SIMILPhysics Reference text

CHAPTER II

MOTION IN A STRAIGHT LINE

Trigonometry

Trigonometrical ratios of an angle

Consider a right angled triangle ABC, the angle at B being 90°. The angle at C is θ. The side AB opposite to the angle θ is called opposite side and BC is called adjacent side and AC is the hypotenuse. From these three sides six ratios can be formed and they are called trigonometrical ratios.

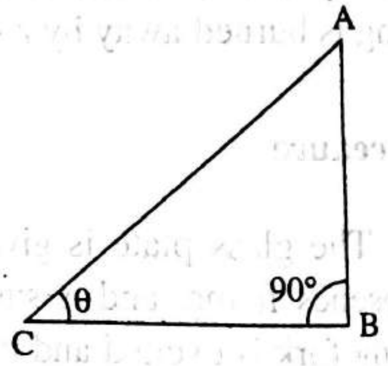


Fig. 5

1. Sine $\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{AB}{AC} =$ Sin θ

2. Cosine $\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{BC}{AC} =$ Cos θ

3. Tangent $\theta = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{AB}{BC} = \text{Tan}\,\theta = \sin\theta / \cos\theta$

4. Cosecant $\theta = \dfrac{\text{hypotenuse}}{\text{opposite side}} = \dfrac{AC}{AB} = \text{Cosec}\,\theta = 1/\sin\theta$

5. Secant $\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{AC}{BC} = \text{Sec}\,\theta = 1/\cos\theta$

6. Cotangent $\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{BC}{AB} = \text{Cot}\,\theta = \cos\theta/\sin\theta$

The value of the above ratios depend only on the angle and does not depend on the length of the sides. The above ratios are interrelated.

$$\text{Cosec}\,\theta = \frac{1}{\text{Sin}\,\theta};\ \text{Sec}\,\theta = \frac{1}{\text{Cos}\,\theta};\ \text{Cot}\theta = 1/\tan\theta$$

and

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1.$$

In the above triangle, $\angle A = 90 - \theta$

$$\text{Sin}\,(90-\theta) = \frac{BC}{AC} = \text{Cos}\,\theta$$

$$\text{Cos}\,(90-\theta) = \frac{AB}{AC} = \text{Sin}\,\theta$$

The following relations are also useful.

$$\text{Sin}\,(90+\theta) = \text{Cos}\,\theta; \quad \text{Cos}\,(90+\theta) = -\text{Sin}\,\theta$$
$$\text{Sin}\,(180-\theta) = \text{Sin}\,\theta; \quad \text{Cos}\,(180-\theta) = -\text{Cos}\,\theta$$

The values of trigonometrical ratios of some important angles

Angle	0	30	45	60	90
Sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

The trigonometrical ratios of all angles can be obtained from the mathematical tables.

Calculus: (A) Differentiation

Consider a function $y = f(x)$. Suppose x is increased by a small quantity Δx. Then the corresponding change Δy in y will also be small. The ratio $(\Delta y/\Delta x)$ gives the rate of change of the function $y = f(x)$ with respect to x. When $\Delta x = 0$, $(\Delta y/\Delta x)$ takes up the indeterminate form. But we can see that $(\Delta y/\Delta x)$ may tend to a limiting value as $\Delta x \to 0$ (Δx tends to zero). This limiting value of $(\Delta y/\Delta x)$ as $\Delta x \to 0$ is the *differential coefficient* or *derivative* of y with respect to x and is denoted by the symbol (dy/dx).

Thus $(dy/dx) = Lt_{\Delta x \to 0}(\Delta y/\Delta x)$. [reads limit when Δx tend to zero $(\Delta y/\Delta x)$]. The derivate of y is some times denoted by y'.

The process of finding the differential coefficient is called differentiation. (dy/dx) is the first derivative of y with respect to x.

Some basis rules in differentiation

1. The derivative of a constant c is zero; $\frac{d}{dx}(c) = 0$.
2. *Product rules*

   If $y = uv$, where u and v are both functions of x, then $(dy/dx) = \frac{d}{dx}(uv) = u(dv/dx) + v(du/dx)$
3. *Quotient rule*

   If $y = (u/v)$, where u and v are both functions of x, then

$$(dy/dx) = \frac{d}{dx}(u/v) = [v(du/dx) - u(dv/dx)] \div v^2$$

Some standard results

1. $\frac{d}{dx}(x^n) = nx^{n-1}$ ex: $\frac{d}{dx}(x^3) = 3x^2$
2. $\frac{d}{dx}(cx^n) = cnx^{n-1}$ ex: $\frac{d}{dx}(4x^3) = 4 \times 3x^2 = 12x^2$
3. $\frac{d}{dx}(\sin x) = \cos x$; *i.e.*, $\frac{d}{d\theta}(\sin\theta) = \cos\theta$
4. $\frac{d}{dx}(\cos x) = -\sin x$; *i.e.*, $\frac{d}{d\theta}(\cos\theta) = -\cos\theta$.
5. $\frac{d}{dx}(\tan x) = -\sec^2 x$ or $\frac{d}{d\theta}(\tan\theta) = -\sec^2\theta$
6. $\frac{d}{dx}(\log_e x) = 1/x$
7. $\frac{d}{dx}(e^x) = e^x$
8. $\frac{d}{dt}(x^n) = nx^{n-1}(dx/dt)$; ex: $\frac{d}{dt}(x^3) = 3x^2(dx/dt)$
9. $\frac{d}{dt}(\sin\theta) = \cos\theta(d\theta/dt)$; ex: $\frac{d}{dt}(\sin\omega t) = \cos\omega t \times \frac{d}{dt}(\omega t) = \cos\omega t \times \omega = \omega\cos\omega t$ where ω is a constant.

Successive differentiation

Consider the functional relation $y = f(x)$. When y is differentiated with respect to x once, we get the first differential of y with respect to x which is denoted by (dy/dx) or y'. (dy/dx) may again be differentiated with respect to x. This derivative of the first derivative is called the *second derivative* of y with respect to x and is denoted by (d^2y/dx^2) or y''.

Examples:

(i) Velocity, $v = (ds/dt)$, the first derivative of s with respect to t; acceleration, $a = \frac{d}{dt}(v) = (dv/dt) = \frac{d}{dt}(ds/dt) = (d^2s/dt^2)$, the second derivative of s with respect to t.

(ii) $\frac{d}{dt}(\sin\omega t) = \cos\omega t \times \omega = \omega\cos\omega t$

$$\therefore \quad \frac{d^2(\sin\omega t)}{dt^2} = \frac{d}{dt}(\omega\cos\omega t) = -\omega^2\sin\omega t$$

(B) INTEGRATION

The process of integration is just the reverse of differentiation. If $\frac{d}{dx}(f(x)] = \phi(x)$, then $f(x)$ is called the integral of the function $\phi(x)$ with respect to x. This is written as $\int \phi(x)dx = f(x)$, reads integral $\phi(x)dx$ equal $f(x)$.

Some standard results

1. $\frac{d}{dx}(x^n) = nx^{n-1}$; $\quad \therefore \int nx^{n-1} = x^n$
2. $\int x^n dx = \frac{x^{n+1}}{n+1}$; ex (i) $\int x^3 dx = x^4/4$

   (ii) $\int 4x^3 = 4 \times x^4/4 = x^4$
3. $\int \sin\theta \, d\theta = -\cos\theta$
4. $\int \cos\theta \, d\theta = \sin\theta$
5. $\int (1/x)dx = \log_e x$
6. $\int e^x dx = e^x$

Note

It is to be noted that sign Σ is used for summation of discrete values, while $\int$ is used for summation of a continuous function. It means that the method of integration is used to sum up the effect of a continuously varying function. For example, work done by a variable force, $W = \int F ds$.

Constant of integration (C) - Indefinite integrals

If $\frac{d}{dx}[f(x)] = \phi(x)$, then $\int \phi(x)dx = f(x)$. Also $\frac{d}{dx}[f(x) + C] = \frac{d}{dx}[f(x)] + \frac{dC}{dx} = \frac{d}{dx}[f(x)] = \phi(x)$; where C is a constant and so $dC/dx = 0$.

$$\therefore \int \phi(x)dx = f(x) + C$$

$$Ex: \quad \int x^n dx = \left(\frac{x^{n+1}}{n+1}\right) + C$$

In all the indefinite integrals, a constant of integrals C is supposed to be present. Hence we should add a constant C in the result of all indefinite integrals.

Definite integrals

If $\int f(x)dx = F(x) + C$, then the value of the integral when $x = b$ is $F(b) + C$ and its value when $x = a$ is $F(a) + C$. Hence the value of the integral when $x = b$ minus the value of the integral when $x = a$ is $[F(b) + C] - F(a) + C] = F(b) - F(a)$. This value $F(b) - F(a)$ is called the definite integral of $f(x)$ between the limits a and b. This is denoted by $\int_a^b f(x)dx = [F(x)]_a^b$. Here a and b are called the limits of integration, b being the upper limit and a the lower limit. $\int_a^b f(x)dx$ reads integral $f(x)dx$ between the limits a to b.

When we calculate the definite integral, the arbitrary constant of integral C gets cancelled.

It is to be noted that $\int_a^b f(x)dx$ is the summation of the function of x for all values of x from a to b.

Examples:

(i) $$\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b = \frac{1}{(n+1)}\left[b^{n+1} - a^{n+1}\right]$$

(ii) $$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]^3 = \frac{1}{3}\left[3^3 - 1^3\right] = \frac{1}{3} \times 26 = 26/3$$

(ii) $$\int_0^3 x^3\,dx = \left[\frac{x^4}{4}\right]^4 = \frac{1}{4}[4^4 - 0) = 256.$$

VELOCITY AND ACCELERATION

The concept of motion and rest depends on observer and object. *If the position of an object in space changes with time relative to an observer it is said to be in motion; other-wise it is at rest.*

The line joining the instantaneous positions of the particle is called its *path*.

Concept of point object

If the distance travelled by an object is very large compared to the size of the object, it can be considered a point object.

For example, a train whose length is nearly a kilometre may be treated as a point object if it travels very long distances. In describing its journey over many hours we may ignore its length and simply think it as a point object. Even moon going round the earth is so small in comparison with the size of the orbit that it can be considered a point object.

Motion in one, two and three dimensions

If the motion of an object is restricted to a straight line it is said to execute *one dimensional motion.*

A train running along a straight track or a bus on a straight road are examples of bodies in one dimensional motion.

If the motion of an object is restricted to a plane it is said to be in *two dimensional motion.*

The motion of a boat on a lake or a coin along a surface are examples of two dimensional motion.

An object moving in space is said to be in *three dimensional motion.*

A butterfly flying in air or motion of gas molecules in space are examples of three dimensional motion.

Distance and displacement

The length of the path traversed by a particle in a certain interval of time is called ***distance*** *travelled by that particle.*

Displacement *of an object is the change of position along a particular direction and it is given by the vector drawn from the initial position to the final position.*

If the particle moves from A to B as shown in the figure 1, the distance travelled is Δs while displacement is $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.

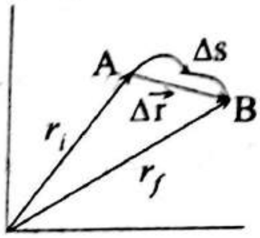
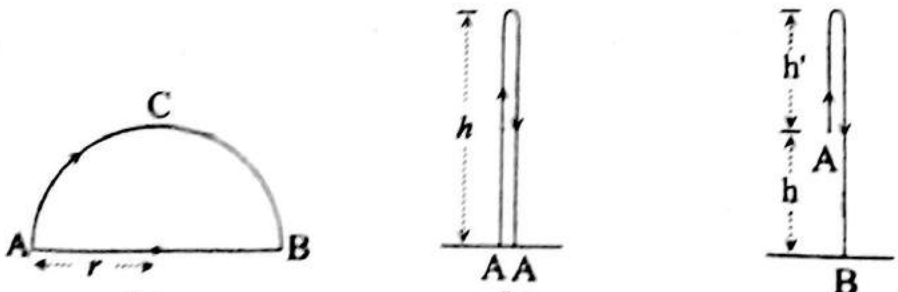


Fig. 6

Regarding distance and displacement it is worth noting that:

1. Distance is a scalar, while displacement is a vector both having the same dimension (L) and unit (m)
2. The magnitude of the displacement of an object between two points is the shortest distance between those two points
3. For a moving particle distance can never be zero or negative while displacement can be zero, positive or negative
4. For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means, that the body is moving towards the initial position.

In fig 2(a), a body moves along a semicircular path of radius r along ACB. Fig 2(b) shows a body thrown vertically upto a height h returns to the point of projection. Fig 2(c) shows a body projected vertically up from a height h above the ground went upto an additonal height h' and then falling to the ground.



(a) Distance = πr Displacement = $2r$

(b) Distance = $2h$ Displacement = 0

(c) Distance = $h+2h'$ Displacement = $-h$

Fig. 7

Speed

It is the time rate of change of position. It is the distance travelled in unit time

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time}}; \quad \text{Unit: } ms^{-1}; \quad \text{Dimensions: } LT^{-1}$$

Average speed

It is the ratio of the total distance travelled by the object to the total time taken.

$$\text{Average speed, } \overline{v} = \frac{\text{total distance travelled}}{\text{total time}}$$

Note:

1. If a particle travels at speeds v_1 and v_2 for intervals t_1 and t_2 respectively, then,

$$\text{Total distance} = v_1t_1 + v_2t_2; \quad \text{Total time} = t_1 + t_2$$

$$\text{Average speed, } \overline{v} = \frac{(v_1t_1 + v_2t_2)}{(t_1 + t_2)}. \quad \text{If } t_1 = t_2 = t,\ \overline{v} = (v_1 + v_2)/2$$

2. If a particle travels a distance S_1 with a speed v_1 and then S_2 with a speed v_2 then,

$$\text{Total distance} = S_1 + S_2; \quad \text{Total Time} = \frac{S_1}{v_1} + \frac{S_2}{v_2}.$$

$$\overline{v} = (S_1 + S_2)/\left(\frac{S_1}{v_1} + \frac{S_2}{v_2}\right).$$

$$\text{If } S_1 = S_2 = S,\ \overline{v} = 2v_1v_2/(v_1 + v_2)$$

Example

II.1. A car covers the first half of the distance between two places at a speed of 40 km/h and the second half at 60 km/h. What is the average speed of the car?

Let $2x$ be the total distance covered.

$$\text{Total time} = \frac{x}{40} + \frac{x}{60} = \frac{3x + 2x}{120} = \frac{5x}{120} = \frac{x}{24}\ \text{h}$$

$$\text{Average speed, } \overline{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2x}{x/24} = 48\ \text{km/h}$$

$$\text{Or, } \overline{v} = 2v_1v_2/(v_1 + v_2) = 2 \times 40 \times 60/(40 + 60) = \mathbf{48\ km/h}$$

Motion along a straight line

Origin, unit and sense of passage of time

In any process involving time, some instant of time is assigned the value zero time. To represent the passage of time along a line, zero time is taken as the origin. The unit of time may be second, minute or hour as per convenience. Events occurred before zero time will be assigned a negative number of time units while those that occur after will carry a positive number of time units.

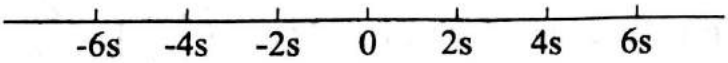


Fig. 8

The origin of the time-axis can be shifted to any point on the time axis. The time interval between two points on the axis is not affected by the shift in the origin of the time axis.

Origin, unit and direction for position measurement

To represent the motion of a body along a straight line, some point O is taken as the origin and some unit of length is chosen. At any instant t, the position of object is given a real number positive or negative written as $x(t)$. This is called *position co-ordinate*.

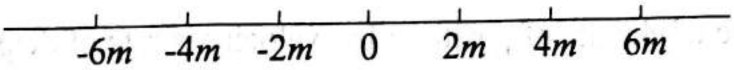


Fig. 9

The distance measured to the right of the origin is taken *positive* and distance measured to the left of the origin is taken *negative*.

In the case of motion along a vertical line the distance covered upwards from the origin is taken as positive and that downwards from the origin is taken as negative.

The origin of the position axis can be shifted to any point on the axis.

Velocity

It is the time rate of displacement. It is the distance travelled in unit time in a specified direction.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}; \quad \text{Unit: ms}^{-1}; \quad \text{Dimensions}: LT^{-1}$$

Average velocity

If the displacement of a particle in a certain interval of time is known, the average velocity is taken as the ratio of displacement to the time taken.

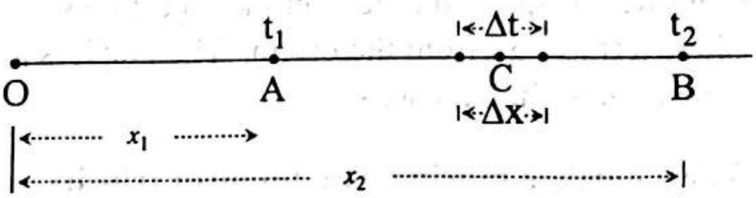


Fig. 10

Let O be the origin. At any instant t_1 let the object be at A and at any time t_2 let it be at B. Let $OA = x_1$ and $OB = x_2$.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity

The velocity of a particle at any instant of its motion is called instantaneous velocity. To calculate the velocity at any instant C (Fig. 5), we note a small displacement Δx in an infinitesimally small interval of time Δt, including the point C. The average velocity during the interval of time Δt is $\overline{v} = (\Delta x/\Delta t)$. The velocity at the instant C is, $v = (\Delta x/\Delta t)$, limit when Δt tends to zero.

$$\text{Instantaneous velocity, } v = Lt_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

Thus instantaneous velocity is the time derivative of displacement.

Uniform velocity

A particle is said to move with uniform velocity if it makes equal displacements in equal intervals of time, however small these intervals are. The velocity of the particle will be the same at every instant. *The average and instantaneous velocities have the same value in uniform motion.*

Distinction between speed and velocity

1. Speed is the rate of change of position; but velocity is rate of displacement
2. Both speed and velocity have the same unit and dimensions
3. Speed is a scalar quantity while velocity is a vector
4. Velocity can be positive or negative but speed can only be positive
5. Speed is greater than or equal to the velocity.

Note: A particle moving with uniform velociy moves with uniform speed; but a particle moving with uniform speed need not move with uniform velocity. For example, a particle moving round a circle with uniform speed has non uniform velocity since its direction is changing continuously.

Position-time graph

1. For a stationary object

An object is said to be stationary if its position does not change with respect to time. Let the object be stationary at position $x(t) = x_0$ from the origin. The graph is a straight line parallel to the time axis.

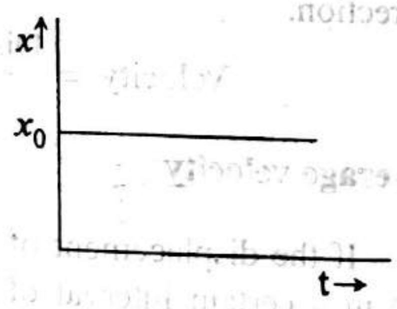


Fig. 11

2. For a particle moving with uniform velocity

At the instant we begin observing the particle *i.e.*, at $t = 0$, let the particle be at $x = x_0$ from the origin. The graph AB is a straight line inclined to the time axis. At $t = t_1$, let the particle be at x_1 and when $t = t_2$, let it be at x_2 from the origin. The velocity of the particle is given by

$$\frac{x_2 - x_1}{t_2 - t_1} = \frac{DE}{CE} = \tan\alpha,$$ the slope of the position-time graph. Thus the slope of the position-time graph represents the velocity of the particle.

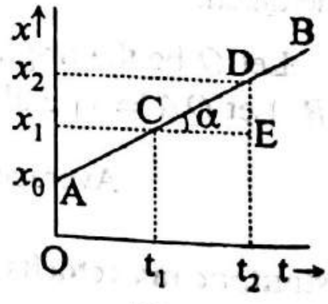


Fig. 12

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Velocity-time graph for uniform motion

A graph drawn with velocity on the y axis and time on the x-axis is called velocity-time graph. For a particle moving with uniform velocity, the graph is a straight line parallel to the time axis.

Let v be the uniform velocity of the particle. The velocity time graph helps us to calculate the displacement of the particle in a given time interval geometrically.

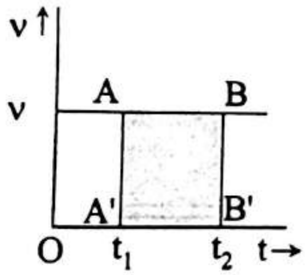

Fig. 13

Let A and B be two points on the graph corresponding to instants t_1 and t_2. Then $AA' = BB' = v$ and $A'B' = t_2 - t_1$.

Displacement during the time interval $t_2 - t_1$ = uniform velocity × time

$$= v(t_2 - t_1) = AA' \times A'B' = \text{area of the rectangle } ABB'A'$$

Equation for uniform motion

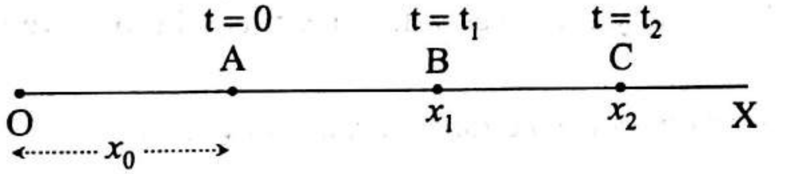

Fig. 14

Consider a particle moving with uniform velocity v along a straight line OX with O as origin. At $t = 0$, let the particle be at A when $x = x_0$. At time $t = t_1$, let it be at x_1 and at time $t = t_2$, let it be at x_2. Then $AB = vt_1$ and $AC = vt_2$. Also,

$$x_1 = x_0 + vt_1 \quad \text{and} \quad x_2 = x_0 + vt_2 \quad \text{Then, } x_2 - x_1 = v(t_2 - t_1)$$

If $x_2 - x_1 = S$ and $t_2 - t_1 = t$, then, $\mathbf{S = vt}$

Examples

II.2. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man over the intervals of time (i) 0 to 30 minute (ii) 0 to 50 minute (iii) 0 to 40 minute? **[NCERT]**

$$\text{Time taken to reach the market} = \frac{2.5\,\text{km}}{5\,\text{km/h}} = \frac{1}{2}\,\text{h} = 30\,min$$

$$\text{Time taken for return journey} = \frac{2.5\,\text{km}}{7.5\,\text{km/h}} = \frac{1}{3}\,\text{h} = 20\,min$$

Average velocity in 30 min = $2.5/(30/60)$ = **5 km/h**

Average speed in 30 min = **5 km/h**

Average velocity in 50 min = **0**, since displacement is zero in 50 min

$$\text{Average speed} = (2.5 + 2.5)/(50/60) = \mathbf{6\,km/h}$$

$$\text{Displacement in 40 min} = 2.5\,\text{km} - \text{ displacement in 10 min}$$

$$= 2.5 - 7.5 \times \frac{10}{60} = 2.5 - 1.25 = 1.25\,\text{km}$$

$$\text{Average velocity} = \frac{1.25}{(40/60)} = \mathbf{1.875\,km/h}$$

$$\text{Average speed} = \frac{2.5 + 1.25}{(40/60)} = \mathbf{5.625\,km/h}$$

Accelerated motion

In nonuniform motion of an object, the velociy is different at different instants *i.e.*, the velocity is a function of time. If the velocity is increasing with time, the particle is said to be *accelerated.* If the velocity decreases with time; it is said to move with *retardation* or *deceleration.*

Acceleration *of an object is time rate of change of velocity.*

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}; \quad \text{Unit: ms}^{-2} \quad \text{Dimension } = LT^{-2}$$

Acceleration is a vector quantity. It is positive if the velocity is increasing and negative if the velocity is decreasing.

Average acceleration of a particle in a specified time interval is ratio of the change in velocity to the interval of time

$$\text{Average acceleration, } \overline{a} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration

The acceleration of an object at a given instant of time or at a given point of motion is called its instantaneous acceleration. It is defined as the limiting value of average acceleration in a small interval of time around that instant when the time interval approaches zero.

$$\text{Instantaneous acceleration, } Lt_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Uniform acceleration

A particle is said to move with uniform acceleration if the velocity changes by equal amounts in equal intervals of time, however small these intervals may be. Here the acceleration at every instant is constant.

KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Consider a particle moving along a straight line with uniform acceleration 'a'. At the instant $t = 0$ let the particle be at A and its velocity be v_0. At $t = t$, let the particle be at B and its velocity then be v_t. Let the position coordinates of A and B be x_0 and x_t respectively such that
$x_t - x_0 = S$.

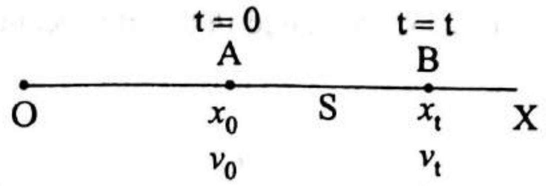

Fig. 15

1. Velocity-time relation: $v_t = v_o + at$

According to the definition of acceleration,

$$\text{Acceleration, } a = \frac{\text{change in velocity}}{\text{time}} = \frac{v_t - v_0}{t}$$

$$v_t - v_0 = at \quad ; \mathbf{v_t = v_0 + at}$$

2. Position-time relation: $x_t - x_o = v_o t + (1/2)at^2$

If the velocity of a particle is constant, the displacement = velocity × time .

If it moves with uniform acceleration, the displacement is given by;

$S =$ average velocity × time

Since acceleration is uniform, average velocity $= \dfrac{(v_0 + v_t)}{2}$

$$\therefore S = \frac{(v_0 + v_t)}{2} \times t$$

$$\text{But} \quad v_t = v_0 + at; \quad S = \frac{(v_0 + v_0 + at)}{2} \times t = \frac{(2v_0 + at)}{2} \times t$$

$$\therefore \mathbf{S = v_0 t + (1/2)at^2}$$

$$\text{But } S = x_t - x_0 \therefore \quad \mathbf{x_t - x_0 = v_0 t + \frac{1}{2}at^2}$$

3. Position-velocity relation: $v_t^2 - v_o^2 = 2aS$

Acceleration = change in velocity/time; $a = (v_t - v_0)/t \therefore v_t - v_0 = at$ (i)

Displacement, $S =$ Average velocity × time $= \dfrac{(v_t + v_0)t}{2}$

$$\frac{(v_t + v_0)t}{2} = S \quad ; v_t + v_0 = \frac{2S}{t} \qquad \text{(ii)}$$

Multiplying eqns (i) and (ii), $\mathbf{v_t^2 - v_0^2 = 2aS}$; Or, $\mathbf{v_t^2 - v_0^2 = 2a(x_t - x_0)}$

4. Distance travelled in a particular second: $S_n = v_o + (n - 1/2)a$

Consider a particle moving with uniform acceleration a. At $t = 0$, let v_0 be the velocity.

Distance travelled in the nth second, S_n = Distance travelled in n seconds − Distance travelled in $(n - 1)$ seconds

$$i.e., S_n = v_0 n + \frac{1}{2}an^2 - [v_0(n-1) + \frac{1}{2}a(n-1)^2]$$
$$= v_0 n + \frac{1}{2}an^2 - [v_0 n - v_0 + \frac{1}{2}a(n^2 - 2n + 1)]$$
$$= v_0 n + \frac{1}{2}an^2 - v_0 n + v_0 - \frac{1}{2}an^2 + an - \frac{1}{2}a$$
$$= v_0 + an - \frac{1}{2}a = v_0 + (n - 1/2)a$$

Note: More commonly used symbols: $v_0 = u$ and $v_t = v$. Then the equations are:-

$$\mathbf{v = u + at; \quad S = ut + \frac{1}{2}at^2; \quad v^2 - u^2 = 2aS; \quad S_n = u + (n - 1/2)a}$$

Examples

II.3. A car travelling at a speed 54 km/hr is brought to rest in 90 s. Find the retardation and distance travelled by the car before coming to rest.

$u = 54\,\text{km/hr} = 15\,\text{ms}^{-1}$; $v = 0$; $t = 90\,s$; $a = ?$; $S = ?$

$v = u + at$;, $0 = 15 + a \times 90$; $a = -15/90 = -1/6\,\text{ms}^{-2}$

$\therefore$ Retardation $= (1/6)\text{ms}^{-2}$

Distance travelled $= ut + \frac{1}{2}at^2$

$= 15 \times 90 + \frac{1}{2} \times -1/6 \times 90^2 = 1350 - 675 = \mathbf{675\,m}$

II.4. A car starting from rest acquires a speed 25 ms⁻¹ in 10 s after which it maintains this speed for 10 s. Find (a) the acceleration (b) distance travelled during acceleration (c) total distance travelled.

(a) $u = 0$; $v = 25\,\text{ms}^{-1}$; $t = 10\,s$; $a = ?$

$v = u + at$; $25 = 0 + a \times 10$; $10a = 25$; $a = \mathbf{2.5\,ms^{-2}}$

(b) $S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.5 \times 10^2 = \mathbf{125\,m}$

(c) Distance travelled at constant speed $= S = v \times t = 25 \times 10 = 250\,m$

Total distance travelled $= 125 + 250 = \mathbf{375\,m}$

II.5. A body travels 2 m in the first 2 s and 2.2 m in the next 4 s? What will be the velocity at the end of the seventh second from start?

Let u be the initial velocity and a the uniform acceleration.

$$\text{Since } S = ut + \frac{1}{2}at^2; \quad 2 = u \times 2 + \frac{1}{2}a \times 2^2; \quad u + a = 1 \qquad (1)$$

$$\text{Also, } 4.2 = u \times 6 + \frac{1}{2}a \times 6^2; \quad u + 3a = 0.7 \qquad (2)$$

From (1) and (2), $a = \mathbf{-0.15\,ms^{-2}}$ and $u = \mathbf{1.15\,ms^{-1}}$

Velocity at the end of the seventh second;

$$v = u + at = 1.15 - (0.15) \times 7 = \mathbf{0.10\,ms^{-1}}$$

II.6. The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. Show that the velocity with which the middle point of the train passes the same point is $\sqrt{(u^2 + v^2)/2}$

Let x be the length of the train and a its acceleration.

$$\text{Then, } v^2 - u^2 = 2ax \qquad (1)$$

Let V be the velocity of the train when the midpoint of the train passes the same point.

$$\text{Then, } V^2 - u^2 = 2ax/2 \;\; ; V^2 - u^2 = ax \qquad (2)$$

From (1) and (2), $v^2 - u^2 = 2(V^2 - u^2)$; $\quad V = \sqrt{(\mathbf{u^2 + v^2})/2}$

II.7. A body moves in a straight line along X-axis; its distance from the origin is given by the equation $x = 8t - 3t^2$, where x is in metres and t in seconds. Find the displacement of the particle when its velocity is zero.

$$x = 8t - 3t^2; \quad \text{velocity} \quad v = \frac{dx}{dt} = 8 - 6t$$

When velocity is zero: $0 = 8 - 6t$; $\quad t = 4/3\,s$

Displacement in 4/3 s; $\quad x = 8 \times 4/3 - 3 \times (4/3)^2 = \mathbf{5.33\,m}$

II.8. A body is moving along the X-axis with constant acceleration of $4\,ms^{-2}$. At time $t = 0$, it is at $x = 5$ m and has velocity $v(0) = 3\,ms^{-1}$. (a) Find the position and velocity at $t = 2s$. (b) Where is the body when its velocity is $5\,ms^{-1}$?

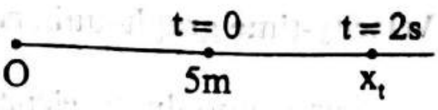

Fig. 16

(a) $v = u + at = 3 + 4 \times 2 = 11\,ms^{-2}$

$$x_t - x_0 = ut + \frac{1}{2}at^2 = 3 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 14\,m$$

$$x_t = x_0 + 14 = 5 + 14 = \mathbf{19\,m}$$

(b) $v^2 = u^2 + 2aS = u^2 + 2a(x_t - x_0)$

$5^2 = 3^2 + 2 \times 4(x_t - x_0); \quad x_t - x_0 = \frac{25-9}{8} = 2$

$x_t - 5 = 2; \quad x_t = \mathbf{7\,m}$

II.9. *A hundred metre sprinter increases her speed from rest uniformly at the rate of 1 ms^{-2} up to 75 m and covers the last 25 m with uniform speed. How much time does she take to cover the first half and second half of the run?

Let t_1, t_2 and t_3 be the time taken to cover 50 m, 75 m and 100 m

For accelerated motion upto 75 m

$$v^2 = u^2 + 2aS; \quad v^2 = 0 + 2 \times 1 \times 75; \quad v = \sqrt{150} = \mathbf{12.24\,ms^{-1}}$$

$$v = u + at; \quad 12.24 = 0 + 1 \times t_2; \quad t_2 = \mathbf{12.24\,s}$$

During the last quarter she is running at constant speed

$$\text{Time taken to cover last 25 m} = \frac{25}{12.24} = \mathbf{2.04\,s}$$

$$\text{Time taken to cover the whole distance} = 12.24 + 2.04 = \mathbf{14.28\,s}$$

Time taken to cover the first half

$$S = ut + \frac{1}{2}at^2; \quad 50 = 0 + \frac{1}{2} \times 1 \times t_1^2; \quad t_1 = \mathbf{10\,s}$$

Time taken to cover the second half $= 14.28 - 10 = \mathbf{4.28\,s}$

Position-time graph for uniformly accelerated motion

For uniformly accelerated motions, $x - t$ graphs have parabolic shapes

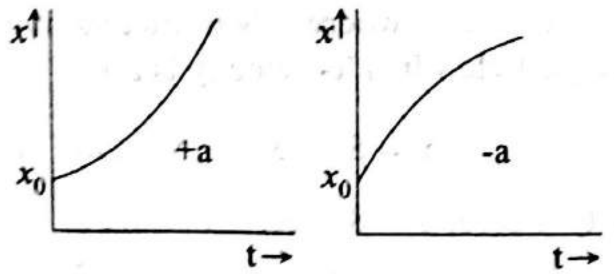

Fig. 17

Velocity-time graph–uniformly accelerated motion

For uniformly accelerated motion, the velocity-time graph is a straight line inclined to the time axis. Let v_1 and v_2 be the velocities of the particle at instants $t = t_1$ and $t = t_2$

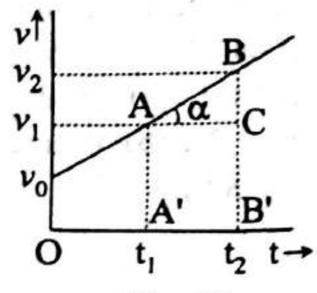

Fig. 18

$$\text{Acceleration, } a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{BC}{AC} = \tan\alpha$$

Thus the slope of the velocity-time graph gives the acceleration of the particle.

To show that the area enclosed by the velocity-time graph represents the displacement

The velocity-time curve for a general case is given in the figure 14. As the slope of the curve changes from time to time the acceleration is not a constant. Whatever may be the acceleration it can be shown that the area under the curve gives the displacement.

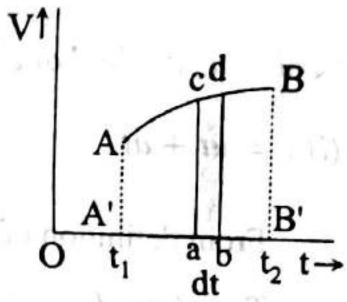

Fig. 19

Imagine the time interval to be divided into small intervals like 'ab'. During such interval the velocity can be considered to have a constant value given by the ordinate 'ac' or 'bd'.
Distance covered during this interval

$$= v \times dt = ac \times ab = \text{ area of the strip } abcd$$

Considering similar intervals, the total displacement during the interval $(t_2 - t_1)$ is equal to the area $ABB'A'$.

Examples

II.10. Fig. 15 shows the velocity time curve for a particle travelling along a straight line. Find the distance covered in 6 s and displacement in 6 s.

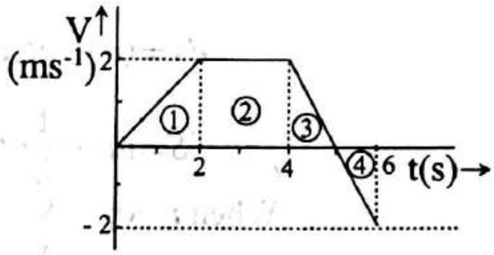

Fig. 20

$$\text{Distance covered} = \text{area 1} + \text{area 2} + \text{area 3} + \text{area 4}$$

$$= \frac{1}{2} \times 2 \times 2 + 2 \times 2 + \frac{1}{2} \times 1 \times 2 + \frac{1}{2} \times 1 \times 2 = \mathbf{8\,m}$$

$$\text{Displacement} = \text{area 1} + \text{area 2} + \text{area 3} - \text{area 4}$$

$$= \frac{1}{2} \times 2 \times 2 + 2 \times 2 + \frac{1}{2} \times 1 \times 2 - \frac{1}{2} \times 1 \times 2 = \mathbf{6\,m}$$

Position-time relation: graphical proof

Consider a particle moving with uniform acceleration a. At $t = 0$, let the velocity be v_0. After time t, let v_t be the velocity. On a velocity-time graph, AC represents the graph and BC represents the velocity, $v_t = v_0 + at$. Then, $BD = v_0$ and $DC = at$

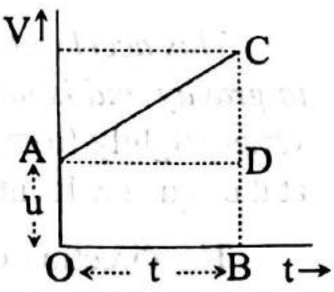

Fig. 21

$$\text{Displacement} = \text{area under the velociy time graph} = \text{area } OACB$$

$$= \text{area of rectangle } OADB + \text{ area of triangle } ADC$$

$$= OA \times OB + \frac{1}{2} AD \times DC = v_0 t + \frac{1}{2} t \times at$$

$$\therefore S = \mathbf{v_0 t + \frac{1}{2} a t^2}$$

VERTICAL MOTION UNDER GRAVITY

It has been found that, in the absence of air resistance, all bodies regardless of their size, mass or position fall near the surface of the earth with constant acceleration.

This acceleration is caused by the earth's attraction and is known as acceleration due to gravity and is identified by the symbol 'g'. Near the surface of the earth the value of g varies slightly from place to place. It has maximum value at the poles and minimum value at the equator. Its mean value is taken as 9.8 ms^{-2}.

The direction of the acceleration due to gravity g is always vertically down towards the centre of the earth.

Note: (i) *Conventionally the direction of the initial velocity is taken as the positive direction.*

Acceleration due to gravity g is always in the *same direction*; *i.e.*, vertically downwards towards the centre of the earth. When a body is projected vertically up, the upward direction is taken as the positive direction. Since g is in a direction opposite to this, g is taken negative. When a body is projected vertically down, the downward direction is taken as positive. Hence g is taken positive.

(ii) If the air resistance is not taken into account, when a body is projected upwards, the time of ascent is equal to the time of descent.

If the air resistance is neglected, the only force that acts on the body is gravitational and the acceleration of the body is the acceleration due to gravity g.

Let u be the velocity of projection.

Time of ascent (t_1):-

$v_0 = u;\quad a = -g;\quad v_t = 0;\quad v_t = v_0 + at;\quad 0 = u - gt_1 \quad \therefore t_1 = u/g$ (i)

Maximum height (h):-

$v_0 = u;\quad v_t = 0;\quad v_t^2 - v_0^2 = 2as;\quad 0 - u^2 = 2gh;\quad h = u^2/2g$

Time of descent (t_2):-

$v_0 = 0;\quad a = +g;\quad s = h = u^2/2g$

$s = v_0 t + (1/2)at^2;$ ie, $u^2/2g = (1/2)g t_2^2 \quad \therefore t_2 = u/g$ (ii)

From the equations (i) and (ii), it is clear that the time of ascent is equal to the time of descent.

(iii) If the air resistance is taken into account, when a body is projected upwards, time of descent is greater than the time of ascent.

Let F_r be the air resistance.

Acceleration during the upward motion, $a_1 = (mg + F_r)/m$

Acceleration during the downward motion, $a_2 = (mg - F_r)/m \quad \therefore a_1 > a_2$. If t_1 and t_2 are the time of ascent and time of descent, $(t_1^2/t_2^2) = (a_2/a_1) \quad \therefore t_2 > t_1$

Examples

II.11. A body is projected vertically upwards with a velocity 49 ms^{-1}. Find (a) the maximum height (b) time of ascent (c) time of flight and (d) velocity when it is at a height of 60 m. [$g = 9.8\ \text{ms}^{-2}$]

(a) $u = +49\ \text{ms}^{-1};\quad v = 0;\quad a = -9.8\ \text{ms}^{-2};\quad S = ?$

$v^2 = u^2 + 2aS;\quad 0 = 49^2 - 2 \times 9.8 \times S;\quad S = \dfrac{49^2}{2 \times 9.8} = \mathbf{122.5\ m}$

(b) $u = +49\ \text{ms}^{-1};\quad v = 0;\quad a = -9.8\ \text{ms}^{-2};\quad t = ?$

$v = u + at;\quad 0 = 49 - 9.8t;\quad t = \dfrac{49}{9.8} = \mathbf{5\ s}$

(c) $u = +49\ \text{ms}^{-1};\quad a = -9.8\ \text{ms}^{-2};\quad S = 0;\quad t = ?$

$S = ut + \dfrac{1}{2}at^2;\quad 0 = 49t - \dfrac{1}{2} \times 9.8t^2;\quad 4.9t = 49;\quad t = \mathbf{10\ s}$

(d) $v^2 = u^2 + 2aS;\quad v^2 = 49^2 - 2 \times 9.8 \times 60 = 2404 - 1176 = 1225$

$v = \sqrt{1225} = \mathbf{\pm 35\ ms^{-1}}$

II.12. A stone is projected vertically up from the top of a tower 73.5 m with velocity $24.5\ ms^{-1}$. Find the time taken by the stone to reach the foot of the tower.

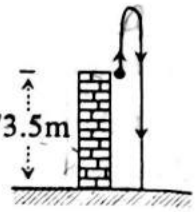

Fig. 22

Let the top of the tower be the origin of coordinate. Distances measured in the vertical direction of projection is positive; opposite direction is negative.

$$S = -73.5\ \text{m};\quad u = +24.5\ \text{ms}^{-1};\quad a = -9.8\ \text{ms}^{-2};\quad t = ?$$

$$S = ut + \frac{1}{2}at^2;\quad -73.5 = 24.5t - 4.9t^2$$

$$4.9t^2 - 24.5t - 73.5 = 0;\quad t^2 - 5t - 15 = 0$$

$$t = 5 \pm \sqrt{25 - 4 \times 1 \times -15} = \frac{5 \pm \sqrt{85}}{2} = \mathbf{7.11\ s}$$

II.13. A stone thrown vertically up from the top of a tower with a velocity $20\ ms^{-1}$ reaches the ground in 6 s. Find the height of the tower.

Let the top of the tower be the origin of coordinates. Initial direction of projection is positive.

$$u = +20\ \text{ms}^{-1};\quad a = -9.8\ \text{ms}^{-2};\quad t = 6\ \text{s};\quad h = ?\quad S = -h$$

$$S = ut + \frac{1}{2}at^2;\quad S = 20 \times 6 - 4.9 \times 6^2 = 120 - 176.4 = -56.4$$

Height of the tower, h = **56.4 m**

II.14. *Rain drops are falling at regular intervals of time from the roof of a building 16 m high. If the first drop reaches the ground at the instant the fifth drop starts falling, find the heights of the various drops at the instant the first drop reaches the ground.

Let the drops fall at a regular interval of t. When the fifth drop starts falling the time elapsed will be t, $2t$, $3t$ and $4t$ for the fourth, third, second and first drops respectively and if the distance covered are S_4, S_3, S_2 and S_1, then,

$$S_1 = \frac{1}{2}g(4t)^2 = 16 \times \frac{1}{2}gt^2;\quad S_1 = 16\ \text{m}\quad \therefore \frac{1}{2}gt^2 = 1\ \text{m}$$

$$S_2 = \frac{1}{2}g(3t)^2 = 9 \times \frac{1}{2}gt^2 = 9\ \text{m}$$

$$S_3 = \frac{1}{2}g(2t)^2 = 4 \times \frac{1}{2}gt^2 = 4\ \text{m}$$

$$S_4 = \frac{1}{2}gt^2 = 1 \times \frac{1}{2}gt^2 = 1\ \text{m}$$

The heights of various drops from the ground are **0, 7 m, 12 m, 15 m** and **16 m**.

II.15. A body freely falling from the top of a tower describes 65.1 m in the last second of its fall. Find the height of the tower. $g = 9.8\ ms^{-2}$

Let h be the height of the tower and t the time taken to cover the whole distance.

$$S = ut + \frac{1}{2}at^2\quad \therefore h = 4.9t^2 \tag{1}$$

Distance travelled in $(t-1)$ seconds $= (h - 65.1)$ m

$$h - 65.1 = 4.9(t-1)^2 = 4.9t^2 - 9.8t + 4.9 \tag{2}$$

Subtracting (2) from (1) $65.1 = 9.8t - 4.9$

$$9.8t = 70; \quad t = \frac{70}{9.8} = \mathbf{7.14\ s}$$

Height of the tower $h = 4.9t^2 = 4.9 \times 7.14^2 = \mathbf{249.8\ m}$

OR

$S_n = u(n - 1/2)a; \quad 65.1 = (t - 1/2)9.8 \quad \therefore t = (70/9.8) = 7.14$ s

$$h = 4.9\ t^2 = 4.9 \times 7.14^2 = \mathbf{249.8\ m}$$

II.16. *A balloon is rising vertically from rest on ground with an acceleration of 4.9 ms^{-2}. At the end of 4 seconds, a stone is let fall from the balloon. How high will the stone rise and after how long will it reach the ground? $g = 9.8$ ms^{-2}

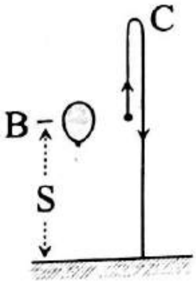

Fig. 23

To find the height of the balloon just when the stone is dropped

$u = 0; \quad a = +4.9\ ms^{-2}; \quad t = 4\ s; \quad S = ?$

$$S = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 4^2 = \mathbf{39.2\ m}$$

To find the velocity of the balloon just when the stone is released

$u = 0; \quad a = +4.9\ ms^{-2}; \quad t = 4\ s; \quad v = ?$

$v = u + at = 4.9 \times 4 = \mathbf{19.6\ ms^{-1}}$

When the stone is released it has the same velocity as the balloon at the time of release. Hence it moves up initially but experiences a retardation 9.8 ms^{-2}

For the upward motion of the stone

$u = +19.6\ ms^{-2}; \quad a = -9.8\ ms^{-2}; \quad v = 0; \quad S = ?$

$v^2 = u^2 + 2as; \quad 0 = 19.6^2 - 2 \times 9.8 \times S; \quad S = \mathbf{19.6\ m}$

$\frac{(20)^2}{2 \times 10}$ $\frac{400}{20}$

Maximum height reached by the stone from ground level

$$= 39.2 + 19.6 = \mathbf{58.8\ m}$$

Time taken by the stone to reach the ground

$u = +19.6\ ms^{-1}; \quad S = -39.2\ m; \quad a = -9.8\ ms^{-2}; \quad t = ?$

$$S = ut + \frac{1}{2}at^2; \quad -39.2 = 19.6 \times t - 4.9t^2$$

$$4.9t^2 - 19.6t - 39.2 = 0 \therefore \quad t^2 - 4t - 8 = 0$$

$$t = \frac{+4 \pm \sqrt{16 - 4 \times 1 \times -8}}{2} = \frac{4 \pm \sqrt{48}}{2} = \mathbf{5.46\ s}$$

II.17. A body is projected vertically up from the foot of a tower 392 m high with a velocity 98 ms^{-1}. At the same instant another body is dropped from the top. Find when and where they will meet? $g = 9.8$ ms^{-2}

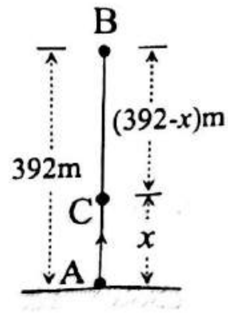

Fig. 24

Let them meet at a height x above the ground t seconds after projection.

For the body A

$$u = +98\ ms^{-1}; \quad a = -9.8\ ms^{-2}; \quad S = x; \quad t = t$$

$$S = ut + \frac{1}{2}at^2; \quad x = 98t - 4.9t^2 \qquad (1)$$

For the second body B

$$u = 0; \quad a = +9.8\ ms^{-2}; \quad S = (392 - x); \quad t = t$$

$$392 - x = 4.9t^2 \qquad (2)$$

Adding (1) and (2) ; $392 = 98t \therefore \quad t = \mathbf{4\ s}$

Substituting the value of t in equation (1)

$$h = 98 \times 4 - 4.9 \times 4^2 = \mathbf{313.6\ m}$$

IMPORTANT POINTS

Speed *is the rate of change of position. It is a scalar quantity*
Velocity *is the rate of displacement. It is a vector quantity*
Acceleration *is the ratio of change in velocity to time* $= \frac{\text{change in velocity}}{\text{Time}} = (v - u)/t$
Equations of motion *: For accelerated motion*
$v = u + at,\ s = ut + \frac{1}{2}at^2,\ v^2 - u^2 = 2as,\ s_n = u + a(n - \frac{1}{2})$
$1\ km/hr = (5/18)\ ms^{-1}$
Conventionally the direction of initial velocity is taken positive
If v_1 and v_2 are the velocities of the body to cover first and second halves of the journey
average velocity $= \frac{\text{Total distance}}{\text{Time}} = 2v_1v_2/(v_1 + v_2)$
If a body travels with velocity v_1 for an interval of time t_1, and with v_2 for an interval of time t_2, then $\overline{v} = (v_1t_1 + v_2t_2)/(v_1 + v_2)$
Displacement time graph*: Slope of the graph at any instant gives the velocity at that instant.*
Velocity time graph*: Slope of the graph at any instant gives the acceleration at that instant.*
Area under velocity time graph gives displacement
Acceleration time graph*: Area under the graph gives the change in velocity*
For vertical projection*: Time of ascent = Time of descent = u/g, neglecting air resistance*
Maximum height reached $= u^2/2g$

$$v_{inst} = (ds/dt); \quad a_{inst} = (dv/dt) = (d^2s/dt^2)$$

EXERCISES

A. Multiple Choice Questions (Chose the best alternative)

1. The displacement time graph of two particle A and B are straight lines inclined at angle 30° and 60° with the time axis. The ratio of their velocities will be
   A. 1:2 B. $1:\sqrt{3}$ C. $\sqrt{3}:1$ D. 1:3 E. 2:1

2. Two bodies of masses m_1 and m_2 are dropped from two different heights h_1 and h_2. The ratio of the times taken by the bodies to reach the ground is
A. $h_1 : h_2$ B. $m_1/h_1 : m_2/h_2$ C. $\sqrt{h_1} : \sqrt{h_2}$ D. $h_1^2 : h_2^2$ E. $h_2 : h_1$

3. A stone thrown vertically upward with a velocity u from the top of a tower reaches the ground will a velocity $3u$. The height of the tower is
A. $(3u^2/g)$ B. $(4u^2/g)$ C. $(6u^2/g)$ D. $(9u^2/g)$ E. (u^2/g)

4. A body released from the top of a tower falls through the first half height of the tower in 2 s. It will reach the ground nearly after
A. 5 s B. 2.82 s C. 4 s D. 2.43 s E. 8 s

5. A body starting from rest moves with constant acceleration for 20 seconds. If the body travels a distance x_1, in the first 10 seconds and x_2 in the next 10 seconds, then the relation between x_1 and x_2 is
A. $x_2 = x_1$ B. $x_2 = 2x_1$ C. $x_2 = 3x_1$ D. $x_2 = 4x_1$ E. none of above

6. A particle travels a certain distance with uniform speed of 6 ms^{-1} and an equal distance with a speed of 4 ms^{-1}. The average speed of the whole journey
A. 10 ms^{-1} B. 5 ms^{-1} C. 4.8 ms^{-1} D. 2.4 ms^{-1} E. none of the above

7. Two bodies are thrown vertically upwards with initial velocities in the ratio 2:3. Then the ratio of the maximum heights attained by them is
A. 2:3 B. 1:1 C. 4:9 D. $\sqrt{2} : \sqrt{3}$ E. 3:2

8. The displacement-time graph shown in the figure represents
A. the body moving with uniform velocity
B. the body moving with uniform acceleration
C. the body moving with uniform acceleration and then with uniform speed
D. the body moving with uniform velocity up to time t and then stops.
E. none of the above

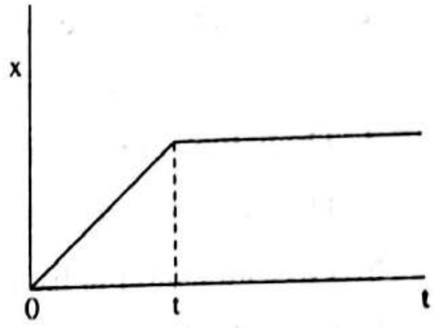

Fig. 25

9. A stone is dropped into a well of depth h. The splash is heard after a time t. If v is the velocity of sound then
A. $t = \sqrt{2h/g}$ B. $\sqrt{g}/2h$ C. $(h/v) + (h/g)/(h/v)$ D. $\sqrt{2h/g} + (h/v)$
E. (h/v)

10. A ball is dropped from a bridge 122.5 m high. After it has fallen for 2 seconds a second ball is thrown straight down after it. What must be the initial velocity of the second ball, so that both the balls hit the surface of water at the same time?
A. 49 ms^{-1} B. 26.1 ms^{-1} C. 9.8 ms^{-1} D. 55.5 ms^{-1} E. 24.5 ms^{-1}

11. Water drops fall at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at the instant the first touches the ground. How far above is the second drop at that instant? (g=10 ms^{-2})
A. 3.75 m B. 4 m C. 1.25 m D. 2.5 m E. none of the above

12. The position of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to
A. a/b B. $2a/3b$ C. $a/3b$ D. $2a/b$ E. zero

13. A body dropped from the top of a tower falls through 40 m during the last two seconds of its fall. The height of the tower is

A. 50 m B. 60 m C. 45 m D. 70 m E. 65 m

14. A particle moving with constant acceleration from A to B along a straight line with velocities u and v at A and B. The velocity at the midpoint is

A. $(u + v)/2$ B. $(v - u)/2$ C. $\sqrt{(u^2 + v^2)/2}$ D. $(v^2 - u^2)/2$ E. zero

15. The velocity of an electron starting from rest is given by $v = 2t$. The distance travelled by the electron in 3 second is

A. 9 m B. 16 m C. 27 m D. 6 m E. 12 m

16. Position–time graph of a body is a straight line parallel to the time axis. What does this imply?

A. uniform velocity B. uniform acceleration C. uniform speed
D. stationary object E. none of the above

Key to multiple choice questions

(1) D (2) C (3) B (4) B (5) C (6) C (7) C (8) D (9) D (10) B
(11) A (12) C (13) C (14) C (15) A (16) D

B. Very short answer questions

1. Distinguish between 'speed' and 'velocity'.
2. What is a velocity–time graph?
3. Draw the velocity–time graph of a body thrown vertically upwards.
4. Define instantaneous velocity and acceleration.
5. Give an example for a body moving with uniform speed, but variable velocity.
6. Can a body have acceleration without velocity? Explain.
7. What is a 'point object'?
8. Distinguish between 'distance' and 'displacement'.
9. Can an object in one dimensional motion with zero speed possess non-zero velocity? Explain.
10. Obtain the equation $v_t = v_0 + at$.
11. Can an object have an eastward velocity while experiencing a westward acceleration? Explain.
12. Can the speed of a particle ever be negative? If so, give an example; if not, explain why?
13. Can an object be increasing in speed as its acceleration decreases? If so, give an example; if not explain why?
14. Can bodies with different velocities have the same acceleration? Explain.
Hint: Yes. Ex: verticel projection.

C. Short answer questions

1. If a body is thrown vertically upwards and the air resistance is not negligible, is the time of rise equal to the time of fall? If not, which is greater? Explain, why?

2. If u is the velocity of a car and a the maximum retardation possible, find the minimum distance in which it can be stopped.

3. A man starts from his home at 9 AM, walks with a speed of 5 km/h on a straight course upto his office 2.5 km away, stays at the office upto 5 PM and returns by a three-wheeler at a speed of 25 km/h. Plot $x - t$ graph of his motion.

4. A boy standing on a stationary lift (open from above) throws a ball upward with a speed of 49 ms^{-1}. How much time does the ball take to return to his hand. If the lift starts moving up with uniform speed of 5 ms^{-1} and again he throws the ball with the same speed, how long does the ball take to return to his hand?

   [Ans: 10 s, 10 s]

   Hint: As the lift moves up with uniform velocity, 5 ms^{-1}, then the initial velocity of the boy and the ball is also 5 ms^{-1}. Relative initial velocity of ball remains the same. Hence time is the same.

5. Show that the area under velocity–time graph represents the distance travelled.

6. Show that the distance travelled in the nth second by a body moving with uniform acceleration a is $S_n = v_0 + a[n - (1/2)]$, where v_0 is the velocity at $t = 0$.

7. A three-wheeler starts from rest, accelerates uniformly at 1 ms^{-2} on a straight road for 10 seconds and then moves with uniform velocity. Plot the distance covered by the body in n^{th} second against n.

   [Hint: $S_n = a(n - \frac{1}{2}) = (n - \frac{1}{2})$] Ans: Straight line

8. Draw the displacement-time graph of a freely falling body.

9. Look at the following graphs (a) to (d). Show that none of them represents one dimensional motion of a particle. [NCERT]

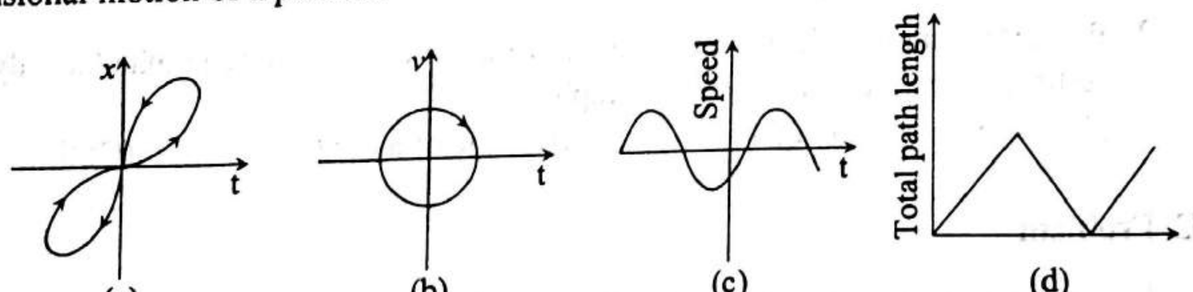

Fig. 26

Ans:

(a) *A particle cannot have two different positions at the same time.*
(b) *A particle cannot have velocities in opposite directions at the same time.*
(c) *Speed is always non-negative*
(d) *Total path cannot decrease with time*

10. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial velocities 15 ms^{-1} and 30 ms^{-1}. Show that the following graph correctly represents the time variation of relative position of the second stone with respect to the first. Give the equations of linear and curved paths. [NCERT]

Ans.

For first stone $x_1 = 15t - 5t^2$ (1)

For the second stone $x_2 = 30t - 5t^2$ (2)

Subtracting, $x_2 - x_1 = 15t$ (3)

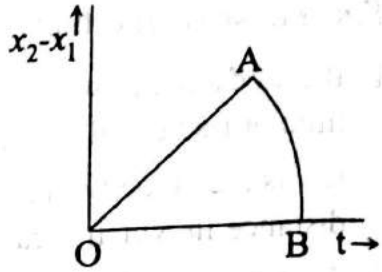

Fig. 27

This is the linear equation for the portion OA. After the first has reaches the ground, $x_1 = -200$ m

$$\therefore x_2 - x_1 = 30t - 5t^2 + 200$$

This represents the curved path AB.

11. The displacement of a particle is proportional to the cube of the time elapsed. How does the acceleration of the body depend on the time elapsed?

12. The driver of a train moving at a speed v_1 sights another train at a distance d ahead of him moving in the same direction with a lower speed v_2. He applies brakes and gives a constant retardation a to his train. Show that there will be no collision if $d > (v_1 - v_2)^2/2a$.

13. The speed time graph of a particle moving along a fixed direction is shown in the figure. Obtain the distance travelled by the particle from $t = 0$ s to 10 s. What is the average speed of the particle over this interval? [Ans: 60 m; 6 ms^{-1}]

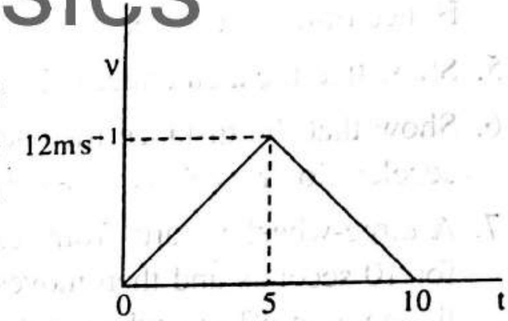

Fig. 28

D. Essays

1. Define uniform velocity and uniform acceleration. Derive the relation $S = v_0t + 1/2at^2$; $v_t^2 - v_0^2 = 2as$.
2. What is meant by velocity time-graph. Draw the velocity–time graph of a body thrown vertically upwards. Mark on the graph
   (A) the maximum height (B) time of ascent (C) time of flight.

E. Problem

1. A particle travels half a distance at 12 km/h and the remaining half at 18 km/h. Calculate the average speed.

   [Ans: 14.4 km/h]

2. A particle moving with an initial velocity of 60 ms^{-1} is brought to rest in a distance of 120 m. Assuming the deceleration uniform, calculate the retardation and the time interval.

   [Ans: 15 ms^{-2}; 4 s]

3. Starting from rest from one end of a runway, a jet airliner acquires a speed of 90 ms^{-1} in one minute. Find (A) the acceleration (B) distance travelled and (C) speed at the end of the first 40 s.

   [Ans: 1.5 ms^{-2}; 2700 m; 60 ms^{-1}]

4. A uniformly accelerated body travelling along a straight line with initial velocity $2\ ms^{-1}$ passes over 24 m from start in 3 seconds. Calculate the uniform acceleration and the velocity it will acquire when it has passed over 4 m from the start.
[Ans: $4\ ms^{-2}$; $6\ ms^{-1}$]

5. A train starting from rest is accelerated uniformly and reaches a speed of 72 km/h in 40 s. It travels at this speed for 5 minutes and is brought to rest in 20 s subjecting it to uniform retardation. Draw a velocity–time graph and find the total distance travelled.
[Ans: 6600 m]

6. A bullet strikes a uniform plank with a velocity of $400\ ms^{-1}$ and comes out with half the velocity. What would be the velocity if the plank were only half thick.
[Ans: $316.2\ ms^{-1}$]

7. A train passes three points A, B, C at 24, 36, 54 km/h respectively with uniform acceleration. If the distance $AB = 2$ km find the distance BC.
[Ans: 4.5 km]

8. A particle moving with a certain velocity is subjected to a retardation $4\ ms^{-2}$. If the particle returns to the starting point in 12 s, calculate the initial velocity.
Hint: $u = ?$; $a = -4\ ms^{-1}$; $t = 12$ s, $S = 0$ [$24\ ms^{-1}$]

9. A person is running at his maximum speed of $4\ ms^{-1}$ to catch a train. When he is 6 m from the door of the train, it starts moving at a constant acceleration of $1\ ms^{-2}$. How long does he take to catch the train?
[Ans: 2 s or 6 s]

10. In a 100 m race which a sprinter clears in 11 s calculate his initial uniform acceleration and its duration, if his speed remains constant at $10\ ms^{-1}$ thereafter.
[Ans: $5ms^{-2}$; 2 s]

11. A body moving with uniform acceleration has velocities $20\ ms^{-1}$ and $30\ ms^{-1}$ when passing points P and Q in its path. Find the velocity midway between P and Q.
[Ans: $25.5\ ms^{-1}$]

12. A stone dropped into a well hits the water surface in 4 seconds. How deep is the well and with what velocity does the stone hit the water surface?
[Ans: 78.4 m; $39.2\ ms^{-1}$]

13. A ball dropped on an anvil from a height 3.6 m is found to rise up 2.5 m after rebounding. Calculate the velocity with which the ball (A) strikes the anvil (B) leaves the anvil.
[Ans: $8.4\ ms^{-1}$; $7\ ms^{-1}$]

14. A stone is thrown vertically upwards with a velocity $14.7\ ms^{-1}$. Calculate (A) the greatest height (B) time taken to reach the highest point (C) time of flight (D) velocity with which it strikes the ground.
[Ans: 11.025 m; 1.5 s; 3 s; $14.7\ ms^{-1}$]

15. Two balls *A* and *B* are thrown simultaneously, *A* vertically upwards with a speed of $20\ ms^{-1}$ from the ground, and *B* vertically downwards from a height of 40 m with the same speed and along the same line of motion. When and where will they meet?
[Ans: 1 s, 15.1 m]

16. A body let fall from the top of a tower falls through 7/16 of its height during the last second of its fall. What is the height of the tower?
[Ans: 78.4 m]

17. A stone thrown vertically up went up 98 m and came down . How long was it in air?

[Ans: 8.945 s]

18. A body is dropped from a point 4.9 m above a window 1.5 m high. Find the time taken by the body to pass against the window.

[Ans: 1/7 s]

19. A stone is dropped from a rising balloon when it is at a height 61.25 m above the ground and it reaches the ground in 5 s. What was the velocity of the balloon just at the moment the stone was dropped?

[Ans: 12.25 ms^{-1}]

20. A parachutist bails out from an aeroplane flying horizontally and after dropping through a distance of 40 m, opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long was he in air? At what height did he bail out from the plane?

[Ans: 15.86 s; 235 m]

21. A juggler throws balls into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws 'n' balls each second? Acceleration due to gravity $= g$.
Hint: Time taken by each ball to reach the highest point $t = (1/n)$ s. Then $u = g/n$; $S = h =$? or consider the downward motion, $u = 0, t = 1/n, S = h =$?

[Ans: $h = g/2n^2$]

22. The position of an object moving along the x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5\ ms^{-2}$ and t is measured in second. What is its velocity at $t = 0$ s and $t = 2$ s? What is the average velocity between $t = 2$ s and $t = 4$ s?

[NCERT]

Hint: $x = a + bt^2 \quad v = (dx/dt) = 2bt$
At $t = 0$ s, $v = 0$ m/s; At $t = 2$ s, $v = 2 \times 2.5 \times 2 =$ **10 ms^{-1}**
At $t = t_1 = 2$ s, $x = x_1 = 8.5 + 2.5 \times 2 = 18.5$ m;
At $t = t_2 = 4$ s, $x = x_2 = 8.5 + 2.5 \times 4^2 = 48.5$ m; $\therefore S = x_2 - x_1 = 30$ m
$t = t_2 - t_1 = 4 - 2 = 2$ s $\therefore \overline{v} = (x_2 - x_1)/(t_2 - t_1) = 30/2 =$ **15 m/s**

23. Prove that the distance transversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers, begining with unity. (This is known as Galileo's law of odd numbers) [NCERT]

Hint: Let τ be the equal intervals of time and $S_1, S_2, S_3, \ldots$ be the displacements during the first, second, third ... intervals of time. Since $v_0 = 0$; $S = (1/2)$ at^2.

$$\therefore S_1 = (1/2)a\tau^2 \ldots (1) \quad S_1 + S_2 = (1/2)a(2\tau)^2 = (1/2)a \times 4\tau^2$$
$$\therefore S_2 = (1/2)a \times 3\tau^2; \quad S_1 + S_2 + S_3 = (1/2)a(3\tau)^2 = (1/2)a \times 9\tau^2$$
$$\therefore S_3 = (1/2)a \times 5\tau^2; \quad \therefore S_1 : S_2 : S_3 : \ldots = 1 : 3 : 5 : \ldots$$

24. A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistorey bulding. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise? (b) How long will it be in air before it hits the ground? ($g = 10\ ms^{-2}$).

[NCERT]

[Ans: 45 m from the ground; 5 s]