Chapter–3

Motion in a Plane

CBSE CLASS XI NOTES

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MOTION IN A PLANE

Sunil

Physical Quantities in general may be divided into two main classes.

Scalar Quantities

Quantities having only magnitude, no direction

Eg; current, temperature, speed, distance, work

Vector Quantities

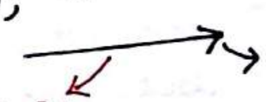
Quantities having both magnitude and direction.

Eg; Force, velocity, displacement, acceleration, current density. etc.

Representation of a vector.

A letter with overhead arrow

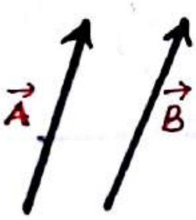
eg; $\vec{A}$

or → 

head (direction)

Tail (magnitude)

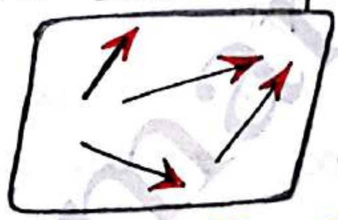
Equal vectors



Two vectors are said to be equal when they are equal (identical) in magnitude and direction

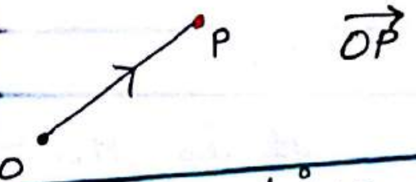
Coplanar Vector

The vectors in the same plane.



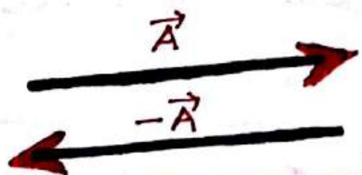
Position vector

A vector representing the position of a point with respect to an arbitrary origin is called position vector.



Negative vector

Vector having same magnitude but opposite to the direction of a given vector.



Modulus of a vector

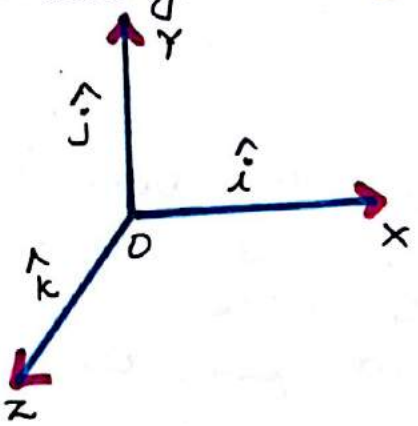
It gives the magnitude of a vector. $|\vec{A}| = A$

Unit Vector.

It is the vector with unit magnitude and gives the direction of the given vector.

$$\hat{A} = \frac{\vec{A}}{|A|}$$

The unit vectors of x, y, z are $\hat{i}, \hat{j}, \hat{k}$ respectively.



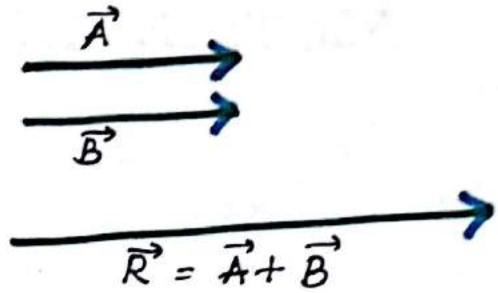
Zero vector

It is that vector which has zero magnitude and an arbitory direction. $\vec{O}$

eg; acceleration of a moving body with uniform velocity.

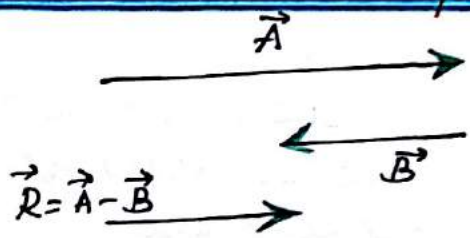
Addition of Vectors.

Two vectors acting in the same direction.



* If two vectors are acting in same direction, the resultant vector magnitude is equal to the sum of magnitudes of individual vectors and the direction is same as that of individual vectors.

Substraction of Vectors



$$\vec{R} = [\vec{A} + (-\vec{B})]$$

$$\vec{R} = \vec{A} - \vec{B}$$

* If two vectors acting in opposite direction, the magnitude of the resultant vector is the difference between the two vectors.
* The direction of the resultant vector is the direction of the bigger vector.

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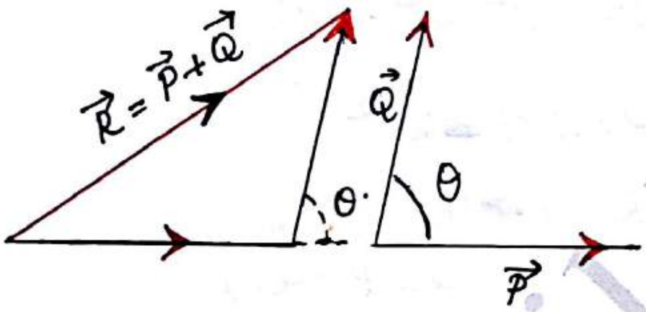
Inclination of vectors.

* when two vectors are inclined at an angle 'θ', they can be added by using triangle law of vectors or parallelogram law of vectors.

Triangle law of vectors.

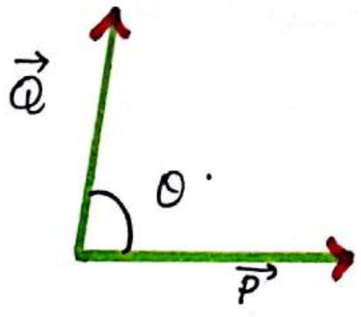


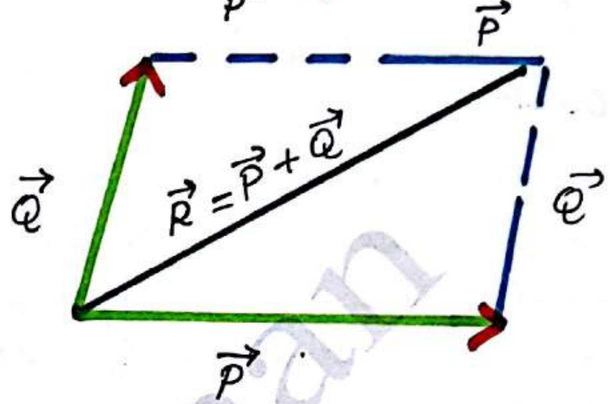
statement

If two sides of trainsgle represents the two vectors in magnitude and direction taken in the same order, then the third side of the trainsgle taken in the opposite order gives the resultant vector which is equal to the sum of two vectors.

Parallelogram law of vectors

* state and prove

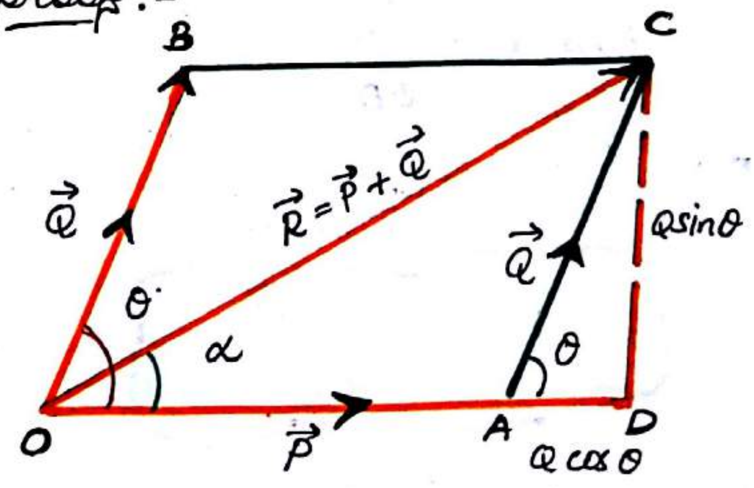




* If two vectors are represented by two sides of a parallelogram in magnitude and direction, then the diagonal passing through the point of intersection of two vectors gives the resultant, which is equal to the vector sum of two vectors.

Analytical method of vector Addition.

proof :-



* $\vec{OA}$ - represents $\vec{P}$.
* $\vec{OB}$ - represents $\vec{Q}$
* vector sum $\vec{R}$ is represented by the diagonal $\vec{OC}$ of the parallelogram OACB
* draw CD ⊥r to OA
* let 'θ' be the angle between the given two vectors '$\vec{P}$' and '$\vec{Q}$' and 'α' be the angle between the resultant $\vec{R}$ and $\vec{P}$

* $OC^2 = OD^2 + CD^2$

$$OC^2 = (OA + AD)^2 + CD^2$$

$$OC^2 = OA^2 + AD^2 + 2 \cdot OA \times AD + CD^2$$

$$OC^2 = OA^2 + AC^2 + 2 \cdot OA \times AD$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$AD^2 + CD^2 = AC^2$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

It gives the magnitude of the resultant.

Direction of $\vec{R}$

$$\tan\alpha = \frac{CD}{OD}$$

$$= \frac{CD}{OA + AD}$$

$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

special cases:-

* If $\vec{P}$ and $\vec{Q}$ are in the same direction

$$\theta = 0^\circ$$

$$\cos 0^\circ = 1$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ}$$

$$R = \sqrt{(P+Q)^2} \qquad R = P + Q$$

* If '$\vec{P}$' and $\vec{Q}$ are in opposite direction.

$$\theta = 180^\circ, \quad \cos 180^\circ = -1$$

$$\cos\theta = -1$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{P^2 + Q^2 - 2PQ}$$

$$R = \sqrt{(P-Q)^2}$$

$$R = P - Q$$

$$R = P - Q$$

* If $\vec{P}$ and $\vec{Q}$ are ⊥r to each other.

$$\theta = 90^\circ$$

$$\cos 90^\circ = 0$$

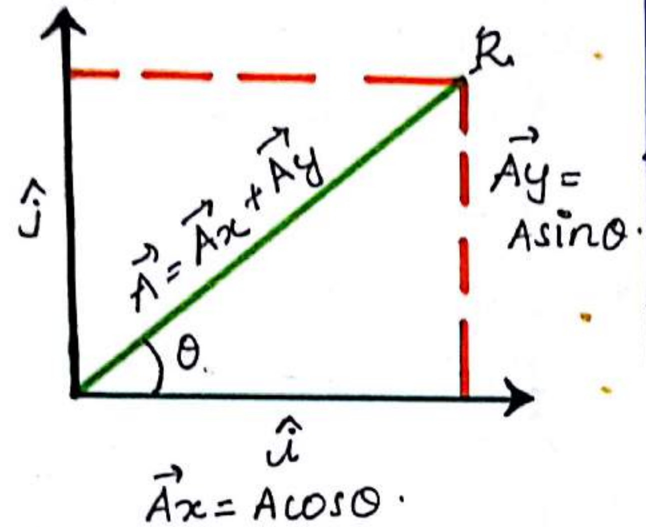
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$R = \sqrt{P^2 + Q^2}$$

$$R = \sqrt{P^2 + Q^2}$$

Resolution of a vector into 2 rectangular components in a plane?

Let us resolve a vector $\vec{A}$ into 2 components in the direction of 'x' and 'y' axis. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along OX and OY Respectively. Let OR represent $\vec{A}$.



* According to parallelogram law of vectors.

$$\vec{A} = \vec{Ax} + \vec{Ay}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

* $A_x = A\cos\theta$

  $A_y = A\sin\theta$

* magnitude of $\vec{A} = A$

$$A = \sqrt{A_x^2 + A_y^2}$$

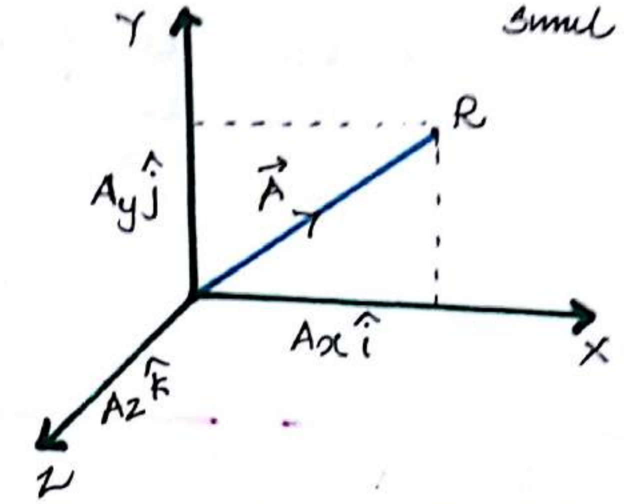
* Direction of $\vec{A}$

$$\boxed{\tan\theta = \frac{A_y}{A_x}}$$

* magnitude of vector A when rectangular components is 3 dimension

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_3\hat{k}$$



Laws of Vector Addition.

(1) commutative law

vector addition is commutative

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

(2) Associative law

vector addition is associative

$$\boxed{\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}}$$

(3) Distributive law

vector addition is distributive

$$m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$$

$$(m+n)\vec{A} = m\vec{A} + n\vec{A}$$

Multiplication of Vectors.

Multiplication of a vector $\vec{A}$ by a scalar number λ gives a vector whose magnitude is changed by a factor 'λ' but its direction is same as $\vec{A}$.

* Modulus of $\lambda\vec{A}$

$$|\lambda\vec{A}| = \lambda|\vec{A}| = \text{magnitude}$$

Substraction of Vectors

$$\vec{P}-\vec{Q} = \vec{P}+(-\vec{Q})$$

* magnitude of resultant

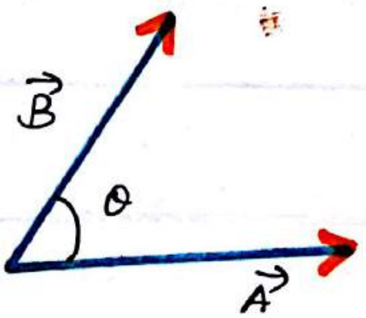
$$R=\sqrt{P^2+Q^2-2PQ\cos\theta}$$

* Direction of the resultant

$$\tan\theta = \frac{Q\sin\theta}{P-Q\cos\theta}$$

Scalar Product or Dot product

Scalar product can be defined as the product of the magnitude of two vectors and the cosine of the angle between the two vectors



$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

characteristics.

* scalar product is commulative

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

* If two vectors are at right angle $\theta = 90°$

$$\vec{A}\cdot\vec{B} = AB\cos 90°$$

$$\vec{A}\cdot\vec{B} = 0 \text{ if } \vec{A} \perp \vec{B}$$

* If two vectors are parallel $\theta = 0°$

$$\vec{A}\cdot\vec{B} = AB\cos 0°$$

$$\vec{A}\cdot\vec{B} = AB \text{ if } \vec{A} \parallel \vec{B}$$

* $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

since $\theta = 0°$.

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

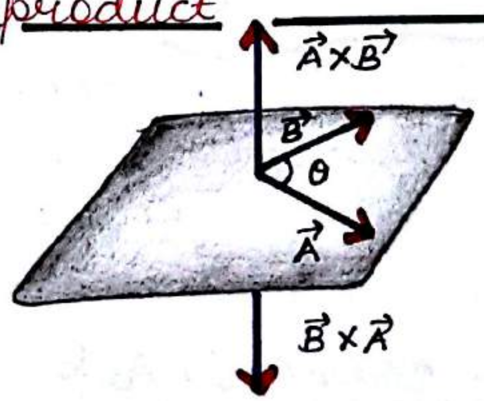
since $\theta = 90°$

* In cartesian co-ordinate

$$\vec{A}\cdot\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$\therefore \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

vector product or cross product



The vector products of two vectors $\vec{A}$ and $\vec{B}$ is written as $\vec{A} \times \vec{B}$ and is a vector whose magnitude is equal to the product of the magnitudes of the given vectors and the sign of the angle between them.

$$\vec{A} \times \vec{B} = AB \sin\theta$$

<u>characteristics</u>

* $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
  but $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$
* The vector product of two collinear vectors is zero. $\vec{A} \times \vec{B} = 0$ if $(\theta = 0 \text{ or } \pi)$
* $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
  $\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{k} = \hat{i}$; $\hat{k} \times \hat{i} = \hat{j}$
* If $\vec{A} \perp \vec{B}$ $\vec{A} \times \vec{B} = AB\hat{n}$
  $(\theta = 90°)$
* In cartesian co-ordinates

$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$i(A_yB_z - A_zB_y) - j(A_xB_z - A_zB_x) + k(A_xB_y - B_xA_y)$$

PROJECTILE MOTION

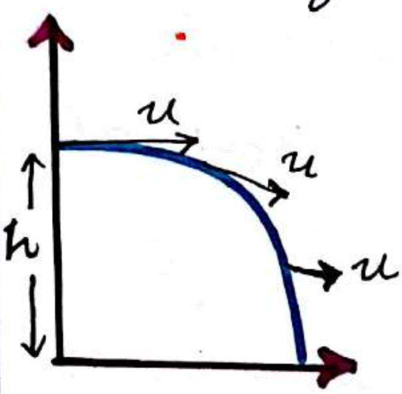
projectile is the name given to a body which after having been given an initial velocity is allowed to move under the influence of gravity alone.

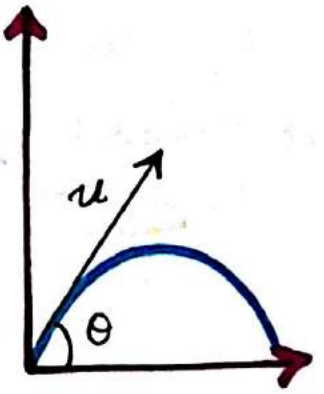
Eg; Javeline.

* <u>methods of projecting a projectile</u>

(1) projecting a body horizontally from a certain height.

(2) Body can be projected at an angle with the horizontal.





Body projected at an angle with the horizontal.

* consider a particle projected with an initial velocity 'u' at an angle 'θ' from the horizontal.

* **velocity resolved into two components**

Horizontal velocity $u_x = u\cos\theta$.

vertical velocity $u_y = u\sin\theta$

* **Displacement**

Horizontal displacement in time t

$$x = u_x t$$

$$\boxed{x = u\cos\theta\, t}$$

* **vertical displacement**

$$s = ut + \frac{1}{2}at^2$$

$$y = u_y t + \frac{1}{2}(-g)t^2$$

$$\boxed{y = u\sin\theta\, t - \frac{1}{2}gt^2}$$

* projectile is subjected to two velocities.

a) uniform horizontal velocity.

$$\boxed{V_x = u_x = u\cos\theta}$$

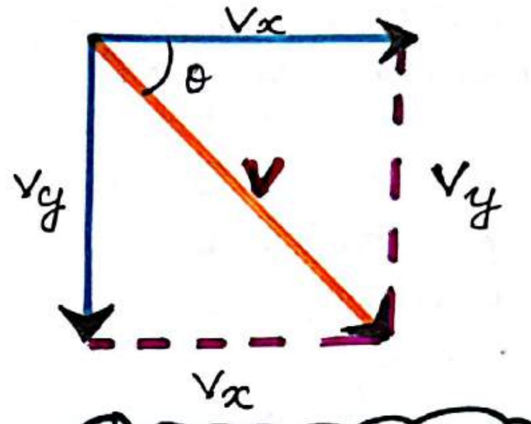
b) non uniform vertical velocity

$$v = u + at$$

$$v_y = u_y + (-g)t$$

$$\boxed{v_y = u\sin\theta - gt}$$

* Resultant velocity.



$$\boxed{V = \sqrt{V_x^2 + V_y^2}}$$

* Direction of resultant velocity

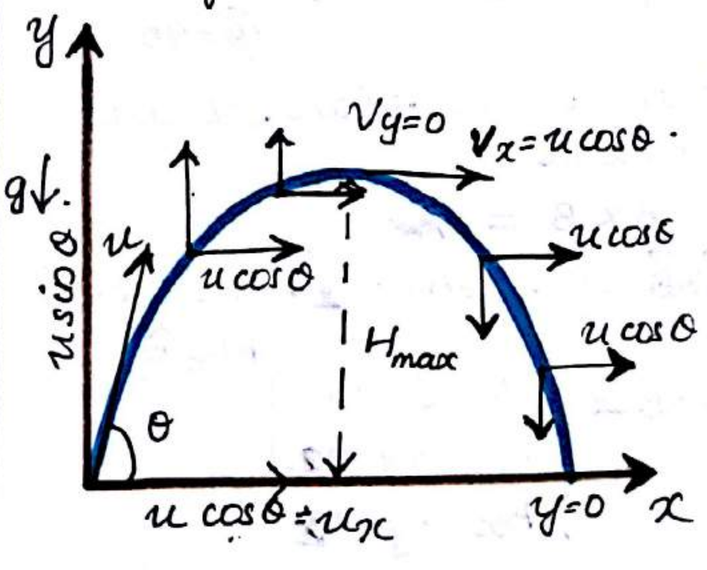
$$\boxed{\tan\theta = \frac{V_y}{V_x}}$$

(a) Maximum height (hmax).

(b) Time of flight (t_f)

(c) Horizontal Range (R)

(d) Maximum horizontal range (Rmax)



[figure]

(a) Maximum height reached (h_{max})

At maximum height, vertical component of velocity is zero [$v_y = 0$]

$$v^2 = u^2 + 2aS$$

$$v_y^2 = u_y^2 + 2(-g)\,h_{max}$$

$$v_y^2 = u_y^2 - 2gh_{max}$$

$$0 = u_y^2 - 2gh_{max} \quad \text{.. (1)}$$

initial vertical velocity

$$u_y = u\sin\theta$$

$$(1) \Rightarrow 0 = (u\sin\theta)^2 - 2gh_{max}$$

$$0 = u^2\sin^2\theta - 2gh_{max}$$

$$2gh_{max} = u^2\sin^2\theta$$

$$\boxed{\therefore h_{max} = \frac{u^2\sin^2\theta}{2g}}$$

(b) Time of flight (t_f)

It is the time taken by the projectile to return to the ground.

displacement $S = 0$

Initial vertical velocity $u_y = u\sin\theta$

$a = -g$

time of flight $t = t_f$

$$S = ut + \frac{1}{2}at^2$$

$$0 = u_y t_f + \frac{1}{2}(-g)t_f^2$$

$$0 = u\sin\theta\, t_f - \frac{1}{2}g t_f^2$$

$$0 = u\sin\theta - \frac{1}{2}g t_f$$

$$\therefore \frac{1}{2}g t_f = u\sin\theta$$

$$\boxed{\therefore t_f = \frac{2u\sin\theta}{g}}$$

(c) Horizontal Range [R]

It is the horizontal distance covered by the body between the point of projection and the point where the projectile reaches the ground.

$$x = ut$$

$$R = u_x t_f$$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

Maximum horizontal range (R_{max})

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

* Range will be maximum when $\sin 2\theta = 1$

$\therefore 2\theta = 90^\circ$

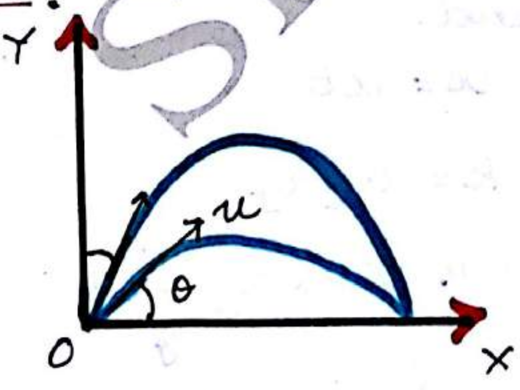
$\theta = 45^\circ$

$$\boxed{R_{max} = \frac{u^2 \sin 90}{g}}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

* when a body is projected with 45° with the horizontal, it can cover the maximum horizontal distance.

S.T horizontal range is same for two angles?



when a body is projected with a velocity 'u' making an angle 'θ' with the horizontal, the horizontal range

$$\boxed{R_1 = \frac{u^2 \sin 2\theta}{g}} \quad \ldots (1)$$

when it is projected with velocity 'u' at an angle $(90-\theta)$, then horizontal range is.

$$R_2 = \frac{u^2 \sin 2[90-\theta]}{g}$$

$$R_2 = \frac{u^2 \sin (180-2\theta)}{g}$$

$$\boxed{R_2 = \frac{u^2 \sin 2\theta}{g}} \quad \ldots (2)$$

$\therefore (1) = (2)$

$$\boxed{R_1 = R_2}$$

* Range is same for θ and $[90-\theta]$

S.T the path of a projectile is a parabola?

Let 'u' be the initial velocity of projection and 'θ' the angle of projection.

Initial vertical component $= u \sin\theta$

Horizontal component $= u \cos\theta$

after 't' seconds, the vertical displacement,

$y = u\sin\theta \times t - (\frac{1}{2}) g t^2$ ... (1)

The horizontal displacement

$x = u\cos\theta \times t$

$\therefore t = \frac{x}{u\cos\theta}$ ... (2)

(2) in (1)

$\Rightarrow y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$

$\Rightarrow y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$

* This is of the form $y = bx - cx^2$ and is the equation of a parabola.

CIRCULAR MOTION

objects moving in circular path are said to have circular motion.

Angular displacement (θ)

The angle swept over by the radius vector in a given interval of time is

called angular displacement."

* measured in radian
* vector Quantity

Angular velocity (ω)

It is the time rate of change of angular displacement.

$\omega = \frac{\Delta\theta}{\Delta t}$ $\Delta \rightarrow$ delta

* measured in $\frac{\text{radians}}{\text{seconds}}$ [rad/s or rad s^{-1}]
* vector Quantity
* same direction of θ.

Instantaneous Angular velocity.

It is the angular velocity of the particle at an instant

$\omega = \lim_{\Delta t \to 0} \left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt}$

Relation b/w linear velocity and ω

consider a particle moving round a circle of centre 'O' and radius 'r',

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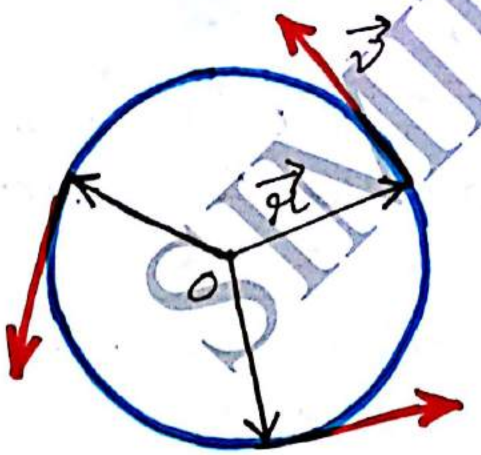
CIRCULAR MOTION

* **Define uniform circular motion give some examples.**

If a particle moves along a circular path with a constant speed, then its motion is said to be a uniform circular motion

eg. motion of the tip of
* the second hand of a clock.
* Rim of a wheel.
* uniform circular motion is an accelerated motion.

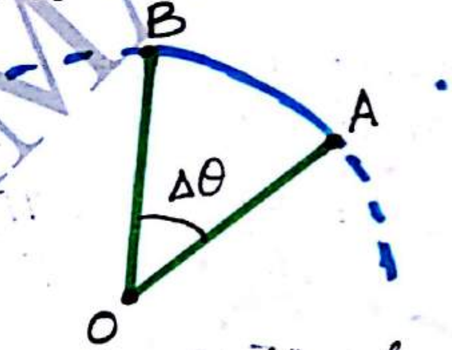


* Direction of velocity changes at every point
* speed of the body remains the same.

* **Define the terms:**
(1) Angular displacement
(2) Angular velocity
(3) Time period
(4) Frequency
in connection with a circular motion.

(1) Angular displacement

The angle swept out by the radius vector in a given interval of time is called angular displacement.



$\Delta\theta \rightarrow$ Angular displacement.

* SI unit radian

(2) Angular velocity:

The time rate of change of Angular displacement of a particle is called angular velocity.

Angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$

* S.I unit $rad \cdot s^{-1}$ or $\frac{rad}{s}$

(3) Time period

It is the time taken by the particle to describe one complete revolution. T, unit $\rightarrow$ S

(4) Frequency

It is defined as the number of revolutions made by an object in one second.

$$\nu = \frac{1}{T}$$

$\nu \rightarrow$ frequency (nu)

* Relation between ω, T and ν

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

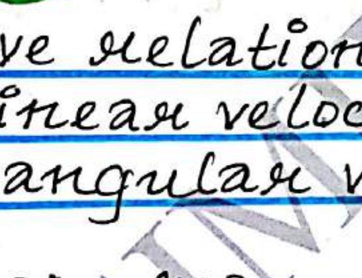
$$\omega = 2\pi\nu \quad \because \frac{1}{T} = \nu$$

* Derive relation between linear velocity and angular velocity

OR

S.T $v = r\omega$

suppose a particle moves from A to B in time Δt covering a distance ΔS along arc AB.



$\therefore$ angular displacement $\quad \because \left(\Delta\theta = \frac{\text{Arc}}{\text{radius}}\right)$

$$\Delta\theta = \frac{\Delta S}{r}$$

$\div$ by Δt

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta S}{\Delta t} \quad \dots (1)$$

taking limit $\Delta t \rightarrow 0$ on both sides

* $\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega \quad \dots (2)$

* $\frac{\Delta S}{\Delta t} = \frac{dS}{dt} = v \quad \dots (3)$

(2) & (3) in 1

$$\omega = \frac{1}{r} v$$

$$\therefore v = r\omega$$

linear velocity = angular velocity × radius

* Angular acceleration (α)

It is the time rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt}$$

* relation between linear and angular acceleration.

OR S.T $a = r\alpha$

we know

$V = r\omega$

differentiating

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$\therefore$ $a = r\alpha$ $\because$ $\frac{d\omega}{dt} = \alpha$, $\frac{dv}{dt} = a$

imp→

* Define centripetal acceleration. Derive an expression for the centripetal acceleration.

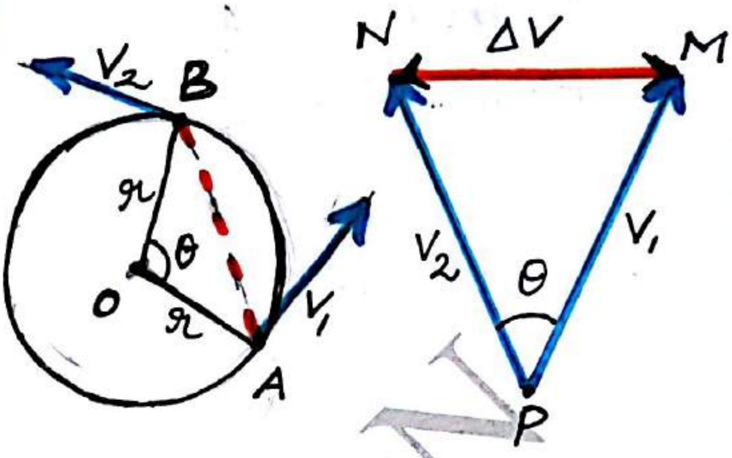
The acceleration acting on an object undergoing uniform circular motion is called centripetal acceleration.

consider a particle executing uniform circular motion around a circle of radius r.

V_1 → velocity at position A

V_2 → velocity at position B.

Δt → time interval



* Draw MN, It is the change in velocity

Δgle LMN and Δgle OAB are similar

$$\therefore \frac{AB}{OA} = \frac{MN}{PM}$$

$$\frac{AB}{r} = \frac{\Delta V}{V} \quad \ldots (1)$$

but $AB = V\Delta t$

$$\therefore \frac{V\Delta t}{r} = \frac{\Delta V}{V}$$

$$\frac{V^2}{r} = \frac{\Delta V}{\Delta t}$$

$$\frac{V^2}{r} = a$$

$\because$ $\frac{\Delta V}{\Delta t} = a$, $V = r\omega$

$$a = \frac{V^2}{r}$$

$$a = \frac{r^2\omega^2}{r} = r\omega^2$$

$$\therefore a = r\omega^2$$

short cut

$$F = ma \quad \text{--- (1)}$$

$$F = \frac{mv^2}{r} \quad \text{--- (2)}$$

$$\therefore ma = \frac{mv^2}{r}$$

$$a = \frac{v^2}{r}. \qquad v = r\omega$$

$$\therefore a = \frac{r^2\omega^2}{r} = r\omega^2$$

$$\therefore \boxed{a = r\omega^2}$$