$

(3) $v^2 = u^2 + 2as$



Let a body moves with initial velocity 'u' and uniformly accelerated reaches 'v' in time 't'.

acceleration = $\dfrac{\text{change in velocity}}{\text{time}}$

$$a = \frac{BD}{AD}$$

$$a = \frac{v-u}{t}$$

$$\boxed{at = v-u} \quad ---\ ①$$

$$\boxed{v = u+at} \quad ----\ ②$$

Area under v-t graph gives displacement

S = area OADC + area ABD

$$S = OA \times OC + \frac{1}{2} AD \times BD.$$

$$S = ut + \frac{1}{2} t \times (v-u).$$

$$S = ut + \frac{1}{2}(v-u)t \quad ---\ ③$$

① in ③ $\Rightarrow S = ut + \frac{1}{2} at \times t$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

Area under vt graph = displacement

S = area of trapezium

S = Area OABC

$$S = \frac{1}{2}(OA + BC)\,AD$$

$$S = \frac{1}{2}(u+v)t.$$

$$S = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$$

$$S = \frac{1}{2}(v+u)\left(\frac{v-u}{a}\right).$$

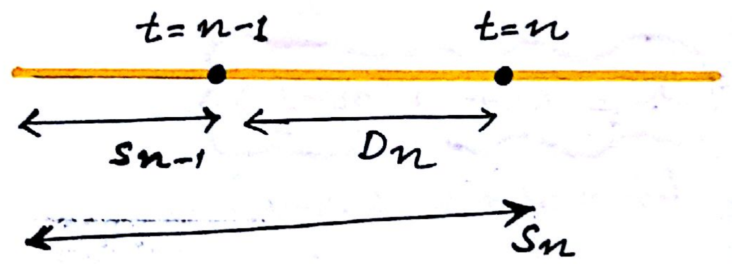
$$S = \frac{1}{2}\left(\frac{v^2-u^2}{a}\right).$$

$$2aS = v^2 - u^2.$$

$$\boxed{v^2 - u^2 = 2aS}$$

$$\boxed{v^2 = u^2 + 2aS}$$

Obtain the expression for the distance travelled in n^{th} second.



$u \rightarrow$ Initial velocity
$v \rightarrow$ final velocity

S_n and S_{n-1} :- Distances travelled by an object in 'n' and 'n-1' seconds respectively.

Scanned by CamScanner

D_n - Distance travelled in n^{th} second.

$$S = ut + \frac{1}{2}at^2$$

$$S_n = un + \frac{1}{2}an^2 \quad \ldots (1)$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \quad \ldots (2)$$

$$D_n = S_n - S_{n-1}$$

$$= un + \frac{1}{2}an^2 - [u(n-1) + \frac{1}{2}a(n-1)^2]$$

$$= un + \frac{1}{2}an^2 - [un - u + \frac{1}{2}a(n^2 - 2n + 1)]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + \frac{1}{2}2an - \frac{a}{2}$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{1}{2}a$$

$$D_n = u + an - \frac{1}{2}a$$

$$D_n = u + a(n - \tfrac{1}{2})$$

$$D_n = u + a\frac{(2n-1)}{2}.$$

$$D_n = u + \frac{a}{2}(2n-1)$$

** Formula's & Imp Questions.

1. $v = \frac{x}{t}$

2. Average speed = $\frac{\text{Total distance}}{\text{Total time.}}$

$$v_{avg} = \frac{\text{Total distance}}{\text{total time}}$$

3. Instataneous velocity = $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

4. $a = \frac{v}{t}$

4. average acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

5. Instataneous acceleration

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

6. $\left.\begin{array}{l} v = u + at \\ S = ut + \frac{1}{2}at^2 \\ v^2 = u^2 + 2aS \end{array}\right\}$ eqtn of motion

7. Distance travelled in n^{th} second.

S_n or

$$D_n = u + \frac{a}{2}(2n-1)$$

Simil

Define relative velocity explain relative velocity in one dimension?

The velocity of one body with respect to another body is called relative velocity.

Relative velocity of A w.r.t B

$$\boxed{\bar{V}_{AB} = \bar{V}_B - \bar{V}_A}$$

Eg: consider two trains moving on parallel tracks in the same direction with same speed. To an observer standing on the ground, both the trains are moving with reasonable speed. But to an observer on the other train, the other train appears to be stationary.

* Relative velocity of one train w.r.t observer in the other train is zero.

Relative velocity in One dimension [1D]

consider two objects A and B

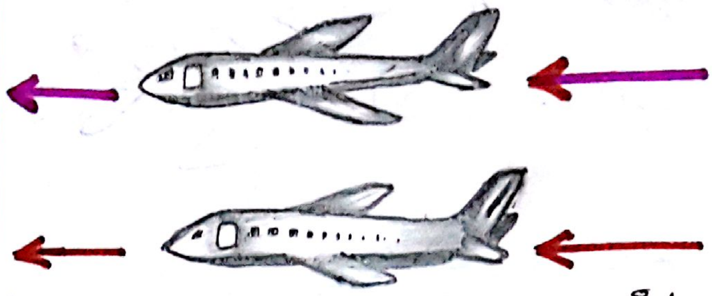
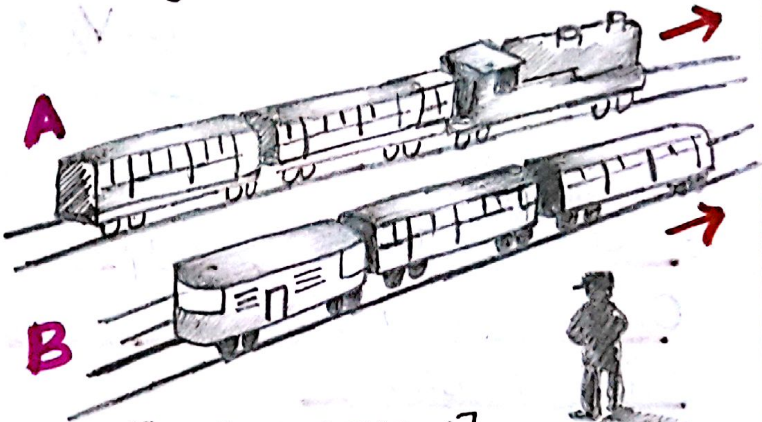


fig [relative velocity]



[1 D MOTION]

moving with uniform velocity V_A and V_B along parallel straight tracks in the same direction.

* Let the position of A be $x_A(0)$ at $(t=0)$
* position of B be $x_B(0)$ at $t=0$
* position of A and B be $x_A(t)$ and $x_B(t)$ at $(t=t)$

$$x_A(t) - x_A(0) = V_A t \quad \ldots (1)$$

$$x_B(t) - x_B(0) = V_B t \quad \ldots (2)$$

substract (1) from (2)

② − ① ⇒

$$x_B(t) - x_A(t) - [x_B(0) - (-x_A(0))] = V_B t - V_A t$$

$$x_B(t) - x_A(t) - x_B(0) + x_A(0) = V_B t - V_A t$$

$$\boxed{x_B(t) - x_A(t) = x_B(0) - x_A(0) + (V_B - V_A)t}$$

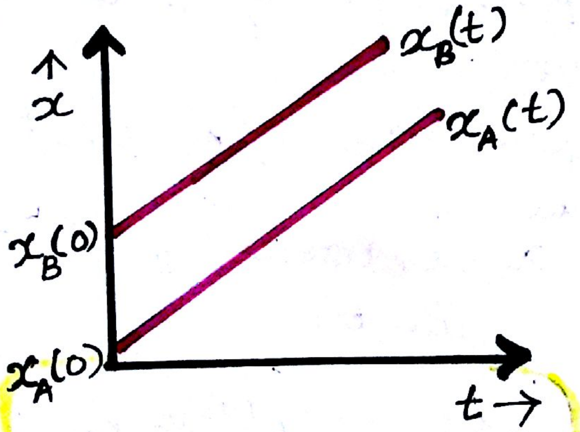
. . . ③

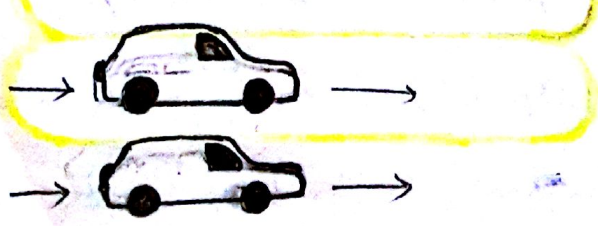
Position time graphs – One dimensional Relative motion

(a) when two bodies are moving with same velocities $V_A = V_B$

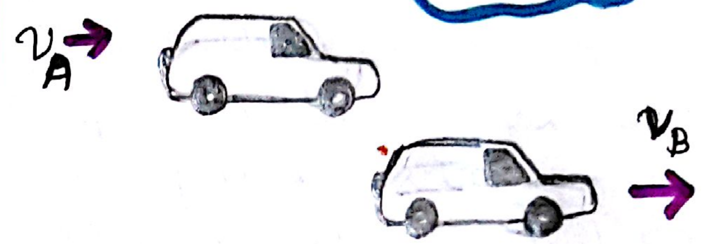
③ ⇒ $x_B(t) - x_A(t) = x_B(0) - x_A(0)$

It is a constant hence position time graphs are parallel.





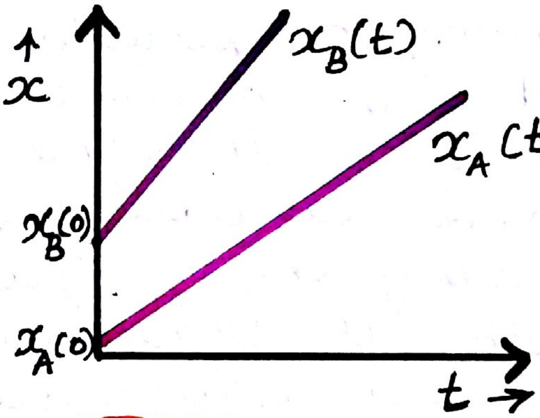
(b) when two bodies moving with different velocities $V_B > V_A$



we know that

$$x_B(t) - x_A(t) = x_B(0) - x_A(0) + (V_B - V_A)t$$

∴ $x_B(t) - x_A(t)$ increases with time



(c) If $V_B > V_B$

$$x_B(t) - x_A(t) = x_B(0) - x_A(0) + (V_B - V_A)t$$

∴ $x_B(t) - x_A(t) \downarrow$ and they meet