Guidlines to NCERT Exercises

4.1. *State for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.*

Ans. Scalars : volume, mass, speed, density, number of moles and angular frequency.

Vectors : Acceleration, velocity, displacement and angular velocity.

4.2. *Pick out the two scalar quantities in the following list : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.*

Solution. Work and current.

4.3. *Pick out the only vector quantity in the following list : Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.*

Ans. Impulse.

4.4. *State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :*

(*a*) *Adding any two scalars.*

(*b*) *Adding a scalar to a vector of the same dimensions.*

(*c*) *Multiplying any vector by any scalar.*

(*d*) *Multiplying any two scalars.*

(*e*) *Adding any two vectors.*

(*f*) *Adding a component of a vector to the same vector.*

Ans. (*a*) **No.** Only two such scalars can be added which represent the same physical quantity.

(*b*) **No.** A scalar cannot be added to a vector even of same dimensions because a vector has a direction while a scalar has no direction *e.g.*, speed cannot be added to velocity.

(*c*) **Yes.** We can multiply any vector by a scalar. For example, when mass (scalar) is multiplied with acceleration (vector), we get force (vector) *i.e.*, $\vec{F} = m\vec{a}$.

(*d*) **Yes.** We can multiply any two scalars. When we multiply power (scalar) with time (scalar), we get work done (scalar) *i.e.*, $W = Pt$.

(*e*) **No.** Only two vectors of same nature can be added by using the law of vector addition.

(*f*) **No.** A component of a vector can be added to the same vector only by using the law of vector addition. So the addition of a component of a vector to the same vector is not a meaningful algebraic operation.

4.5. *Read each statement below carefully and state with reasons, if it is true or false :*

(*a*) *The magnitude of a vector is always a scalar.*

(*b*) *Each component of a vector is always a scalar.*

(*c*) *The total path length is always equal to the magnitude of the displacement vector of a particle.*

(*d*) *The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.*

(*e*) *Three vectors not lying in a plane can never add up to give a null vector.*

Ans. (*a*) **True.** The magnitude of a vector is a pure number and has no direction.

(*b*) **False.** Each component of a vector is also a vector.

(*c*) **False.** The displacement depends only on the end points while the path length depends on the actual path. The two quantities are equal only if the direction of motion of the object does not change. In all other cases, path length is greater than the magnitude of displacement.

(*d*) **True.** This is because the total path length is either greater than or equal to the magnitude of displacement over the same interval of time.

(*e*) **True.** This is because the resultant of two vectors will not lie in the plane of third vector and hence cannot cancel its effect to give null vector.

4.6. *Establish the following vector inequalities geometrically or otherwise :*

(*a*) $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$ (*b*) $|\vec{a}+\vec{b}| \ge |\vec{a}|-|\vec{b}|$

(*c*) $|\vec{a}-\vec{b}| \le |\vec{a}|+|\vec{b}|$ (*d*) $|\vec{a}-\vec{b}| \ge |\vec{a}|-|\vec{b}|$

When does the equality sign above apply ?

Ans. (*a*) If θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta}$$

Now $|\vec{a}+\vec{b}|$ will be maximum when

$$\cos\theta = 1 \text{ or } \theta = 0$$

$$\therefore \quad |\vec{a}+\vec{b}|_{max} = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos 0^\circ}$$

$$= \sqrt{(|\vec{a}|+|\vec{b}|)^2} = |\vec{a}|+|\vec{b}|$$

Hence $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

The equality sign is applicable when $\theta = 0°$ *i.e.*, when $\vec{a}$ and $\vec{b}$ are in the same direction.

(*b*) Again

$$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta}$$

The value of $|\vec{a}+\vec{b}|$ will be minimum when

$$\cos\theta = -1$$

or $$\theta = 180°$$

$$\therefore \quad |\vec{a}+\vec{b}|_{min} = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos 180°}$$

$$= \sqrt{(|\vec{a}|-|\vec{b}|)^2} = |\vec{a}|-|\vec{b}|$$

Hence $|\vec{a}+\vec{b}| \ge |\vec{a}|-|\vec{b}|$

The equality sign is applicable when $\theta = 180°$ *i.e.*, when $\vec{a}$ and $\vec{b}$ are in opposite directions.

(*c*) If θ is the angle between $\vec{a}$ and $\vec{b}$, then the angle between $\vec{a}$ and $-\vec{b}$ will be $(180°-\theta)$, as shown in Fig. 4.100.

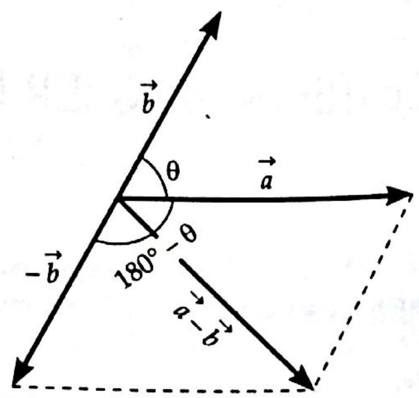

Fig. 4.100

$$\therefore \quad |\vec{a}-\vec{b}| = |\vec{a}+(-\vec{b})|$$

$$= \sqrt{|\vec{a}|^2+|-\vec{b}|^2+2|\vec{a}||-\vec{b}|\cos(180°-\theta)}$$

$$= \sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta}$$

$$[\because |-\vec{b}| = |\vec{b}|, \cos(180°-\theta) = -\cos\theta]$$

$|\vec{a} - \vec{b}|$ will be maximum when $\cos\theta = -1$ or $\theta = 180°$

$$\therefore \quad |\vec{a} - \vec{b}|_{max} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 180°}$$

$$= \sqrt{(|\vec{a}| + |\vec{b}|)^2} = |\vec{a}| + |\vec{b}|$$

Hence $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$

The equality sign is applicable when $\theta = 180°$.

(*d*) $|\vec{a} - \vec{b}|$ will be minimum when

$\cos\theta = 1$ or $\theta = 0°$

$$\therefore \quad |\vec{a} - \vec{b}|_{min} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 0°}$$

$$= \sqrt{(|\vec{a}| - |\vec{b}|)^2} = |\vec{a}| - |\vec{b}|$$

Hence $|\vec{a} - \vec{b}| \ge |\vec{a}| - |\vec{b}|$

The equality sign is applicable when $\theta = 0°$.

4.7. *Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, which of the following statements are correct :*

(*a*) *$\vec{a}, \vec{b}, \vec{c}$, and $\vec{d}$ must each be a null vector,*

(*b*) *The magnitude of $(\vec{a} + \vec{c})$ equals the magnitude of $(\vec{b} + \vec{d})$,*

(*c*) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$, and $\vec{d}$,

(*d*) $\vec{b} + \vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear ?

Ans. (*a*) $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ need not each be a null vector. The resultant of four non-zero vectors can be a null vector in many ways *e.g.*, the resultant of any three vectors may be equal to the magnitude of fourth vector but has the opposite direction. Hence the statement $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ must each be a null vector, is **not correct**.

(*b*) Because $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, hence $\vec{a} + \vec{c} = -(\vec{b} + \vec{d})$ *i.e.*, the magnitude of $(\vec{a} + \vec{c})$ is equal to the magnitude of $(\vec{b} + \vec{d})$ but their directions are opposite. Hence the given statement is **correct**.

(*c*) Because, $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$ or $\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$. Hence, magnitude of vector $\vec{a}$ is equal to magnitude of vector $(\vec{b} + \vec{c} + \vec{d})$. The sum of the magnitudes of vectors $\vec{b}, \vec{c}$ and $\vec{d}$ may be greater than or equal to that of vector $\vec{a}$ (or vector $\vec{b} + \vec{c} + \vec{d}$). Hence the statement that the magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}$, $\vec{c}$ and $\vec{d}$ is **correct**.

(*d*) Because $\vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$, hence $(\vec{b} + \vec{c}) + \vec{a} + \vec{d} = 0$. The resultant sum of three vectors $\vec{b} + \vec{c}$, $\vec{a}$ and $\vec{d}$ can be zero only if $\vec{b} + \vec{c}$ is in plane of $\vec{a}$ and $\vec{d}$. In case $\vec{a}$ and $\vec{d}$ are collinear, $\vec{b} + \vec{c}$ must be in line of $\vec{a}$ and $\vec{d}$. Hence the given statement is **correct**.

4.8. *Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.101. What is the magnitude of the displacement vector for each ? For which girl is this equal to the actual length of path skated ?*

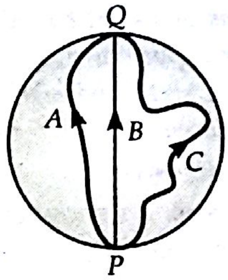

Fig. 4.101

Solution. Displacement of each girl = $\vec{PQ}$

Magnitude of displacement vector for each girl

$= |\vec{PQ}| = 2 \times \text{radius} = 2 \times 200 = $ **400 m.**

For girl *B*, the magnitude of displacement vector = actual length of path.

4.9. *A cyclist travels from centre O of a circular park of radius* 1 *km and reaches point P. After cycling along* 1/4 *th of the circumference along PQ, he returns to the centre of the park along QO. If the total time taken is* 10 *minutes, calculate (i) net displacement (ii) average velocity and (ii) average speed of the cyclist.*

Ans. (*i*) Net displacement is zero as both initial and final positions are same.

(*ii*) Average velocity $= \dfrac{\text{displacement}}{\text{time taken}}$

As displacement is zero, average velocity of the cyclist is also **zero.**

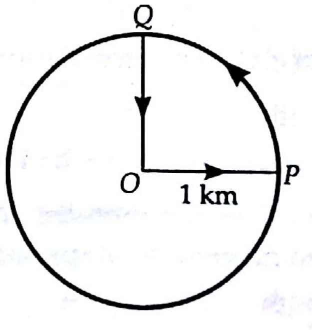

Fig. 4.102

(*iii*) Total distance covered

$$= OP + \text{Arc } PQ + OQ = r + \frac{2\pi r}{4} + r$$

$$= 1 + \frac{2 \times 22 \times 1}{7 \times 4} + 1 = \frac{25}{7} \text{ km}$$

$$\text{Time taken } = 10 \text{ min} = \frac{1}{6} \text{ h}$$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

$$= \frac{25/7 \text{ km}}{1/6 \text{ h}} = \mathbf{21.43 \text{ kmh}^{-1}}.$$

4.10. *On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.*

Ans. As shown in Fig. 4.103, suppose motorist starts from point *A*.

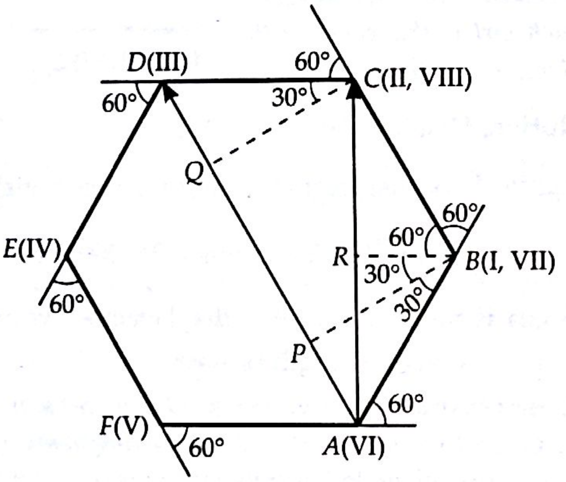

Fig. 4.103

Clearly, he will follow the hexagonal path *ABCDEFA*. The orders of the turns taken by him are indicated at the vertices of the hexagon.

(*i*) *At the third turn*, the motorist will be at *D*. The magnitude of displacement $\vec{AD}$ will be

$$= AP + PQ + QD$$

$$= AB \sin 30° + BC + CD \sin 30°$$

$$= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2} = 1000 \text{ m} = \mathbf{1 \text{ km}}.$$

The direction of $\vec{AD}$ is 60° left of the initial direction $\vec{AB}$.

Total path length

$$= AB + BC + CD = 500 \times 3 = 1500 \text{ m} = \mathbf{1.5 \text{ km}}.$$

(*ii*) *At the sixth turn*, the motorist comes back to the starting point *A*, so magnitude of displacement is *zero*.

Total path length

$$= AB + BC + CD + DE + EF + FA$$

$$= 500 \times 6 = 3000 \text{ m} = \mathbf{3 \text{ km}}.$$

(*iii*) *At the eighth turn*, the motorist will be at C. The magnitude of his displacement $\vec{AC}$ is

$$|\vec{AC}| = AR + RC = AB \sin 60° + BC \sin 60°$$

$$= 500 \times \frac{\sqrt{3}}{2} + 500 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} = \mathbf{866 \text{ m}}.$$

The direction of $\vec{AC}$ is 30° left of the initial direction $\vec{AB}$.

Total path length = $500 \times 8 = 4000$ m = **4 km**.

4.11. *A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (i) the average speed of the taxi and (ii) the magnitude of average velocity ? Are the two equal ?*

Solution. Magnitude of displacement = 10 km

Total path length = 23 km

$$\text{Time taken} = 28 \text{ min} = \frac{28}{60} \text{ h} = \frac{7}{15} \text{ h}$$

(*i*) $$\text{Average speed} = \frac{\text{Total path length}}{\text{Time taken}}$$

$$= \frac{23 \text{ km}}{\frac{7}{15} \text{ h}} = \mathbf{49.3 \text{ kmh}^{-1}}.$$

(*ii*) Magnitude of average velocity

$$= \frac{\text{Displacement}}{\text{Time taken}} = \frac{10 \text{ km}}{\frac{7}{15} \text{ h}} = \mathbf{21.43 \text{ kmh}^{-1}}.$$

Clearly, the average speed and magnitude of the average velocity are not equal. They will be equal only for straight path.

4.12. *Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the relative velocity of rain with respect to the woman ? What is the direction in which she should hold her umbrella to protect herself from the rain ?*

Solution. The situation is shown in Fig. 4.104.

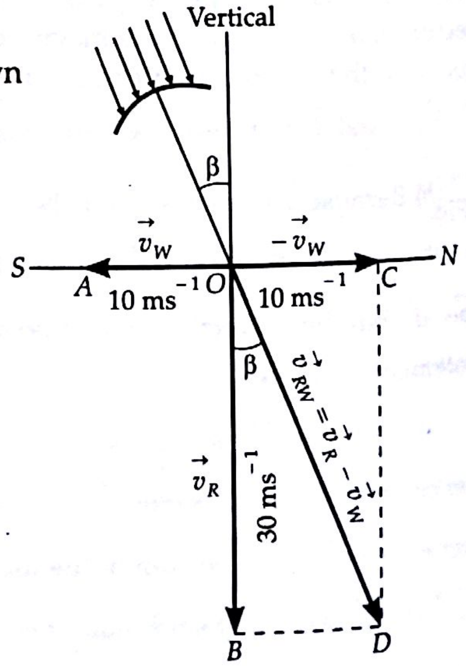

Fig. 4.104

Here

$\vec{OA} = \vec{v}_W$ = velocity of woman cyclist

= 10 ms^{-1}, due south

$\vec{OB} = \vec{v}_R$ = velocity of rain

= 30 ms^{-1}, vertically downward

$\vec{OC} = -\vec{v}_W$

= Opposite velocity of the woman cyclist.

$\vec{OD} = \vec{v}_R + (-\vec{v}_W) = \vec{v}_R - \vec{v}_W = \vec{v}_{RW}$

= Velocity of rain relative to woman cyclist

$v_{RW} = OD = \sqrt{OC^2 + OB^2} = \sqrt{10^2 + 30^2}$

$= 10\sqrt{10} = \mathbf{31.6\ ms^{-1}}$

If OD makes angle β with the vertical, then

$$\tan\beta = \frac{BD}{OB} = \frac{OC}{OB} = \frac{10}{30} = 0.3333 \quad \text{or} \quad \beta = \mathbf{18°26'}.$$

The woman should hold her umbrella at 18°26′ with the vertical in the direction of her motion *i.e.*, towards south.

4.13. *A man can swim with a speed of 4 kmh^{-1} in still water. How long does he take to cross the river 1 km wide, if the river flows steadily at 3 kmh^{-1} and he makes his strokes normal to the river current ? How far from the river does he go, when he reaches the other bank ?*

Solution. In Fig. 4.105, $\vec{v}_M$ and $\vec{v}_R$ represent the velocities of man and river. Clearly $\vec{v}$ is the resultant of these velocities. If the man begins to swim along AB, he will be deflected to the path AC by the flowing river.

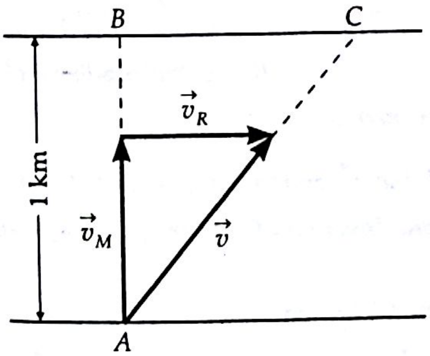

Fig. 4.105

Time taken to cover distance AC with velocity $\vec{v}$ will be same as the time taken to cover distance AB with velocity $\vec{v}_M$.

∴ Time taken by the man to cross the river is

$$t = \frac{AB}{v_M} = \frac{1\text{ km}}{4\text{ km h}^{-1}} = \frac{1}{4}\text{ h} = \mathbf{15\ min.}$$

Distance through which the man goes down the river is

$$BC = v_R \times t = 3\text{ kmh}^{-1} \times \frac{1}{4}\text{ h} = \mathbf{0.75\ km.}$$

4.14. *In a harbour, wind is blowing at the speed of 72 kmh^{-1} and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 kmh^{-1} to the north, what is the direction of the flag on the mast of the boat ?*

Solution. When the boat is stationary, the flag flutters along N-E direction. This shows that velocity of the wind is along N-E direction. When the boat moves, the flag flutters along the direction of relative velocity of wind w.r.t the boat. Thus $\vec{v}_W = \vec{OA}$ = wind velocity = 72 kmh^{-1}, due N-E direction

Boat velocity = $\vec{v}_B = \vec{OB}$ = 51 kmh^{-1}, due north

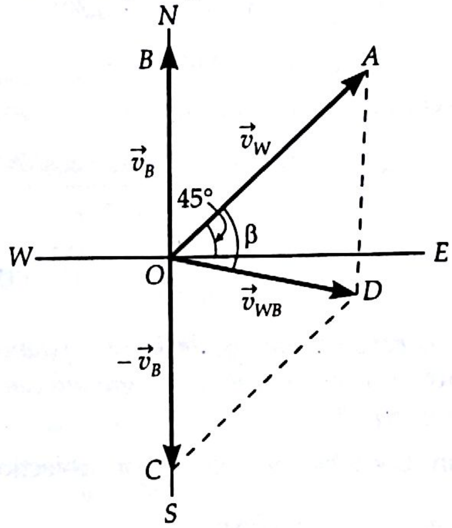

Fig. 4.106

Relative velocity of wind w.r.t. boat is given by

$$\vec{v}_{WB} = \vec{v}_W - \vec{v}_B = \vec{v}_W + (-\vec{v}_B)$$

$$= \vec{OA} + \vec{OC} = \vec{OD}$$

Clearly, the flag will flutter in the direction of $\vec{OD}$ on the mast of the moving boat.

Angle between $\vec{v}_W$ and $-\vec{v}_B$, $\theta = 45° + 90° = 135°$

If $\vec{v}_{WB}$ makes angle β with $\vec{v}_W$, then

$$\tan\beta = \frac{v_B \sin\theta}{v_W + v_B\cos\theta} = \frac{51\sin 135°}{72 + 51\cos 135°}$$

$$= \frac{51\sin 45°}{72 + 51(-\cos 45°)} = \frac{51 \times \frac{1}{\sqrt{2}}}{72 - 51 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\sqrt{2} - 51} = 1.0037$$

or $\beta = 45.01°$

Angle w.r.t east direction = 45.01° − 45° = 0.01°

Hence the flag will flutter almost in the east direction.

4.15. *The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ?*

Solution. Here $H = 25$ m, $u = 40$ ms^{-1}

If the ball is thrown at an angle θ with the horizontal, then maximum height of flight,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\therefore \quad 25 = \frac{(40)^2 \sin^2 \theta}{2 \times 9.8}$$

or $$\sin^2 \theta = \frac{25 \times 2 \times 9.8}{(40)^2} = 0.306$$

or $$\sin \theta = \sqrt{0.306} = 0.554$$

and $$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - 0.306} = \sqrt{0.694} = 0.833$$

The maximum horizontal distance is given by

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g}$$

$$= \frac{2 \times (40)^2 \times 0.554 \times 0.833}{9.8} = \mathbf{150.7\ m.}$$

4.16. *A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball ?* **[Delhi 98]**

Solution. Let u be the velocity of projection. Then

$$R_{max} = \frac{u^2}{g} = 100 \text{ m}$$

or $$u^2 = 100\,g \quad \text{or} \quad u = \sqrt{100\,g}$$

For upward throw of the ball, we have

$$u = \sqrt{100\,g}, \; v = 0, \; a = -g, \; s = ?$$

As $v^2 - u^2 = 2as$

$$\therefore \quad 0 - 100\,g = 2(-g)\,s$$

or $$s = \frac{-100\,g}{-2g} = \mathbf{50\ m.}$$

Thus the cricketer can throw the same ball to a height of 50 m.

4.17. *A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone ?*

Solution. Here $r = 80$ cm, $\nu = \frac{14}{25}$ rps

$$\therefore \quad \omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-1}$$

The acceleration of the stone is

$$a = r\omega^2 = 80 \times \left(\frac{88}{25}\right)^2 = \mathbf{991.2\ cm\ s^{-2}}.$$

This acceleration is directed along the radius of the circular path towards the centre of the circle.

4.18. *An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.*

Solution. Here $r = 1$ km $= 1000$ m

$$v = 900 \text{ kmh}^{-1} = \frac{900 \times 5}{18} = 250 \text{ ms}^{-1}$$

Centripetal acceleration,

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5 \text{ ms}^{-2}$$

$$\therefore \quad \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = \mathbf{6.38}$$

4.19. *Read each statement below carefully and state, with reasons, if it is true or false :*

(*a*) *The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.*

(*b*) *The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.*

(*c*) *The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.*

Solution. (*a*) **False.** The net acceleration of a particle in circular motion is towards the centre only if its speed is constant.

(*b*) **True.** A particle released at any point of its path will always move along the tangent to the path at that point.

(*c*) **True.** For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence the acceleration vector averaged over one complete cycle is a null vector.

4.20. *The position of a particle is given by*

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k} \text{ m}$$

where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle. (b) What is the magnitude and direction of velocity of the particle at t = 2 s ? **[Delhi 10]**

Solution. (*a*) Given :

$$\vec{r}(t) = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k} \text{ m}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k})$$

$$= \mathbf{3.0\,\hat{i} - 4.0\,t\,\hat{j}}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\,\hat{i} - 4.0t\,\hat{j}) = \mathbf{-4.0\,\hat{j}}.$$

(*b*) At $t = 2$ s, $\quad \vec{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$

The magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + (-8)^2} = \sqrt{73} = \mathbf{8.54\ ms^{-1}}.$$

The direction of velocity is given by

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{8}{3}\right)$$

$$= -\tan^{-1}(-2.6667)$$

$$\simeq -70° \text{ with } x\text{-axis.}$$

4.21. *A particle starts from the origin at $t = 0$ s with a velocity of $10.0\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})\ ms^{-2}$. (a) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at the time ? (b) What is the speed of the particle at the time ?*

Solution. (*a*) Here initial velocity,

$$\vec{v_0} = 10.0\hat{j}\ \text{ms}^{-1}$$

Acceleration,

$$\vec{a} = (8.0\hat{i} + 2.0\hat{j})\ \text{ms}^{-2}$$

The position of the particle at any instant t will be

$$\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}\ t^2$$

$$= 10.0\hat{j}\ t + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})t^2$$

or $x(t)\hat{i} + y(t)\hat{j} = 4.0t^2\hat{i} + (10.0t + 1.0t^2)\hat{j}$

$\therefore$ $x(t) = 4.0t^2$

$y(t) = 10.0t + 1.0t^2$

Given $x(t) = 16$ m, $t = ?$

$4.0t^2 = 16$

$\Rightarrow$ $t = 2$ **s.**

At $t = 2$ s, $y = 10.0 \times 2 + 1.0 \times 2^2 = 24$ m.

(*b*) Velocity,

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}[4.0t^2\hat{i} + (10.0t + 1.0t^2)\hat{j}]$$

$$= 8.0t\hat{i} + (10.0 + 2.0t)\hat{j}$$

At $t = 2$ s, $\vec{v} = 16.0\hat{i} + 14.0\hat{j}$

Speed, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 14^2}$

$$= \sqrt{256 + 196} = \sqrt{452} = \mathbf{21.26\ ms^{-1}}$$

4.22. (*a*) *If $\hat{i}$ and $\hat{j}$ are unit vectors along X- and Y-axis respectively, then what is the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?*

(*b*) *Find the components of $\vec{a} = 2\hat{i} + 3\hat{j}$ along the directions of vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.* **[Delhi 14]**

Solution. (*a*) In Fig. 4.107, $\vec{OA} = \hat{i}$, $\vec{AB} = \hat{j}$, $\vec{AC} = -\hat{j}$.

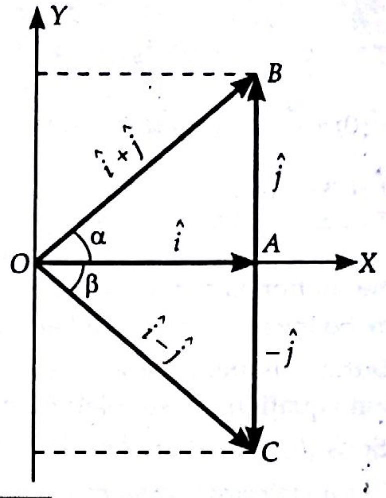

Fig. 4.107

Applying Δ law of vector addition in ΔOAB,

$$\vec{OB} = \vec{OA} + \vec{AB} = \hat{i} + \hat{j}$$

$\therefore$ $|\hat{i} + \hat{j}| = OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$

Direction of $\hat{i} + \hat{j}$ is given by

$$\tan\alpha = \frac{BA}{OA} = \frac{1}{1} = 1$$

$\therefore$ $\alpha = 45°.$

Again, $\vec{OC} = \vec{OA} + \vec{AC} = \hat{i} + (-\hat{j}) = \hat{i} - \hat{j}$

$\therefore$ $|\hat{i} - \hat{j}| = OC = \sqrt{OA^2 + AC^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$

Direction of $\hat{i} - \hat{j}$ is given by

$$\tan\beta = \frac{AC}{OA} = \frac{-1}{1} = -1$$

$\therefore$ $\beta = -45°.$

(*b*) Given $\vec{a} = 2\hat{i} + 3\hat{j}$ and let $\vec{b} = \hat{i} + \hat{j}$ and $\hat{c} = \hat{i} - \hat{j}$. Then component of $\vec{a}$ in the direction $\vec{b}$

$$= (a\cos\theta)\hat{b}$$

$$= \frac{ab\cos\theta}{b}\frac{\vec{b}}{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

$$= \frac{(2\hat{i} + 3\hat{j})\cdot(\hat{i} + \hat{j})}{[\sqrt{1^2 + 1^2}]^2}(\hat{i} + \hat{j})$$

$$= \frac{2\times1 + 3\times1}{2}(\hat{i} + \hat{j}) = \frac{5}{2}(\hat{i} + \hat{j}).$$

Component of $\vec{a}$ in the direction of $\bar{c}$

$$= \frac{\vec{a}\cdot\vec{c}}{|\vec{c}|^2}\vec{c} = \frac{(2\hat{i} + 3\hat{j})\cdot(\hat{i} - \hat{j})}{[\sqrt{1^2 + 1^2}]^2}(\hat{i} - \hat{j})$$

$$= \frac{2\times1 + 3\times(-1)}{2}(\hat{i} - \hat{j}) = -\frac{1}{2}(\hat{i} - \hat{j}).$$

4.23. *For any arbitrary motion in space, which of the following relations are true :*

(a) $\vec{v}_{av} = \dfrac{\vec{v}(t_1)+\vec{v}(t_2)}{2}$ (b) $\vec{v}_{av} = \dfrac{\vec{r}(t_2)-\vec{r}(t_1)}{t_2-t_1}$

(c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$ (d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)\,t + \frac{1}{2}\,\vec{a}t^2$

(e) $\vec{a}_{av} = \dfrac{\vec{v}(t_2)-\vec{v}(t_1)}{t_2-t_1}$.

Ans. As the motion is arbitrary, the acceleration may not be uniform. So the relations (*c*) and (*d*) cannot be true.

For an arbitrary motion, the average velocity cannot be defined as in equation (*a*), so relation (*a*) is not true.

Only relations (*b*) and (*e*) are true.

4.24. *Read each statement below carefully and state, with reasons and examples, if it is true or false :*

A scalar quantity is one that (a) is conserved in a process, (b) can never take negative values, (c) must be dimensionless, (d) does not vary from one point to another in space, (e) has the same value for observers with different orientations of axes.

Solution. (*a*) **False.** Kinetic energy (scalar) is not conserved in an inelastic collision. Moreover, vector quantities like linear momentum, angular momentum, etc., are also conserved.

(*b*) **False.** Scalar quantities such as electric potential, temperature, etc., can take negative values.

(*c*) **False.** Scalar quantities like mass, density, energy, etc., are not dimensionless.

(*d*) **False.** Density (scalar) varies from point to point in the atmosphere.

(*e*) **True.** The mass (scalar) of a body as measured by different observers with different orientations of axes has the same value.

4.25. *An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft ?* **[Central Schools 12]**

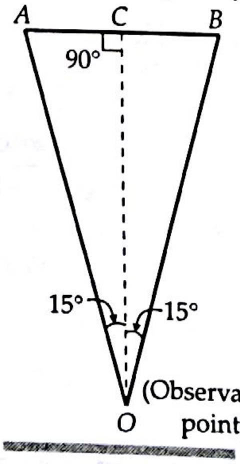

Fig. 4.108

Solution. Let *A* and *B* represent the two aircraft positions separated 10 s apart (Fig. 4.108).

Then $\tan 15° = \dfrac{x}{3400}$

or $x = 3400 \tan 15°$

$= 3400 \times 0.2679$

$= 910.86$ m.

Speed of aircraft

$= \dfrac{910.86 \text{ m}}{5 \text{ s}} = \mathbf{182.2\ ms^{-1}}$.

4.26. *A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.*

Solution. In addition to magnitude and direction, each vector also has a definite location in space. For example, a velocity vector has definite location at every point of uniform circular motion.

A vector can vary with time. For example, increase in velocity produces acceleration.

Two equal vectors $\vec{a}$ and $\vec{b}$ having different locations may not produce identical physical effects. For example, two equal forces (vectors) acting at two different points may not produce equal turning effects.

4.27. *A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?*

Solution. No, anything that has both magnitude and direction is not necessarily a vector. It must obey the laws of vector addition.

Rotation is not generally considered a vector even though it has magnitude and direction because the addition of two finite rotations does not obey commutative law. However, infinitesimally small rotations obey commutative law and hence an infinitesimally small rotation is considered a vector.

4.28. *Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.*

Solution. Out of these, only a plane area can be associated with a vector. The direction of this *area vector* is taken normal to the plane.

4.29. *A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit a target 5 km away ? Assume the muzzle speed to be fixed and neglect air resistance.* **[Central Schools 14]**

Solution. In the first case, $R = 3$ km $= 3000$ m, $\theta = 30°$

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

$\therefore \quad 3000 = \dfrac{u^2 \sin 60°}{g}$

or $\dfrac{u^2}{g} = \dfrac{3000}{\sin 60°} = \dfrac{3000 \times 2}{\sqrt{3}} = 2000\sqrt{3}$

Maximum horizontal range,

$R_{max} = \dfrac{u^2}{g} = 2000\sqrt{3}$ m $= 3464$ m $= \mathbf{3.46\ km.}$

But distance of the target (5 km) is greater than the maximum horizontal range of 3.46 km, so the target cannot be hit by adjusting the angle of projection.

4.30. *A fighter plane flying horizontally at an altitude of 1.5 km with a speed 720 kmh^{-1} passes directly overhead an antiaircraft gun. At what angle from the vertical should the gun be fired for the shell muzzle speed 600 ms^{-1} to hit the plane ? At what maximum altitude should the pilot fly the plane to avoid being hit ? Take $g = 10$ ms^{-2}.*

Solution. Speed of plane = 720 kmh^{-1} = 200 ms^{-1}.

The shell moves along curve OL. The plane moves along PL. Let them hit after a time t.

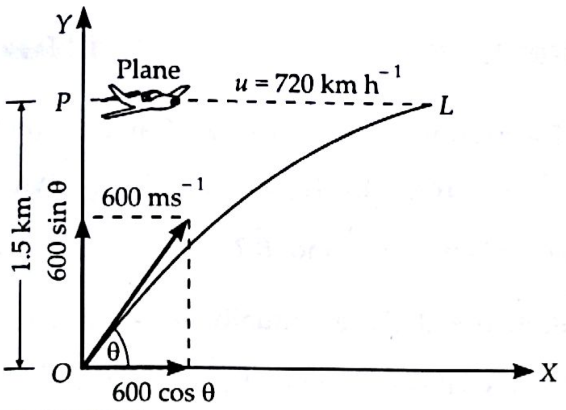

Fig. 4.109

For hitting, horizontal distance travelled by the plane

= Horizontal distance travelled by the shell.

or Horizontal velocity of plane × t

= Horizontal velocity of shell × t

$$200 \times t = 600 \cos\theta \times t$$

$$\cos\theta = \frac{200}{600} = \frac{1}{3} \quad \text{or} \quad \theta = 70°30'$$

The shell should be fired at an angle of 70°30′ with the horizontal or 19°30′ with the vertical.

The maximum height of flight of the shell is

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g}$$

$$= \frac{(600)^2 \times (1-\frac{1}{9})}{2 \times 10} = 16000 \text{ m} = \mathbf{16\ km.}$$

Thus the pilot should fly the plane at a minimum altitude of 16 km to avoid being hit by the shell.

4.31. *A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate 0.5 ms^{-2}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?*

Solution. Here r = 80 m

$$v = 27 \text{ kmh}^{-1} = \frac{27 \times 5}{18} \text{ ms}^{-1} = 7.5 \text{ ms}^{-1},$$

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80} = 0.7 \text{ ms}^{-2}$

Suppose the cyclist applies brakes at the point A of the circular turn, then, tangential acceleration a_T (negative) will act opposite to velocity.

Given $a_T = 0.5 \text{ ms}^{-2}$

As the accelerations a_C and a_T are perpendicular to each other, so the net acceleration of the cyclist is

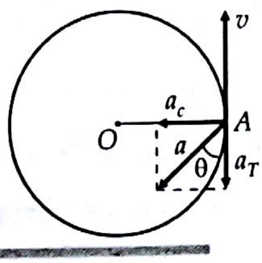

Fig. 4.110

$$a = \sqrt{a_C^2 + a_T^2} = \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.49 + 0.25} = \sqrt{0.74} = \mathbf{0.86\ ms^{-2}.}$$

If θ is the angle between the total acceleration and the velocity of the cyclist, then,

$$\tan\theta = \frac{a_C}{a_T} = \frac{0.7}{0.5} = 1.4 \quad \text{or} \quad \theta = \mathbf{54°28'}.$$

4.32. (*a*) *Show that for a projectile the angle between the velocity and the X-axis as a function of time is given by*

$$\theta(t) = \tan^{-1}\frac{(v_{oy} - gt)}{v_{ox}}$$

(*b*) *Show that the projection angle θ_0 for a projectile launched from the origin is given by :*

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Ans. (*a*) As shown in Fig. 4.111, suppose the projectile is thrown with a velocity v_0 at an angle θ_0 with the X-axis.

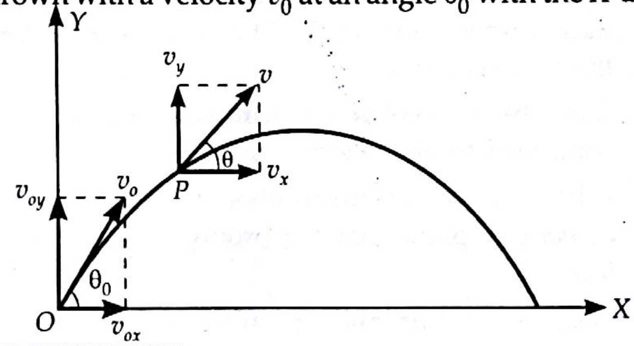

Fig. 4.111

Let v_{ox} and v_{oy} be the components of velocity v_0 along X- and Y-directions respectively.

At any time t, suppose the particle is at point P. Its velocity v has components v_x and v_y along X- and Y-directions. Then

$$v_x = v_{0x} \qquad \text{[Horizontal component remains unchanged]}$$

and $$v_y = v_{0y} - gt$$

If the velocity v makes angle θ with X-axis, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}} \qquad \therefore \quad \theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

(*b*) Maximum height attained by the projectile ,

$$h_m = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Horizontal range of the projectile,

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{v_0^2 \times 2\sin\theta_0 \cos\theta_0}{g}$$

$$\therefore \quad \frac{h_m}{R} = \frac{v_0^2 \sin^2\theta_0}{2g} \times \frac{g}{v_0^2 \times 2\sin\theta_0\cos\theta_0} = \frac{\tan\theta_0}{4}$$

or $$\tan\theta_0 = \frac{4h_m}{R}$$

Hence, $$\theta_0 = \tan^{-1}\frac{4h_m}{R}.$$