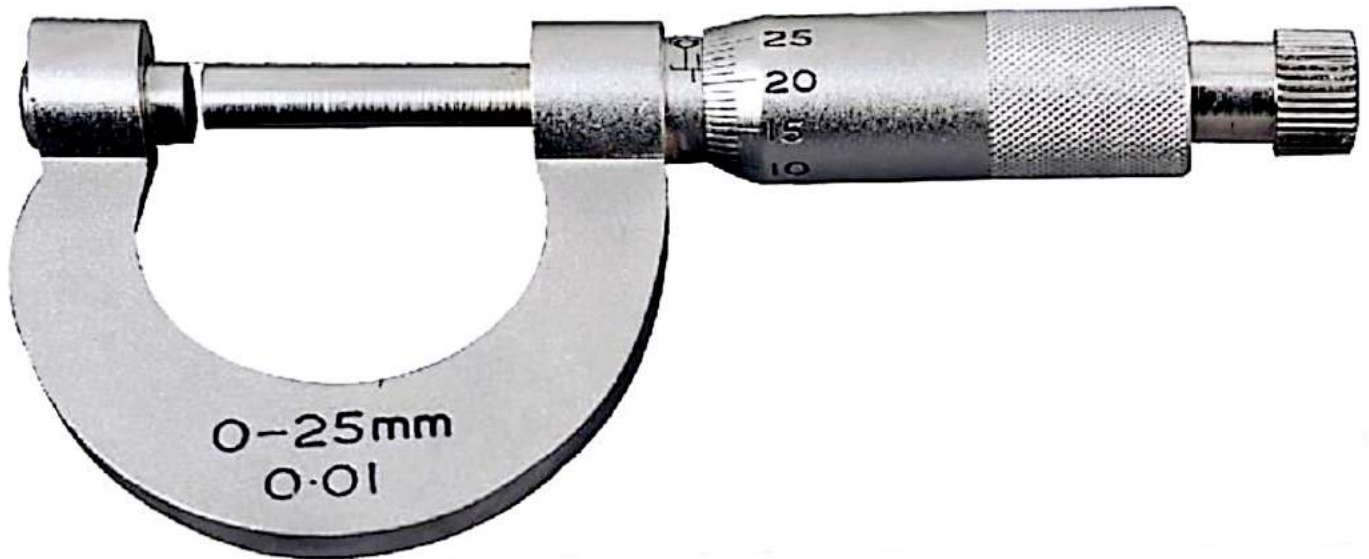


Chapter-1

# Physical World and Measurement



**CBSE CLASS XI NOTES**

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## Syllabus [course structure]

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Physics :- study of matter and energy. It is also the systematic study of the universe.

Classical physics (pre-1900)

Includes study of mechanics, gravitation, heat, sound, light, electricity and magnetism

Modern Physics (post-1900)

Includes the study of quantum mechanics, atomic physics, nuclear physics, molecular physics, relativity, electronics, particle physics and plasma physics.

Scientific method

- (i) Observation of relevant facts
- (ii) proposal of theory based on these observations
- (iii) Testing of the proposed theory by experiments.
- (iv) Modification if necessary.

Physics Technology and society.

1, Steam engine	Laws of thermodynamics
2, Nuclear reactor	Nuclear fission.
3, Rocket propulsion	Newtons laws of motion
4, Radio & T.V	propagation of e.m waves
5, aeroplane	Bernoulli's principle.
6, computer	Digital Logic.

Fundamental forces in Nature.

1, Gravitational force.

It is the weakest force in nature and is always attractive.

$$F = G \frac{m_1 m_2}{r^2}$$

2, Electromagnetic or electrostatic force.

It is the force between electrically charged particles.

→ can be attractive or repulsive.

3, Strong Nuclear force

It is the str-

strongest force. always attractive.

→ Force b/w nucleons

n-n

p-p

p-n

→ charge independent.

→ very short range

4. Weak Nuclear force.

come into play during  $\beta$  (Beta) decay of radioactive nucleus.

### Conservation laws

1. Law of conservation of linear momentum.

In the absence of an external force, the total linear momentum of a system remains constant.

2. Law of conservation of angular momentum

In the absence of an external torque, the total angular momentum of a system remains constant.

3. Law of conservation of Energy.

## Physical Quantity

The quantity in physics which can be measured directly or indirectly in terms of laws of physics

### units

The measurement of physical quantity involves its comparison with a chosen standard of same kind. This reference standard is called a unit.

### Fundamental Physical Quantity

Quantities which cannot be defined in terms of other physical quantities.

The fundamental units are the units of fundamental quantities. eg; Length, Mass and Time.

### Derived Units/physical Qty.

Physical quantities which can be <sup>expressed</sup> defined in terms of fundamental quantities are called derived quantities

units :- Derived units  
eg; velocity, acceleration



## systems of unit

- (1) CGS system - centimetre, gram second
- (2) FPS - foot, pound, second.
- (3) MKS - Metre, kilogram second.

## SI units

- |                        |             |
|------------------------|-------------|
| 1, Mass                | kg.         |
| 2, Length              | m           |
| 3, Time                | s           |
| 4, Temperature         | K           |
| 5, Electric current    | A           |
| 6, Luminous Intensity  | cd (candle) |
| 7, Amount of substance | mol         |

## Supplementary Units

- 1, Plane angle - radians (rad)  

- 2, Solid angle - steradian (sr)  


## Advts of SI

- (1) It is comprehensive
- (2) coherent
- (3) International acceptance.

light year

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

## Astronomical unit

$$(AU) = 1.50 \times 10^{11} \text{ m}$$

$$\text{Angstrom unit (Å)} = 10^{-10} \text{ m}$$

$$\text{Fermi (fm)} = 10^{-15} \text{ m}$$

$$10^{12} \text{ tera - T} \quad 10^{-1} \text{ deci d}$$

$$10^9 \text{ giga - G} \quad 10^{-2} \text{ centi c}$$

$$10^6 \text{ mega - M} \quad 10^{-3} \text{ milli m}$$

$$10^3 \text{ kilo - k} \quad 10^{-6} \text{ micro - } \mu$$

$$10^2 \text{ hecto - h} \quad 10^{-9} \text{ nano - n}$$

$$10^1 \text{ deka - da} \quad 10^{-12} \text{ pico - p}$$

$$10^{-15} \text{ femto - f}$$

$$10^{-18} \text{ atto - a}$$

## Some length measurements

(i) Measurement of large distance by angular measurement  $\alpha = \theta$ .

(ii) Parallax method.

$$\theta = \frac{\alpha}{r}$$

(iii) Reflection method.

→ SONAR [Sound Navigation and Ranging]  
→ RADAR [Radio detector and Ranging]

## Dimensions

The dimension of a physical quantity are the powers to which the fundamental quantities (mass, length, time...) can be raised.

eg;  $k M^x L^y T^z$

$x, y, z \rightarrow$  How many times

$k \rightarrow$  constant.

## Dimensional Equation

The expression which indicates the units of a physical quantity in terms of the fundamental units is called dimensional equation.

eg;

$$\begin{aligned} \text{Area} &= \text{Length} \times \text{length} \\ &= L \times L = L^2 \\ &= M^0 L^2 T^0 \end{aligned}$$

$$\text{Volume} = L^3 = M^0 L^3 T^0$$

$$\begin{aligned} \text{Density} &= \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} \\ &= M L^{-3} T^0 \end{aligned}$$

$$\text{Velocity} = \frac{\text{Disp}}{\text{Time}} = \frac{L}{T} = M^0 L T^{-1}$$

$$\begin{aligned} \text{Acceleration} &= \frac{\text{Vel}}{\text{Time}} = \frac{L T^{-1}}{T} = L T^{-2} \\ &= M^0 L T^{-2} \end{aligned}$$

$$\begin{aligned} \text{Force} &= \text{Mass} \times \text{acceleration} \\ &= M L T^{-2} \end{aligned}$$

$$\begin{aligned} \text{Pressure} &= \frac{\text{Force}}{\text{Area}} = \frac{M L T^{-2}}{L^2} \\ &= M L^{-1} T^{-2} \end{aligned}$$

$$\text{pressure} = \frac{F}{A}$$

$$\begin{aligned} \text{work} &= \text{Force} \times \text{displacement} \\ &= M L T^{-2} \times L = M L^2 T^{-2} \end{aligned}$$

$$\begin{aligned} \text{Power} &= \frac{\text{Work}}{\text{Time}} = \frac{M L^2 T^{-2}}{T} \\ &= M L^2 T^{-3} \end{aligned}$$

$$\begin{aligned} \text{Gas Const } R &= \frac{P V}{T} \\ &= \frac{M L^{-1} T^{-2} \times L^3}{K} \\ &= M L^2 T^{-2} K^{-1} \end{aligned}$$

$$\begin{aligned} \text{Electric charge } q &= \text{current} \times \text{time} \\ &= A T \end{aligned}$$

$$\begin{aligned} \text{Electric potential, } V &= \frac{\text{Work}}{\text{charge}} \\ &= \frac{M L^2 T^{-2}}{A T} \\ &= M L^2 T^{-3} A^{-1} \\ &= M A^{-1} L^2 T^{-3} \end{aligned}$$

note: ① Quantities such as number, angle and trigonometrical ratios are dimensionless.

② Angle has unit but no dimension

Momentum = Mass  $\times$  velocity  
 =  $[M][LT^{-1}]$   
 =  $[MLT^{-1}]$

Pressure stress =  $\frac{\text{Force}}{\text{Area}}$   
 =  $\frac{MLT^{-2}}{L^2}$   
 =  $[ML^{-1}T^{-2}]$

Mass density  
 =  $\frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3}$   
 =  $[ML^{-3}T^0]$

frequency =  $\frac{1}{\text{Time period}}$   
 =  $[M^0L^0T^{-1}]$

acceleration due to gravity =  $\frac{\text{change in velocity}}{\text{Time taken}}$   
 =  $\frac{LT^{-1}}{T} = [LT^{-2}]$   
 =  $[M^0LT^{-2}]$

wave length = distance  
 =  $L$   
 =  $[M^0LT^0]$

Impulse = Force  $\times$  time  
 =  $MLT^{-2} \times T$   
 =  $[MLT^{-1}]$

strain =  $\frac{\text{change in Dimension}}{\text{original Dimension}}$   
 =  $\frac{L}{L} = [M^0L^0T^0]$

surface Tension =  $\frac{\text{Force}}{\text{length}}$   
 =  $\frac{MLT^{-2}}{L} = [MLT^{-2}]$

Electric (I) current =  $[M^0L^0T^0A^1]$

Principle of Homogeneity of dimensions.

The dimensions of all the terms on the two sides of the equation are the same.

uses of Dimensional Analysis.

(1) To check the correctness of an equation  
 An equation is correct only if the dimensions of each term on either side of the equation are equal.

Eg;  $S = ut + \frac{1}{2}at^2$   
 $S = L$   
 $ut = LT^{-1} \times T = L$   
 $\frac{1}{2}at^2 = \frac{1}{2}MLT^{-2} \times T^2 = L$

∴ all has same dimension. so the equation is dimensionally correct.

$$\textcircled{2} \quad v = u + at$$

$$v = LT^{-1}$$

$$u = LT^{-1}$$

$$at = LT^{-2} \times T \\ = LT^{-1}$$

∴ the equation is dimensionally correct.

$$\textcircled{3} \quad v^2 = u^2 + 2as$$

$$v^2 = [LT^{-1}]^2 = [L^2 T^{-2}]$$

$$[u^2] = [LT^{-1}]^2 = [L^2 T^{-2}]$$

$$2as = 2 LT^{-2} \times L \\ = \underline{\underline{L^2 T^{-2}}}$$

∴ Formula is correct

$$\textcircled{4} \quad \frac{1}{2} mv^2 = mgh$$

$$M [LT^{-1}]^2 = [M L^2 T^{-2}] \quad \dots \textcircled{1}$$

$$mgh \Rightarrow M L T^{-2} L \\ = [M L^2 T^{-2}] \quad \dots \textcircled{2}$$

∴  $\textcircled{1} = \textcircled{2}$   
Formula is correct.

(ii) To derive the correct relationship b/w physical quantities.

$$\textcircled{1} \text{ S.T } T = 2\pi \sqrt{\frac{l}{g}}$$

The period of oscillation 't' of a simple pendulum may depend on  
(i) the length of pendulum L, (ii) Mass of bob m  
(iii) acceleration due to gravity g.

$$t = k L^x M^y g^z \dots \textcircled{a}$$

$$T = L^x M^y [LT^{-2}]^z \quad [\because k \text{ is}$$

$T = L^{x+z} M^y T^{-2z}$  dimensionless]  
equating dimensions of L, M & T.

$$x+z=0 \quad \dots \textcircled{1}$$

$$y=0$$

$$-2z=1$$

$$z = -1/2 \quad \dots \textcircled{2}$$

$$\textcircled{1} \Rightarrow x - 1/2 = 0 \quad \therefore x = 1/2$$

$$\therefore T = k L^{1/2} M^0 g^{-1/2}$$

$$T = k \sqrt{\frac{l}{g}} \quad [k = 2\pi \text{ const.}]$$

$$\therefore T = 2\pi \sqrt{\frac{l}{g}}$$

$\textcircled{2}$  The frequency 'n' of a stretched string may depend upon  
(i) length of the vibra-



ting segment  $l$  (ii) the tension in the string  $F$  and (iii) the mass per unit length  $m$ . S.T.

$$v = \frac{k}{l} \sqrt{\frac{F}{m}}$$

$$v \propto l^x F^y m^z$$

$$v = k l^x F^y m^z \quad \text{--- (1)}$$

taking Dimensions

$$T^{-1} = L^x [MLT^{-2}]^y \left[\frac{M}{L}\right]^z$$

$$T^{-1} = L^x [M^y L^y T^{-2y}] \left[\frac{M}{L}\right]^z$$

~~$$T^{-1} = L^x M^y L^y T^{-2y} \left[\frac{M}{L}\right]^z$$~~

$$T^{-1} = L^x [M^y L^y T^{-2y}] [M^z L^{-z}]$$

$$T^{-1} = L^{x+y-z} M^{y+z} T^{-2y}$$

$$\therefore x+y-z=0$$

$$y+z=0$$

$$-2y = -1$$

$$y = 1/2$$

$$\therefore 1/2 + z = 0$$

$$z = -1/2$$

$$\therefore x + \frac{1}{2} + \frac{1}{2} = 0$$

$$x + 1 = 0$$

$$x = -1$$

$$\therefore \text{(1)} \Rightarrow v = k L^{-1} F^{1/2} M^{-1/2}$$

$$\therefore v = \frac{k}{l} \sqrt{\frac{F}{m}}$$

(3) A body of mass 'm' is executing circular motion around a circle of radius  $r$  with speed  $v$ . It

experiences centripetal force  $F$ . P.T  $F = \frac{mv^2}{r}$  dimensionally.

$$F = k m^x v^y r^z \quad \text{--- (1)}$$

dimensions  $\downarrow$

$$MLT^{-2} = M^x [LT^{-1}]^y [L]^z$$

$$MLT^{-2} = M^x L^{y+z} T^{-y}$$

$$x = 1$$

$$y+z=1$$

$$-y = -2$$

$$y = 2$$

$$\therefore y+z=1$$

$$2+z=1$$

$$z = -1$$

$$\therefore \text{(1)} \Rightarrow F = m^1 v^2 r^{-1}$$

$$F = \frac{mv^2}{r}$$

[ii] To convert from one system of units to another system of units using dimension.

(1) convert IN to def reb.

They are units of force in SI & CGS respectively.

$$\text{Force} = MLT^{-2}$$

general formula

$$n_1 M_1^a L_1^b T_1^c = n_2 M_2^a L_2^b T_2^c$$

$$\therefore n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^1 \left[ \frac{L_1}{L_2} \right]^1 \left[ \frac{T_1}{T_2} \right]^{-2}$$

$$n_2 = n_1 \left[ \frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[ \frac{1 \text{ m}}{1 \text{ cm}} \right]^1 \left[ \frac{1 \text{ sec}}{1 \text{ sec}} \right]^{-2}$$

$[n_1 = 1]$

$$= \frac{1000 \text{ g}}{1 \text{ g}} \cdot \frac{100 \text{ cm}}{1 \text{ cm}} \cdot 1$$

$$= 1000 \cdot 100 = 10^5$$

$\therefore 1 \text{ newton} = 10^5 \text{ dyne.}$

(2) 1 Joule to 1 erg.

\* both are units of work

$$[W] = ML^2T^{-2}$$

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^a \left[ \frac{L_1}{L_2} \right]^b \left[ \frac{T_1}{T_2} \right]^c$$

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^1 \left[ \frac{L_1}{L_2} \right]^2 \left[ \frac{T_1}{T_2} \right]^{-2}$$

$[n_1 = 1]$

$$= 1 \left[ \frac{\text{kg}}{\text{g}} \right]^1 \left[ \frac{\text{m}}{\text{cm}} \right]^2 \left[ \frac{\text{sec}}{\text{sec}} \right]^{-2}$$

$$= 1 \left[ \frac{1000 \text{ g}}{\text{g}} \right] \left[ \frac{100 \text{ cm}}{1 \text{ cm}} \right]^2 \cdot 1$$

$$= 1 [1000] [100]^2$$

$$= 1 [1000] [10000]$$

$$= 1 [10000000]$$

$$n_2 = 10^7$$

$$1 \text{ J} = 10^7 \text{ erg}$$

To find dimension of constants like a, b, c.

(1) In Vander Waal's equation  $\left[ P + \frac{a}{V^2} \right] (V - b) = RT$

what are the dimensions of a & b? Here P is pressure, V is volume, T is temperature and R is a gas constant?

$$\left[ P + \frac{a}{V^2} \right] (V - b) = RT$$

As pressure can be added only to pressure

$$\therefore \frac{a}{V^2} = P$$

$$a = PV^2$$

$$a = ML^{-1}T^{-2} [L^3]^2$$

$$= ML^{-1}T^{-2} [L^6]$$

$$a = ML^5T^{-2}$$

again 'b' represents volume

$$b = V$$

$$b = [L^3]$$

$$b = M^0 L^3 T^0$$

[Volume only can be subtracted from vol.]

(2) If  $x = at^2 + bt + ct^2$  where x is metres and t in sec. obtain the unit and dimension of a, b & c?

Dimension of LHS

$$x = L^1$$

Applying principle of Homogeneity

$$R.H.S = L.H.S$$

$$\rightarrow x = a \cdot$$

$$a = \underline{L}$$

$$bt = x$$

$$bT^1 = L$$

$$\rightarrow b = \frac{L}{T} = \underline{LT^{-1}}$$

$$\rightarrow ct^2 = x$$

$$cT^2 = L$$

$$c = \frac{L}{T^2} = \underline{LT^{-2}}$$

Dimension of  $V = LT^{-1}$

$$\therefore L.H.S = R.H.S$$

$$V = at$$

$$LT^{-1} = aT$$

$$a = \frac{LT^{-1}}{T} = \underline{LT^{-2}}$$

$$LT^{-1} = \frac{b}{t+c}$$

$$LT^{-1} = \frac{b}{T+c}$$

$$\frac{T+c}{b} = \underline{LT^{-1}}$$

$$\frac{T}{b} + \frac{c}{b} = \underline{LT^{-1}}$$

(3)  $V = at^2 + bt + c$

L.H.S = R.H.S  $V = m/s$

$$LT^{-1} = at^2$$

$$LT^{-1} = aT^2$$

$$\therefore a = \frac{LT^{-1}}{T^2}$$

$$a = \underline{LT^{-3}} = \underline{M^0LT^{-3}}$$

$$LT^{-1} = bt$$

$$LT^{-1} = bT$$

$$b = \frac{LT^{-1}}{T} \therefore b = \underline{LT^{-2}}$$

$$b = \underline{M^0LT^{-2}}$$

$$c = \underline{LT^{-1}} = \underline{M^0LT^{-1}}$$

$$\frac{T}{b} + \frac{c}{b} = \underline{LT^{-1}}$$

$$\frac{T}{b} = \underline{LT^{-1}}$$

$$b = \frac{T}{L^{-1}T^{-1}} = \underline{L} \therefore b = \underline{M^0L^1T^0}$$

$$\rightarrow \frac{c}{b} = \underline{L^{-1}T^1}$$

$$c = b L^{-1}T^1$$

$$c = L L^{-1}T^1$$

$$c = \underline{T^1} \therefore c = \underline{M^0L^0T^1}$$

### LIMITATIONS OF DIMENSIONAL ANALYSIS

(2) In Mechanics, this method is not suitable if the physical quantity depend on more

(4) If  $V = at + \frac{b}{t+c}$ , obtain  $a, b, c$ ?

than three other physical quantities

- (2) This method does not tell us anything about dimensionless quantities
- (3) This method cannot be employed if R.H.S of equations contains more than one term.
- (4) Quite often it is difficult to guess parameters on which the physical quantity depends.

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### SIGNIFICANT FIGURES

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The significant figures in a measured physical quantity <sup>indicates</sup> the number of digits in which we have confidence in respect of their accuracy.

\* The greater the number of significant figures, the more accurate is the measurement.

#### Rules to determine significant figures

- (1) All the non-zero digits are significant.

(2) The no of significant figures in a number is equal to the no of digits counted from the first non-zero digit on the left to the last digit on the right  
eg; 18.45  $\rightarrow$  4 sign fig

(3) All zero's occurring between two non-zero digits are significant  
eg; 180.045  $\rightarrow$  6 - significant figures

(4) If no starts with decimal all zeros on the left side are not significant, but on right side are significant

eg; 0.0001845  $\rightarrow$  4 sign fig  
0.004030  $\rightarrow$  4 sign fig

(5) If a number has an integral part and decimal part, all zeros in the number are significant

eg; 30.10  $\rightarrow$  4 (sign fig)  
30.00  $\rightarrow$  4 (s.f)

(6) Where there is no decimal part, the last zero's are not significant.

eg; 102000  $\rightarrow$  3. [s.f]

Eq; [s.f.]

300 → 1

$3 \times 10^2$  → 1

$3.0 \times 10^2$  → 2

$3.00 \times 10^2$  → 3

403 → 3

40.3 → 3

4.03 → 3

0.403 → 3

0.0403 → 3

0.04030 → 4

403.0 → 4

40.30 → 4

$1.02 \times 10^3$  → 3

## Errors in measurement.

When we take a measurement with an instrument the measured value is usually different from the true value. There is always an uncertainty in the measurement. This uncertainty is called error.

Accuracy - of a measurement is a measure of how close a measured value is to the true value of the quantity.

Absolute Error

Magnitude of difference between

the true value and a measured value is called the absolute error in the measurement.

$$\Delta a_i = |\bar{a} - a_i|$$

$\bar{a}$  → True value

$\Delta a_i$  → absolute error in the  $i^{\text{th}}$  measurement.

\* It is expressed in the unit of measured values.

Mean Absolute Error.

The arithmetic mean of the absolute errors in various measurements.

$$\bar{\Delta a} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \dots + \Delta a_n}{n}$$

\*  $\Delta a_1, \Delta a_2, \Delta a_3, \dots$  → absolute errors

\* measurement can have any value lying b/w  $\bar{a} + \Delta a$  &  $\bar{a} - \Delta a$

Relative error and percentage error

The relative error is defined as the ratio of mean absolute error to the true value.

$$\text{Relative error} = \frac{\bar{\Delta a}}{\bar{a}}$$

$$\text{Percentage error} = \left( \frac{\bar{\Delta a}}{\bar{a}} \right) \times 100\%$$

eg (2.7) we measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63s, 2.56s, 2.42s, 2.71s and 2.80s. Calculate absolute errors, relative error or percentage error.

$$T_{\text{mean}} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$T_{\text{mean}} = \frac{13.12}{5} = 2.624 \text{ s}$$

$$T_{\text{mean}} = 2.62 \text{ s}$$

absolute errors ( $\Delta a$ )

$$2.63 - 2.62 = 0.01 \text{ s}$$

$$2.56 - 2.62 = -0.06 \text{ s}$$

$$2.42 - 2.62 = -0.20 \text{ s}$$

$$2.71 - 2.62 = +0.09 \text{ s}$$

$$2.80 - 2.62 = +0.18 \text{ s}$$

mean absolute error

$$\text{error } (\bar{\Delta a}) = \frac{[0.01 + 0.06 + 0.20 + 0.09 + 0.18]}{5}$$

$$\bar{\Delta a} = 0.11 \text{ s}$$

$$\text{percentage error} = \frac{0.11}{2.62} \times 100 = 4\%$$

$$\text{relative error} = \frac{0.11}{2.62} = 0.04$$

combination of errors

(i) error in sum

- (ii) error in difference
- (iii) error in product
- (iv) error in a quotient
- (v) error when a quantity is raised to power.

(i) Error in a sum.

If 'z' be the sum of two observed quantities x and y.

$$Z = x + y$$

$$\therefore z \pm \Delta z = (x \pm \Delta x) + (y \pm \Delta y)$$

maximum possible error in z is

$$\Delta z = \Delta x + \Delta y$$

relative error in z is

$$\frac{\Delta z}{z} = \frac{\Delta x + \Delta y}{x + y}$$

% error in z is

$$\frac{\Delta z}{z} \times 100 = \frac{\Delta x + \Delta y}{x + y} \times 100$$

$$\pm (\Delta x + \Delta y) \times 100 / (x + y)$$

(ii) error in a difference

$$Z = x - y$$

maximum possible error in 'z' is

$$\Delta z = \Delta x + \Delta y$$

relative error =  $\frac{\Delta z}{z} = \frac{\Delta x + \Delta y}{x - y}$

$$\frac{\Delta z}{z} = \pm \frac{(\Delta x + \Delta y)}{x - y}$$

percentage error in  $z$  is

$$\frac{\Delta Z}{Z} \times 100 = \pm \frac{(\Delta x + \Delta y)}{x - y} \times 100$$

(iii) Error in a product

$$\therefore z = xy.$$

$$(z \pm \Delta z) = (x \pm \Delta x)(y \pm \Delta y)$$

max possible error

$$\pm \frac{\Delta Z}{Z} = \pm \left( \frac{\Delta x}{x} \right) \pm \left( \frac{\Delta y}{y} \right)$$

maximum relative error

$$\frac{\Delta Z}{Z} = \pm \left[ \left( \frac{\Delta x}{x} \right) + \left( \frac{\Delta y}{y} \right) \right]$$

% error in  $z$  is

$$\left( \frac{\Delta Z}{Z} \right) \times 100 = \pm \left[ \left( \frac{\Delta x}{x} \right) + \left( \frac{\Delta y}{y} \right) \right] \times 100\%$$

iv Error in a Quotient

$$\text{let } z = \frac{x}{y}$$

ma

$$(z \pm \Delta z) = \frac{(x \pm \Delta x)}{(y \pm \Delta y)}$$

maximum possible error

$$\pm \frac{\Delta Z}{Z} = \pm \left( \frac{\Delta x}{x} \right) \pm \left( \frac{\Delta y}{y} \right)$$

relative error

$$\frac{\Delta Z}{Z} = \pm \left( \frac{\Delta x}{x} + \frac{\Delta y}{y} \right)$$

% error in  $z$  is

$$\left( \frac{\Delta Z}{Z} \right) \times 100 = \pm \left[ \frac{\Delta x}{x} + \frac{\Delta y}{y} \right] \times 100\%$$

(v) Error when a quantity is raised to a power.

$$\text{let } z = \frac{x^a}{y^b}$$

max

$$\left( \frac{\Delta z}{z} \right) = \pm a \left( \frac{\Delta x}{x} \right) + b \left( \frac{\Delta y}{y} \right)$$

relative error

$$\frac{\Delta z}{z} = \pm a \left( \frac{\Delta x}{x} \right) + b \left( \frac{\Delta y}{y} \right)$$

% error

$$\left( \frac{\Delta z}{z} \right) \times 100 = \left[ \pm a \left( \frac{\Delta x}{x} \right) + b \left( \frac{\Delta y}{y} \right) \right] \times 100\%$$

Numericals.

(1) The resistance  $R = \frac{V}{I}$  where  $V = (100 \pm 5)V$  and  $I = (10 \pm 0.2)A$ . Find the % error in  $R$ ?

soln:

$$\% \text{ error in } V = 5\%$$

$$\% \text{ error in } I = 2\%$$

$$\therefore \% \text{ in } R = \underline{\underline{5\% + 2\% = 7\%}}$$

(2) Find the relative error in  $z$ , if  $z = A^4 B^3 / CD^{3/2}$  ?

relative error in  $Z$  is

$$\frac{\Delta Z}{Z} = 4\left(\frac{\Delta A}{A}\right) + \left(\frac{1}{3}\right)\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \left(\frac{3}{2}\right)\left(\frac{\Delta D}{D}\right)$$

(3) The period of oscillation of a simple pendulum is  $T = 2\pi\sqrt{l/g}$ . Measured value of  $l$  is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of a pendulum is found to be 90 s. using a wrist watch of 1 s resolution. what is the accuracy in the determination of  $g$ ?

given  $l = 20 \text{ cm}$

$$\Delta l = 1 \text{ mm} = 0.1 \text{ cm}$$

$$T = 90 \text{ s}$$

$$\Delta T = 1 \text{ s}$$

$$T = 2\pi\sqrt{l/g}, \quad T^2 = 4\pi^2 \frac{l}{g}$$

$$g = 4\pi^2 \frac{l}{T^2}$$

$$\frac{\Delta g}{g} = \left(\frac{\Delta l}{l}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$= \frac{0.1}{20} + 2 \times \frac{1}{90}$$

$$= \frac{9 + 40}{1800} = \frac{49}{1800}$$

$$\frac{\Delta g}{g} = 0.027$$

$$\frac{\Delta g}{g} \approx 0.03 //$$

$$\% \text{ error} = 0.03 \times 100$$

$$= 3\%$$

\* The physical quantity 'P' is related to a, b, c and d as follows.

$$P = \frac{a^3 b^2}{\sqrt{c} d}$$

The percentage error in measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively what is the % error in the quantity P.

$$\frac{\Delta P}{P} \times 100 = 3\left(\frac{\Delta a}{a}\right) \times 100 + 2\left(\frac{\Delta b}{b}\right) \times 100 + \frac{1}{2}\left(\frac{\Delta c}{c}\right) \times 100 + \frac{\Delta d}{d} \times 100$$

$$\frac{\Delta P}{P} \times 100 = 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\%$$

$$= 3 + 6 + 2 + 2$$

$$= 13\%$$

$$\therefore \frac{\Delta P \times 100}{P} = 13\%$$

$$\therefore \% \text{ error in } P = 13\%$$

\* The diameter of a wire measured by a screw gauge was found to be 1.328, 1.330, 1.325, 1.326, and 1.334, 1.336 cm. calculate the absolute errors, relative errors and % error.

(\*) True value.

$$\bar{a} =$$



$$\bar{a} = \frac{1.328 + 1.330 + 1.325 + 1.326 + 1.334 + 1.336}{6}$$

$$= \frac{7.979}{6} = \underline{\underline{1.3298}}$$

$$\bar{D} = \underline{\underline{1.330}}$$

(ii) Absolute errors ( $\Delta a$ )

$$1.328 - 1.330 = 0.002$$

$$1.330 - 1.330 = 0.00$$

$$1.325 - 1.330 = 0.005$$

$$1.326 - 1.330 = 0.004$$

$$1.334 - 1.330 = 0.004$$

$$1.336 - 1.330 = 0.006$$

(iii) Mean Absolute error.

$$\Delta \bar{a} = \frac{0.002 + 0.00 + 0.005 + 0.004 + 0.004 + 0.006}{6}$$

$$\Delta \bar{a} = \frac{0.021}{6} = \underline{\underline{0.0035}} = 0.004$$

(iv) Relative error

$$= \frac{\Delta \bar{a}}{\bar{a}} = \frac{0.004}{1.33} = \underline{\underline{0.003}}$$

(v) Percentage error

$$\begin{aligned} \text{error} &= \frac{\Delta \bar{a}}{\bar{a}} \times 100 \\ &= 0.003 \times 100 \\ &= \underline{\underline{0.3\%}} \end{aligned}$$

$$\therefore D = \bar{D} \pm \Delta \bar{D}$$

$$= 1.330 \pm 0.004$$

$$D = \underline{\underline{1.330 \pm 0.004 \text{ cm}}}$$

## Numericals for practice

1, Find the dimensions of  $a/b$  in the relation

$$F = a\sqrt{x} + bt^2$$

where  $F \rightarrow$  force

$x \rightarrow$  distance

$t \rightarrow$  time

$$\text{ans: } [L^{-1/2}, T^2]$$

2, Find the dimensions of  $a$  &  $b$

$$E = \frac{bx^2}{at}$$

$E \rightarrow$  Energy

$x \rightarrow$  distance

$$\text{ans: } [M^{-1} L^2 T^{-1}] \quad t \rightarrow \text{time}$$

3, Find the dimensions of  $a/b$ .

$$P = \frac{a-t^2}{bx}$$

$P \rightarrow$  pressure,  $x \rightarrow$  distance  
 $t \rightarrow$  time.

$$\text{ans: } [MT^{-2}]$$