Chapter–14:

Oscillations

CBSE CLASS XI NOTES

Dr. SIMIL RAHMAN

Periodic Motion

A motion which repeats itself at equal intervals of time is called a periodic motion. (Harmonic motion)

* The time Interval at which the motion is repeated is called time period.

eg. The revolution of the
→ earth round the sun
→ Heart beat of a healthy person.
→ motion of a simple pendulum.

What is meant by oscillatory motion?

If the particles moves back and forth (to and fro) in periodic motion, the motion is said to be oscillatory.

eg. 1.- motion of simple pendulum
2.- motion of prongs of an excited tuning fork.

* All oscillatory motions are periodic but all periodic motions are not oscillatory.

Define the following terms

(a) Time period (b) frequency (c) displacement

(a) Time period

Time period of an oscillatory motion can be defined as the time taken to complete one vibration

* SI unit is second.
* If a particle oscillates N times in time 't', its period

$$T = \frac{\text{time}}{\text{Number of oscillations}}$$

$$\boxed{T = \frac{t}{N}}$$

(b) Frequency

The number of vibrations that occur in a unit time is called frequency of periodic motion

* S.I unit hertz (Hz)

$$\nu = \frac{1}{T}$$

(c) Displacement

It is the

distance of the vibrating particle from its mean position at that instant.

S.I unit – m

Define Simple Harmonic Motion (SHM)?

A particle is said to execute simple harmonic motion when it vibrates periodically in such a way that at any instant the force acting on it is directly proportional to the displacement from mean position and is always acting towards the mean position.

Derive the differential equation of a SHM?

In a simple harmonic motion, force 'F' acts on the vibrating body at any instant is directly proportional to the displacement 'x' and is directed towards the mean position

$F \propto -x$

$F = -kx \quad \cdots (1)$

According to Newton's second law

$F = ma = m\frac{d^2x}{dt^2} \quad \cdots (2)$

$(1) = (2)$

$m\frac{d^2x}{dt^2} = -kx$

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

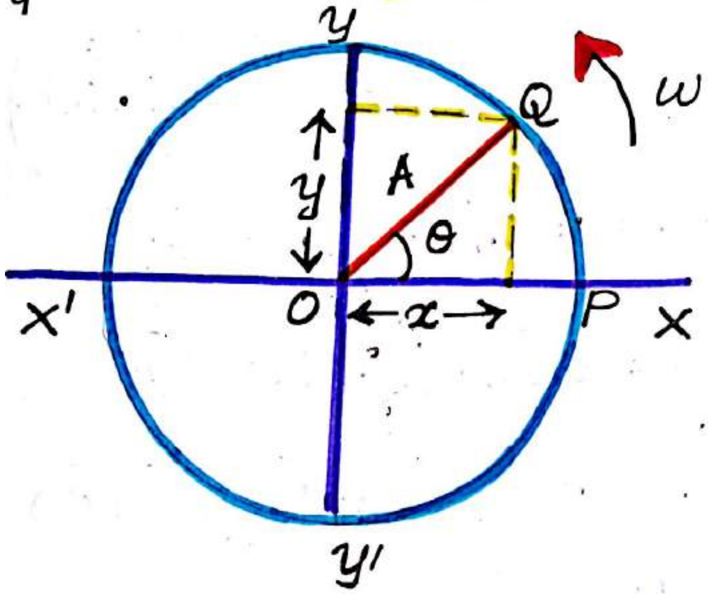
(where $\omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$)

It is the differential equation of SHM.

Show that projection of uniform circular motion on a diameter is SHM. Hence find the expression for period?

Consider a particle moving in anticlockwise direction with uniform angular velocity 'ω' along the circle of radius 'A'.

At time $t = 0$, let the particle be at P. After time t; let the particle be at Q, covered an angular displacement $\theta = \omega t$



from the diagram

Displacement

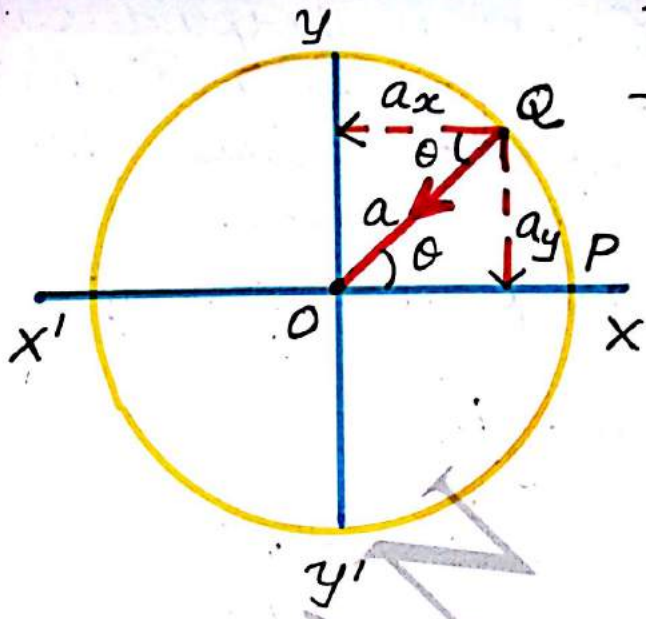
$$\boxed{\begin{aligned} x &= A\cos\theta \\ y &= A\sin\theta \end{aligned}} \quad \ldots\ldots (1)$$

Hence particle executes uniform circular motion, centripetal acceleration acts on it.

$$a = \frac{v^2}{r} = r\omega^2 = A\omega^2$$

$$\boxed{a = A\omega^2} \quad \ldots (2)$$

Acceleration 'a' is resolved into two components a_x and a_y. From figure below



$a_y = -a\sin\theta$

$a_x = -a\cos\theta$

from (2)

$$\boxed{\begin{aligned} a_x &= -A\omega^2\cos\theta \\ a_y &= -A\omega^2\sin\theta \end{aligned}} \quad \ldots (3)$$

(1) in (3)

$$\Rightarrow \boxed{\begin{aligned} a_x &= -\omega^2 x \\ a_y &= -\omega^2 y \end{aligned}}$$

$\therefore a_x \propto -x$

$a_y \propto -y$

$\therefore$ components of acceleration a_x and a_y are proportional to displacements x and y

$\therefore$ projection of uniform circular motion on a diameter is simple Harmonic as the foot of perpendicular along any diameter executes simple Harmonic motion.

Time period $T = \frac{2\pi}{\omega}$.

but $\omega = \sqrt{\frac{k}{m}}$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$

where m – mass of the body
k – force constant

* In the equation $y = A\sin(\omega t+\phi)$, y is displacement at time 't', A, ω, and φ are constants. obtain the expressions for velocity and acceleration. Represent them graphically.?

(a) Displacement

$$y = A\sin(\omega t+\phi) \quad \text{---} (1)$$

It is the distance of the particle from the mean position.

(b) Velocity

The rate of change of displacement of a particle with time is called velocity.

$$y = A\sin(\omega t+\phi)$$

$$V = \frac{dy}{dt} = A\omega\cos(\omega t+\phi)$$

$$V = A\omega\cos(\omega t+\phi) \quad \cdots (2)$$

$\because \cos\theta = \sqrt{1-\sin^2\theta}$

$$\therefore V = A\omega\sqrt{1-\sin^2(\omega t+\phi)}$$

$$V = \omega\sqrt{A^2(1-\sin^2(\omega t+\phi))}$$

$$V = \omega\sqrt{A^2 - A^2\sin^2(\omega t+\phi)}$$

from (1)

$$V = \omega\sqrt{A^2 - y^2} \quad \cdots (3)$$

* At mean position

$y = 0$

$$V = \omega\sqrt{A^2-y^2}$$

$$V = \omega A$$

$$V_{max} = \omega A$$

I

* At extreme position

$y = A$

$$V_{min} = \omega\sqrt{A^2 - A^2} = 0$$

$$\therefore V_{min} = 0$$

(c) Acceleration

Rate of change of velocity of

particle with time is called acceleration.

From eqtn (2)

$$V = A\omega \cos(\omega t + \phi)$$

$$a = \frac{dv}{dt} = -A\omega.\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \sin(\omega t + \phi)$$

From eqtn (1)

$$\boxed{a = -\omega^2 y} \quad \dots\dots (4)$$

* At mean position
$y = 0$

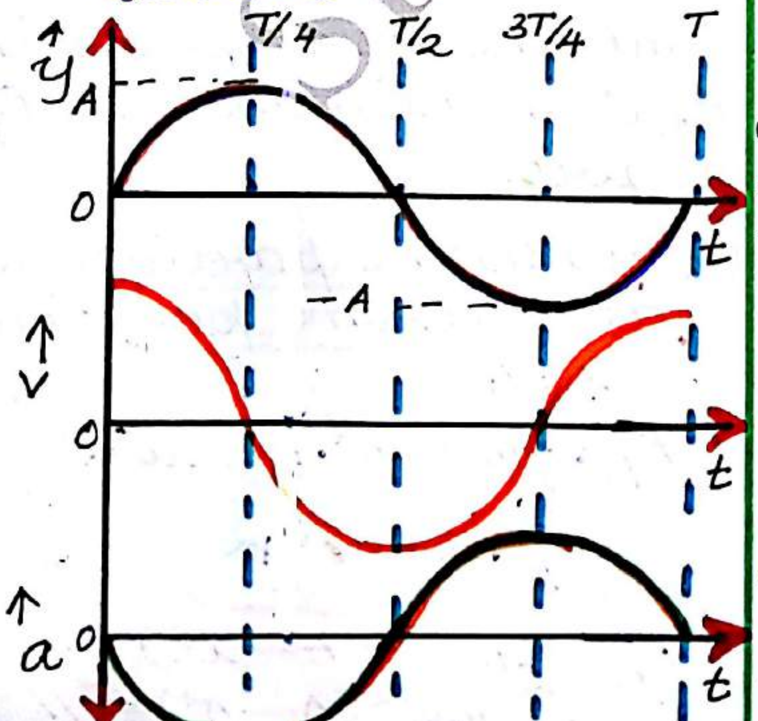
$\therefore \boxed{a_{min} = 0}$

* At extreme position
$y = A$

$$\boxed{a_{max} = -\omega^2 A}$$

II

Graphical Representation (from equations I and II)



* **Derive an expression for the energy of a Harmonic Oscillator?**

A particle executing simple harmonic motion possesses potential Energy E_p and kinetic energy E_K. Total energy at any instant remains constant.

potential Energy (E_p)

It is the work done in displacing the particle against the restoring force from equilibrium position.

acceleration in SHM is

$$a = -\omega^2 x$$

$$\therefore F = ma = -m\omega^2 x$$

$$F = -m\omega^2 x \quad \dots\dots (1)$$

If the particle undergoes further infinitesimally small displacement dx, the small work done 'dw' against the restoring force is

$$dw = -F\,dx$$

$$dw = -(-m\omega^2 x)\,dx$$

$$dw = m\omega^2 x\,dx$$

Total work done when particle is displaced from '0' to 'x'.

$$\int dw = \int_0^x m\omega^2 x\,dx$$

$$W = m\omega^2 \int_0^x x\,dx$$

$$W = m\omega^2 \left[\frac{x^2}{2}\right]_0^x$$

$$W = \frac{1}{2} m\omega^2 x^2.$$

$$\boxed{E_p = \frac{1}{2} m\omega^2 x^2}$$

Kinetic Energy (E_K)

consider a particle of mass 'm' executing SHM with angular frequency ω.

velocity of the particle at any instant

$$V = \omega\sqrt{A^2 - x^2}$$

$$E_K = \frac{1}{2} mv^2$$

$$E_K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\boxed{E_K = \frac{1}{2} m\omega^2 (A^2 - x^2)}$$

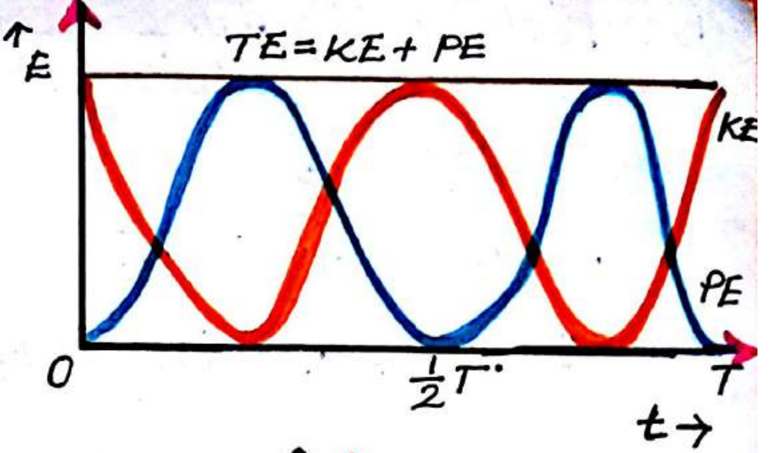
Total Energy (E)

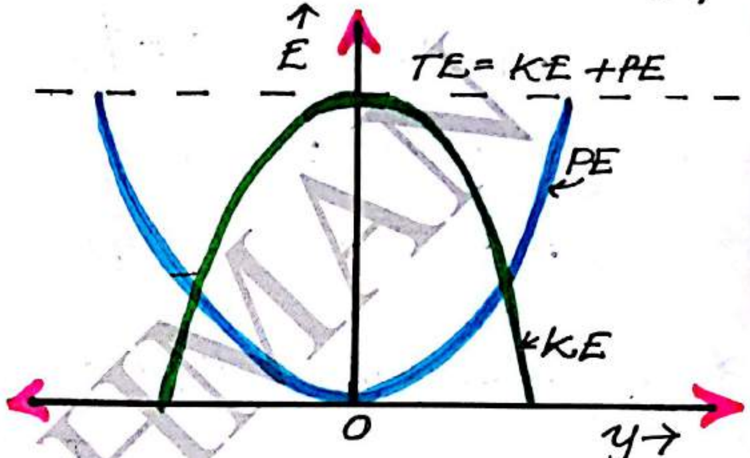
$$E = P.E + K.E$$

$$E = \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\boxed{E = \frac{1}{2} m\omega^2 A^2}$$

graphical representation of energy of a SHM.





(a) when a particle is at mean position $x=0$

$$E_p = \frac{1}{2} m\omega^2 x^2 = 0$$

$$E_p = 0$$

$$E_K = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 A^2$$

$$\boxed{E_K = \frac{1}{2} m\omega^2 A^2}$$

∴ At the mean position E_K is maximum and E_p is zero.

(b) when the particle is at extreme position

$$x = A$$

$$E_p = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 A^2$$

$$\boxed{E_p = \frac{1}{2} m\omega^2 A^2}$$

$$E_K = \frac{1}{2} m\omega^2 (A^2 - x^2)$$
$$= \frac{1}{2} m\omega^2 (A^2 - A^2) = 0$$

State some characteristics of SHM.

1, **Displacement**: Distance of vibrating particle from mean position is called displacement

$$y = A \sin \omega t.$$

2, **Velocity**: velocity of SHM also varies sinusoidally with time.

$$V = A \cos \omega t = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V = \omega\sqrt{A^2 - y^2}$$

3. **Acceleration**: It is directly proportional to displacement and always acts towards mean position.

$$a = -\omega^2 y.$$

4, **Phase**: Argument of cosine or sine function at any instant is called phase of SHM at that instant.

In equation

$$y = A \sin(\omega t + \phi)$$

$(\omega t + \phi)$ gives phase, state of motion of the particle.

5, **Epoch (ϕ) or Initial phase**

It is the phase of the particle at $t = 0$.

phase of particle is $(\omega t \pm \phi)$

$\therefore$ at $t = 0$, phase $= \pm\phi$

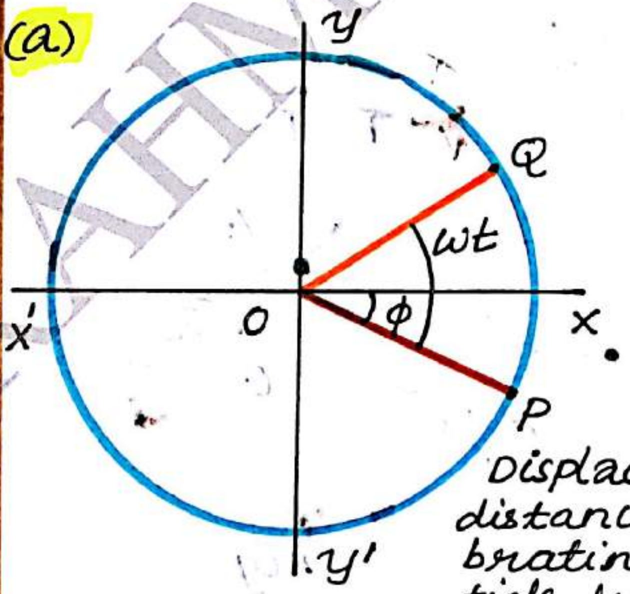
$$\text{phase} = \pm\phi = \text{epoch}.$$

Write the displacement equations X and Y when a particle starts from

(a) Below X-axis in SHM.

(b) Above X-axis in SHM.

(a)

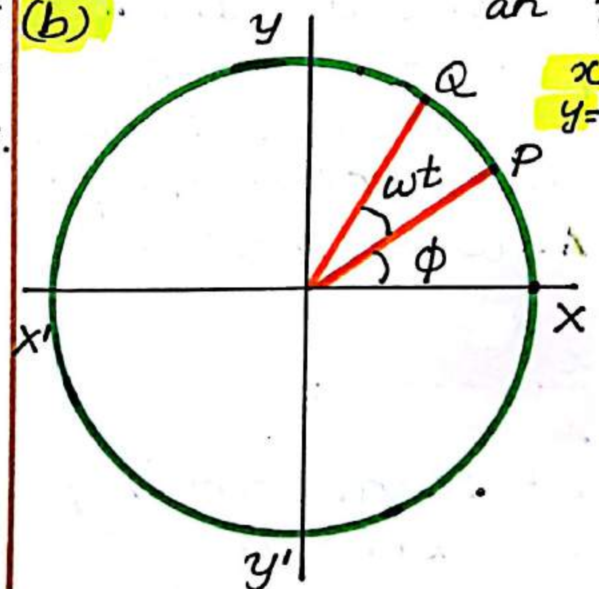


Displacement: distance of vibrating particle from mean position

$x = A\cos(\omega t - \phi)$

$y = A\sin(\omega t - \phi)$

(b)



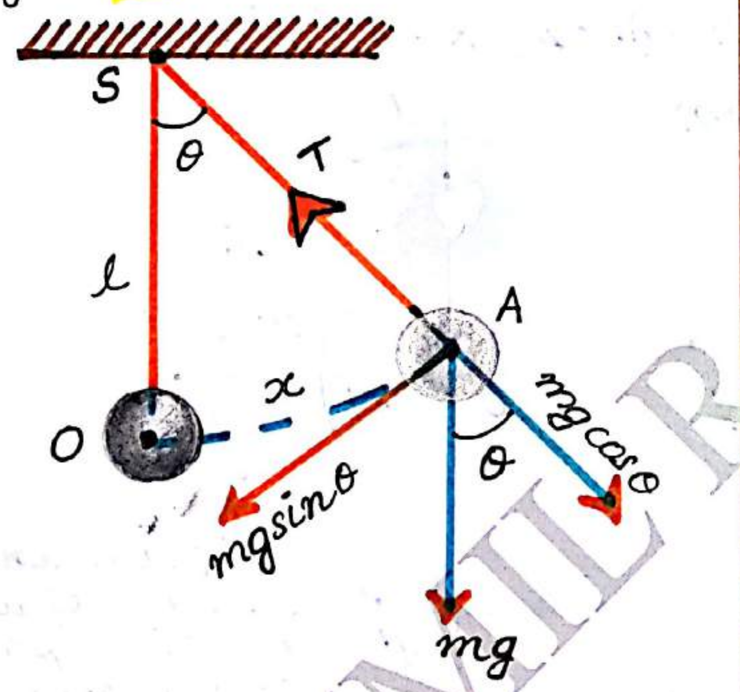
Displacement

$x = A\cos(\omega t + \phi)$

$y = A\sin(\omega t + \phi)$

Show that oscillations of a simple pendulum executes simple harmonic and derive the expression for the time period?

Simple pendulum consists of a bob of mass 'm' attached to the end of light inextensible string of length l.



Consider the bob is displaced through an angle 'θ' from the mean position 'O' to the position A. Such that arc $OA = x$

$$\theta = \frac{x}{l} \quad \ldots \quad (1)$$

* Different forces acting on the bob are

1. Weight of the bob, mg, acts vertically downwards.
2. Tension T acts along its length towards the point of suspension S.

Weight 'mg' can be resolved into two components $mg\cos\theta$ and $mg\sin\theta$. Here, $mg\cos\theta$ balances the tension T in the string

$$\therefore T = mg\cos\theta$$

$$\boxed{T - mg\cos\theta = 0}$$

$mg\sin\theta$ gives restoring force.

$$F = -mg\sin\theta$$

If θ is small $\sin\theta = \theta$

$$\therefore F = -mg\theta \quad \ldots \quad (2)$$

Sub (1) and (2)

$$F = -\frac{mgx}{l} \quad \ldots \quad (3)$$

According to Newton's II law

$$F = ma \quad \ldots \quad (4)$$

(3) = (4)

$$\therefore ma = -\frac{mgx}{l}$$

$$\boxed{a = -\frac{gx}{l}} \quad \ldots \quad (5)$$

Here 'g' and 'l' are constants.

Therefore acceleration is directly proportional to displacement and acts towards mean position $a \propto -x$

$\therefore$ It executes SHM

In SHM $\quad a = -\omega^2 x \quad \ldots (6)$

$(5) = (6)$

$$-\omega^2 x = \frac{-gx}{l}$$

$$\therefore \omega^2 = +\frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}} \quad \ldots (7)$$

$$T = \frac{2\pi}{\omega}$$

Time period $\quad \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

Time period is independent of mass of bob and amplitude of vibration when 'θ' is small.

seconds pendulum

It possesses time period of two seconds.

Show that oscillations of a loaded spring execute SHM and derive time period expression.

when a spring is streched or compressed by a distance x, restoring force acts on it.

$$F \propto -x$$

$$F = -kx \quad \ldots (1)$$

where 'k' is spring constant.

According to Newtons II law.

$$F = ma \quad \ldots (2)$$

$(1) = (2)$

$$ma = -kx$$

$$a = -\frac{k}{m}x \quad \ldots (3)$$

In eqtn (3) $a \propto -x$

$\therefore$ oscillations of loaded spring executes SHM.

It is the acceleration of the body of mass m.

In SHM $\quad a = -\omega^2 x \ldots (4)$

$(3) = (4)$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Time period $\quad T = \frac{2\pi}{\omega}$

$$= 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Define force constant. Give its unit.

$F = -kx$

If $x = 1$, $F = k$ (drop (−) sign)

Force constant of a spring is defined as restoring force per unit displacement of spring.

SI unit N/m

Dimensional Formula MT^{-2}

What is meant by free, forced, resonant, damped and undamped oscillations?

(a) Free oscillation

A body is said to be executing free oscillations, if it vibrates with its natural frequency. In case of spring

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

∴ Natural frequency depends upon mass, dimensions etc.

eg;

1. bob of simple pendulum − vibration.
2. prongs of tuning fork executes free vibration.

(b) Forced vibration

When a body oscillates with the help of an external periodic force with a frequency which is different from natural frequency of the body, its oscillations are called forced oscillations. or vibration

Consider an oscillator 'A' executes free oscillation of natural frequency ν_0. Suppose oscillator A is driven by another one B, whose natural frequency ν. B is called driver oscillator and A is called driven oscillator.

When time passes ν_0 dies out, body starts to vibrate with frequency of driving force ν.

(c) Resonance

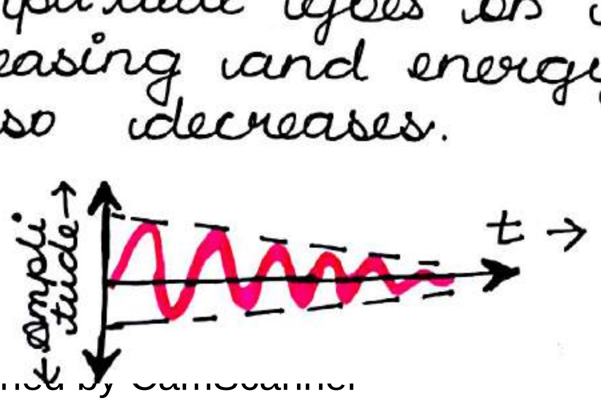
When external frequency becomes

equal to the natural frequency of the vibrating body, amplitude of vibration is maximum. It is called resonance.

eg; when soldiers cross a bridge, they are ordered to go out of steps. If they march on the bridge, frequency of the bridge sometimes becomes equal to the frequency of the steps. Then amplitude of vibration is maximum and hence bridge may get damaged.

(d) Damped Oscillation

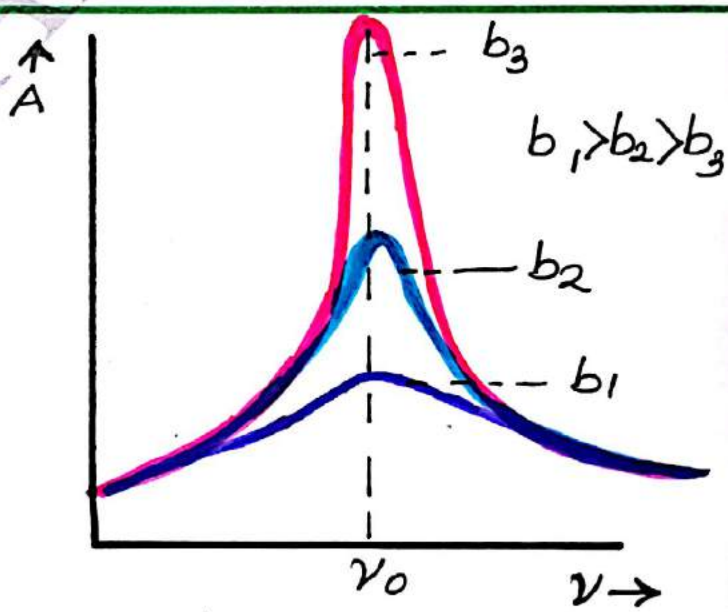
When oscillator oscillates, a part of its energy is used up in overcoming dissipating forces like frictional force, air resistance, viscous force etc. Each cycle, its amplitude goes on decreasing and energy also decreases.



eg; oscillation of simple pendulum in air medium.

(e) Un Damped Oscillation.

If there are no dissipating forces such as friction due to air, the body will produce oscillation of constant amplitude and energy. Such oscillations are called undamped oscillations.



Resonance (fig)

ν_0 — Resonant frequency.