SIMILPhysics Reference text

FUNDAMENTAL FORCES IN NATURE

Every one has a qualitative idea of what is meant by the term 'force'. Most of the every day forces are between macroscopic objects. Familiar examples are the contact forces between bodies, friction, force of compressed or elongated springs, viscous force, surface tension etc. Other examples are electric and magnetic forces. A force of universal nature is the gravitational force.

Different forces occurring in different contexts actually arise from a small number of fundamental forces in nature.

1. Gravitational force

It is the force which holds the earth in its orbit round the sun. It is also the force that holds us on to the earth. It is the force of mutual attraction between any two objects by virtue of their masses. It is the weakest force is nature and is always attractive.

2. Electromagnetic force

It is the force between electrically charged particles. The electric force can be attractive or repulsive. Like charges repel and unlike charges attract. The electrostatic force between two charges varies inversely as the square of the distance between the charges.

Charges in motion produce magnetic field. A moving charge in a magnetic field experiences a force. Thus electric and magnetic fields are inseparable. The electromagnetic force is stronger than gravitational force and it dominates all phenomena at the atomic and molecular scale. It is the electromagnetic force that governs the structure of atoms and molecules. The macroscopic forces like tesion, friction, spring force etc. ultimately originate from electric force.

3. The strong nuclear force

This is the force that binds protons and neutrons in a nucleus. Without this force, the nucleus of an atom would be unstable due to repulsive force between protons within the nucleus. The strong nuclear force is charge independent. Its range is very small, which is of the order of the size of a nucleus. Recent studies indicated that the strong nuclear force is not a fundamental force of nature. It is now known that protons and neutrons are built out of still more elementary constituents called *quarks*. The quark-quark force is now thought to be a fundamental force of nature.

4. The weak nuclear force

This force appears only in certain nuclear processes such as β-decay of a nucleus. In β-decay the nucleus emits an electron and an uncharged particle called antineutrino. If the nucleus decays by emitting a positron, the emission is accompanied by a neutrino. Neutrino and antineutrino experience only weak interactions. So any process involving them is governed by the weak nuclear force. The range of weak nuclear force is extremely small of the order of 10^{-15}m.

Each fundamental force is thought to arise from the exchange of its characteristic field particles. Electromagnetic force is caused by the exchange of *photons* between charged particles. The strong nuclear force arises from the exchange of *mesons*. Gravitational force is thought to be caused by the exchange of an yet undetected particle called *graviton*.

CONSERVATION LAWS
(FOR CBSE ONLY)

In the study of physics we come across certain physical quantities which remain constant with passage of time. We say that the quantity is *conserved*. If we have a conserved quantity at some instant of time we will have the same amount of quantity at a later time even though the quantity might have changed form during the time interval.

In classical physics we often deal with the following conservation laws.

Law of conservation of linear momentum

In the absence of an external force, the total linear momentum of a system remains constant.

Law of conservation of energy

For motion under conservative forces the total mechanical energy is constant. For example, in the case of a freely falling body both kinetic energy and potential energy continuously change with time, but the sum remains constant.

The general law of conservation of energy is true for all forces and for any kind of transformation between different forms of energy.

Finally we have the *law of conservation of electric charge*. According to this electric charge is neither created or destroyed. It can only be transformed or exchanged. For example, when a glass rod is rubbed with silk, the glass rod acquires positive charge, while silk acquires an equal negative charge.

Detailed study of conservation laws is given in subsequent chapters.

UNITS AND MEASUREMENT

Need for measurement

It is difficult to say when man learnt the art of measurement. The changes in length of a shadow during the course of a day, the movement of the sun during the day and of the moon during the night, certainly engaged his attention from early times. Those facts might have given the idea of time. Man also felt the need for measuring length. He might have used his own steps for the purpose. Without actual measurement we cannot make correct judgement about a given object.

With the growth of civilization man developed systems of measurement which are accurate and dependable. With the expanding frontiers of science and development of communication between continents, need of standardisation and acceptance all over the world was felt.

Units for measurement

The laws in physics are expressed in terms of physical quantities. Hence an accurate measurement of these quantities becomes necessary and inevitable in establishing the laws of nature as revealed by experimental observations. In order to measure a physical quantity its value is compared with that of the standard of the same kind. This standard is called **Unit** of a physical quantity.

A **Unit** *must be the same kind as the physical quantity. It should be universally acceptable and must be comparable with the size of the physical quantity to be measured. It must be easily reproducible.*

Fundamental units

A set of independent units of basic physical quantities which can easily be obtained and in terms of which units of all other physical quantities may be expressed is called *fundamental units*. Units of *length, mass* and *time* are taken as fundamental units in mechanics.

Derived units

Physical quantities which can be expressed in terms of fundamental quantities are called derived quantities. The units of such quantities are called derived units.

Systems of units

The common systems of units are:-

1. The CGS System

This is based on the three base units, *centimetre, gram and second* for length, mass and the time respectively.

2. The FPS System

In this system *foot* is the unit of length, *pound* the unit of mass and *second* the unit of time.

3. The MKS System

This is based on *metre* for length, *kilogram* for mass and *second* for time.

4. SI Units

In 1960, the International Committee for Weights and Measures adopted a system of units for all fundamental physical quantities and is called International System of Units or SI Units. It is the purest form of metric system evolved by suitable modification to the traditional metric system.

In SI there are seven fundamental units (base units) and two supplementary units.

Base units in SI

Basic physical quantities	Base unit	Symbol
1. Mass	kilogram	*kg*
2. Length	metre	*m*
3. Time	second	*s*
4. Temperature	kelvin	*K*
5. Electric current	ampere	*A*
6. Luminous intensity	candela	*cd*
7. Amount of substance	mole	*mol*

Advantages of SI

The main advantages of SI are given below:-

(i) *It is comprehensive*

It means that the seven base units cover all disciplines of science and technology.

(ii) The system is coherent

The unit of any derived physical quantity can be obtained as a product or quotient of two or more fundamental units.

(iii) International acceptance

The SI units are internationally accepted.

Definitions of base units

1. Length – metre (m)

The first international standard of length was a bar of a platinum-iridium alloy called standard metre, which was kept at the International Bureau of Weights and Measures near Paris. The distance between two fine lines engraved near the ends of the bar, when the bar was held at a temperature of 0°C and supported mechanically in a prescribed way, was defined to be one metre. Due to war or some other reason this bar may be destroyed. Hence we standardise the metre in terms of wavelength of light or velocity of light in vacuum.

At present (Seventeenth General Conference on Weights and Measures, 1983) *metre is defined as the length of the path traversed by light in vacuum in 1/299,792,458 part of a second.*

This is equivalent to saying that the speed of light is now defined as

$$c = 299,792,458 \text{ ms}^{-1}$$

The metre is also defined in terms of the number of wavelengths of a monochromatic light as follows:-

The metre is a length equal to 1,650,763.73 wavelengths of orange-red light emitted by Krypton-86 atoms in an electric discharge. (Eleventh General Conference on Weights and Measures, 1960).

2. Mass–kilogram (kg)

It is the mass of the platinum-iridium cylinder deposited with the International Bureau of Standards in Serves, France.

3. Time–second (s)

It is the duration of 9,192,631,770 cycles of radiation corresponding to the transition between hyperfine levels of the ground state of Cesium - 133 atoms.

4. Electric current–ampere (A)

It is that current which if maintained in two straight parallel conductors of infinite length placed one metre apart in vacuum would produce between them a force of 2×10^{-7} *newton per unit length.*

5. Temperature–kelvin (K)

The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

6. Luminous intensity–candela (cd)

It is the luminous intensity in the perpendicular direction of a surface of 1/600,000 square metre of a black body at the temperature of freezing platinum under a pressure of 101, 325 Nm^{-2}.

7. Amount of substance–mole (mol)

There is another unit for the amount of substance which is the **mole**. *The mole is the amount of substance of a system which contains as many elementary units as there are carbon atoms in exactly 0.012* kg *of Carbon-12.* The elementary units must be specified and may be an atom, molecule, an ion, an electron etc. or a specified group of entities..

Supplementary units

1. Unit of plane angle–radian (rad)

It is the angle subtended at the centre of a circle by an arc of length equal to its radius.

2. Unit of solid angle–steradian (sr)

It is the solid angle subtended at the centre of a sphere by a portion of its surface area equal to a square with sides of length equal to the radius of the sphere.

Formation of names of decimal multiples and sub–multiples of units

To express magnitude of physical quantities which are inconveniently large or small, we may use prefixes to the units. The standard prefixes are given below.

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{18}	exa-	E	10^{-1}	deci-	d
10^{15}	peta-	P	10^{-2}	centi-	c
10^{12}	tera-	T	10^{-3}	milli-	m
10^{9}	giga-	G	10^{-6}	micro-	μ
10^{6}	mega-	M	10^{-9}	nano-	n
10^{3}	kilo-	k	10^{-12}	pico-	p
10^{2}	hecto-	h	10^{-15}	femto-	f
10^{1}	deka-	da	10^{-18}	atto-	a

Some important practical units of length

1. Light year (*l*y)

It is the distance travelled by light in one year in vacuum.

$$1\ ly = 9.46 \times 10^{15}\ m$$

2. Astronomical unit (AU)

An astronomical unit is the average distance between the sun and the earth

$$1\ AU = 1.50 \times 10^{11}\ m$$

3. Parallactic second or parsec (pc)

One parsec is the distance at which an arc of length 1 AU subtends an angle of 1 second.

$$1 \text{ pc} = 3.08 \times 10^{16} \text{ m}$$

4. Angstrom unit (Å)

$$1 \text{ Å} = 10^{-10} \text{ m}$$

5. Fermi (fm)

$$1 \text{ fm} = 10^{-15} \text{ m}$$

6. Attometre (am)

$$1 \text{ am} = 10^{-18} \text{ m}$$

***Approximate values of some measured lengths**

Distance from the earth to most remote galaxies	$= 3 \times 10^{26}$ m
Mean radius of the orbit of the earth	$= 1.5 \times 10^{11}$ m
Mean distance of earth to moon	$= 3.8 \times 10^{8}$ m
Mean radius of the earth	$= 6.4 \times 10^{6}$ m
Thickness of a paper sheet	$= 1 \times 10^{-4}$ m
Diameter of hydrogen atom	$= 1 \times 10^{-10}$ m
Diameter of atomic nucleus	$= 1 \times 10^{-14}$ m

Approximate mass of various bodies

Sun	$= 2 \times 10^{30}$ kg
Earth	$= 6 \times 10^{24}$ kg
Moon	$= 7 \times 10^{22}$ kg
Hydrogen atom	$= 1.67 \times 10^{-27}$ kg
Electron	$= 9.11 \times 10^{-31}$ kg

Approximate values of some time intervals

Age of earth	$= 1.3 \times 10^{17}$ s
Time for earth to complete one revolution round the sun	$= 3.1 \times 10^{7}$ s
One day	$= 8.64 \times 10^{4}$ s
Life span of an excited atom	$= 10^{-8}$ s
Time for electron to complete one revolution round the nucleus of hydrogen atom	$= 10^{-15}$ s

Some length measurement

1. Measurement of large distance by angular measurement

To measure the distance between two points on an astronomical object we use angular measurements. For example if θ radians is the angle subtended by two diametrically opposite ends of the moon at a point on the earth, then the diameter of the moon $x = r\theta$, where r is the distance of the moon from the earth.

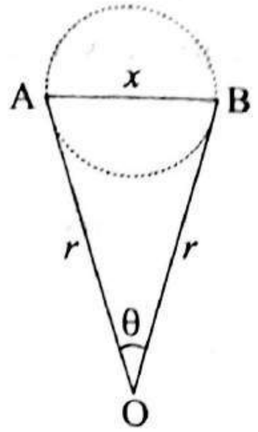

Fig. 3

2. *Parallax Method*

To measure the distance r of a far away object S by parallax method we look at it from two positions A and B separated by a distance x. If θ radians is the angle between the two directions, then

$$\theta = x/r \qquad \therefore r = x/\theta$$

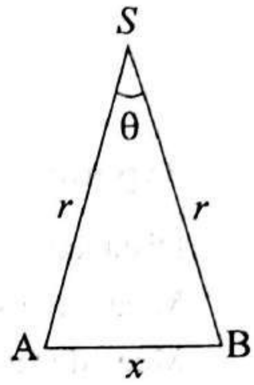

Fig. 4

3. *Reflection Method*

This method is used in RADAR, SONAR etc.

(a) **RADAR (Radio Detection And Ranging)**

Here radio waves are transmitted from a transmitter. It is reflected by an obstacle and is received by the receiver of the transmitting station. If 't' is the time delay in receiving the echo, then the distance of the obstacle is given by $x = ct/2$; where c is the speed of radio waves.

(b) **SONAR (Sound Navigation And Ranging).**

Here instead of radio waves ultrasonic waves are used. Ultrasonic waves are sent through the ocean from the transmitter to a distant object such as a submarine and the reflected waves are received by the receiver.

Example

I.1. Parallax angle of a heavenly body measured from two points diametrically opposite on the equator of the earth is 1.0 minute. If the radius of the earth is 6400 km, find the distance of the heavenly body from the earth.

$$x = 6400 \times 2 \text{ km}; \; r = ?; \; \theta = 1' = \frac{1}{60} \times \frac{\pi}{180} \text{ radian};$$

$$x/r = \theta \qquad \therefore r = x/\theta = 6400 \times 2 \times 60 \times 180/\pi = \mathbf{4.4 \times 10^7 \text{ km}}$$

DIMENSIONAL ANALYSIS

All physical quantities which are purely mechanical in nature can be represented in terms of fundamental quantities mass, length, and time. They are represented by M, L and T. The letters specify only the nature of the units and not the magnitude. The derived units are based on the fundamental units. In such a case the dimensions of such units are expressed in general as $KM^xL^yT^z$; where K is a constant and x, y and z indicate how many times a particular unit is involved.

The dimensions of a physical quantity are the powers to which the fundamental quantities mass, length, time, temperature and electric current are to be raised in order to represent the physical quantity.

The temperature and electric current are represented by K and I or A.

Dimensional equation

The expression which indicates the units of a physical quantity in terms of the fundamental units is called dimensional equation.

For example dimensional equation of velocity is $[V] = M^0LT^{-1}$. The dimensional formulae of some physical quantities are given below.

$$\text{Area} = \text{Length} \times \text{Length} = L \times L = L^2 = M^0L^2T^0$$

$$\text{Volume} = \text{Length} \times \text{Length} \times \text{Length} = L^3 = M^0L^3T^0$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3} = ML^{-3}T^0$$

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{L}{T} = LT^{-1} = M^0LT^{-1}$$

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time}} = \frac{LT^{-1}}{T} = LT^{-2} = M^0LT^{-2}$$

Force	$= \text{Mass} \times \text{Acceleration} = MLT^{-2}$
Pressure	$= \text{Force/area} = MLT^{-2}/L^2 = ML^{-1}T^{-2}$
Work	$= \text{Force} \times \text{Displacement} = MLT^{-2} \times L = ML^2T^{-2}$
Power	$= \frac{\text{Work}}{\text{Time}} = \frac{ML^2T^{-2}}{T} = ML^2T^{-3}$
Gas Constant, R	$= PV/T = ML^{-1}T^{-2} \times L^3/K = ML^2T^{-2}K^{-1}$
Electric charge, q	$= \text{current} \times \text{time} = AT$

Electric potential, $V = \text{work/charge} = ML^2T^{-3}A^{-1}$

Note:- The quantities such as number, angle and trignometrical ratios are dimensionless.

Angle has unit, the radian, but no dimension.

Principle of homogeneity of dimensions

The dimensions of all the terms on the two sides of an equation are the same. In other words the equation must be dimensionally homogeneous.

Uses of Dimensional Analysis

(i) To check the correctness of an equation

An equation is correct only if the dimensions of each term on either side of the equation are equal. In this case the equation is dimensionally correct; but, may or may not be the correct equation for the physical quantity. If the homogeneity of dimensions does not hold good, the equation is definitely incorrect.

Example:-

Check the accuracy of the equation, $S = ut + (1/2)at^2$

Dimension of the term $S = L$; dimension of the term $ut = LT^{-1} \times T = L$ and the dimension of the term $(1/2)at^2 = LT^{-2} \times T^2 = L$. Thus, we find that each term has the same dimensions. So the equation is dimensionally correct.

(ii) To convert a unit from one system into another

Since a physical quantity is expressed in terms of appropriate units of the same nature in all systems, the dimensions should remain the same eventhough the number indicating its magnitude may differ. If a physical quantity of dimensions a, b and c in mass, length and time respectively has magnitude n_1 and n_2 in two systems having fundamental units M_1, L_1, T_1 and M_2, L_2, T_2, respectively then,

$$n_1 M_1^a L_1^b T_1^c = n_2 M_2^a L_2^b T_2^c$$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Example:-

Convert one newton into dynes

They are the units of force in SI and CGS respectively. Dimensional formula for force is MLT^{-2}. Let n_1 newton be equal to n_2 dynes.

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^1 \left(\frac{L_1}{L_2}\right)^1 \left(\frac{T_1}{T_2}\right)^{-2}$$

$$= 1\left(\frac{1\text{ kg}}{1\text{ g}}\right)^1 \left(\frac{1\text{ m}}{1\text{ cm}}\right)^1 \left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2} \qquad [\text{since } n_1 = 1]$$

$$= \frac{1000\text{ g}}{1\text{ g}} \times \frac{100\text{ cm}}{1\text{ cm}} = 1000 \times 100 = 10^5$$

$\therefore$ 1 newton $= 10^5$ dynes

(iii) To derive the correct relationship between physical quantities

When one physical quantity depends on several physical quantities, then the relationship between the quantities can be derived using dimensional method.

Example:-

To derive an expression for the period of oscillation of a simple pendulum *[NCERT]*

The period of oscillation t of a simple pendulum may depend on (1) the length of the pendulum l, (2) the mass of the bob m and (3) the acceleration due to gravity g

Let the form of the equation be, $t = Kl^x m^y g^z$
Taking dimensions on both sides,

$$T = L^x M^y (LT^{-2})^z = L^{(x+z)} M^y T^{-2z} \quad [\because K \text{ is dimensionless}]$$

Equating dimensions of L, M and T,

$$x + z = 0;\ y = 0 \text{ and } -2z = 1 \quad \therefore z = -\frac{1}{2} \text{ and } x = \frac{1}{2}$$

$$\therefore t = Kl^{\frac{1}{2}} m^0 g^{-\frac{1}{2}} = K\sqrt{l/g}$$

The value of the constant K cannot be obtained by dimensional method. Using other methods, the value of the dimensionless quantity K is obtained as 2π.

$$\therefore t = 2\pi\sqrt{l/g}$$

Limitations of dimensional analysis

Eventhough dimensional analysis is a simple and a convenient method to derive an equation for a physical quantity, it has the following limitations.

(i) In mechanics, this method is not suitable if the physical quantity depends on more than three other physical quantities. By equating dimensions of M, L and T on the two sides, we can obtain only three independent equations.

(ii) This method does not tell us anything about dimensionless quantities such as number, angle, trignometrical ratio etc.

(iii) This method cannot be employed if the right hand side of the equation contains more than one term.

(iv) Quite often it is difficult to guess the parameters on which the physical quantity depends.

Examples

I.3. Check the correctness of the equation $P = hdg$, where P is the pressure at a point h below the free surface of a liquid of density d and g the acceleration due to gravity.

The dimensions on LHS and RHS are

$$\text{LHS; } P = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$\text{RHS; } h = L;\ d = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3};\quad g = LT^{-2}$$

$$\therefore hdg = L \times ML^{-3} \times LT^{-2} = ML^{-1}T^{-2}$$

Hence the equation is dimensionally correct.

I.4. The frequency n of a stretched string may depend on (i) length of the vibrating segment l, (ii) the tension in the string F and (iii) the mass per unit length m. Show that $n \propto (1/l)\sqrt{F/m}$

$$n \propto l^x F^y m^z \quad \therefore n = K l^x F^y m^z; \text{ where K is a constant.} \qquad \text{(i)}$$

Taking dimensions on both sides,

$$T^{-1} = L^x(MLT^{-2})^y(ML^{-1})^z = M^{y+z}L^{x+y-z}T^{-2y} \qquad \text{(ii)}$$

Equating dimensions of M, L and T of the equation (ii), we get,

$$y + z = 0; \quad x + y - z = 0; \quad \text{and} \quad -2y = -1$$

$$\therefore y = 1/2; \quad z = -1/2 \quad \text{and} \quad x = -1.$$

Hence equation (i) becomes, $n = Kl^{-1}F^{1/2}m^{-1/2} = K(1/l)\sqrt{F/m}$

$$\therefore \mathbf{n} \propto (1/l)\sqrt{\mathbf{F/m}}$$

I.5. A unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between sun and the earth in terms of the new unit if light takes 8 minutes and 20 seconds to cover this distance? [NCERT]

$$c = 1 \text{ unit/s}; \quad t = 8'20'' = 500 \text{ s}$$

Distance between the sun and the earth,

$$s = c \times t = 1 \times 500 = \mathbf{500 \text{ units}}$$

I.6. A famous relation in Physics relates moving mass m to rest mass m_0 in terms of its speed v and the speed of light c. A boy recalls the relation almost correctly but

forgets where to put the constant c. He writes as $m = m_0/\sqrt{1 - v^2}$. Guess where to put c? **[NCERT]**

The dimensions of physical quantities appearing on both sides of an equation must be equal.

The denominator must be dimensionless. The factor at v^2 must be dimensionless *i.e.*, this must be v^2/c^2.

$$\therefore \text{ The correct relation is, } \mathbf{m = m_0/\sqrt{1 - v^2/c^2}}$$

I.7. Derive Stoke's formula for the viscous force acting on a spherical ball moving through a viscous fluid.

The Stoke's formula gives an expression for the viscous force (F) acting on a small sphere moving through a homogeneous viscous fluid.

The viscous force (F) acting on the sphere depends on:-

(1) η, the viscosity of the liquid

(2) r, the radius of the sphere

(3) v, the velocity of the sphere.

$\therefore F = K\eta^x r^y v^z$; where K is a constant and x, y and z are the powers of η, r and v to be calculated.

Putting dimensions on both sides we get,

$$MLT^{-2} = (ML^{-1}T^{-1})^x (L)^y (LT^{-1})^z = M^x L^{-x+y+z} T^{-x-z}$$

Since, according to the homogeneity of dimensions, the powers of M, L, and T on both sides of the equation are to be equal,

$$x = 1; \quad -x + y + z = 1 \quad \text{and} \quad -x - z = -2$$

Solving these three equations, we get $x = 1$; $y = 1$ and $z = 1$.

$$\therefore F = \eta^1 r^1 v^1 = K\eta r v$$

The value of K, determined experimentally or by calculation, is 6π.

$$\mathbf{F = 6\pi \eta r v}$$

SIGNIFICANT FIGURES

Measurements made by any instrument are not absolutely correct. The numerical value of every measurement is approximate. The extent of approximation depends on the least count of the instrument used for the measurement. So it is important to express the result of a measurement in such a way that the number of digits used for recording it should indicate the accuracy of the measurement.

The significant figures in a measured physical quantity indicates the number of digits in which we have confidence in respect of their accuracy. It is those digits that are reliable plus the last digit that is uncertain.

The greater the number of significant figures, the more accurate is the measurement.

If the length of an object measured with a metre scale graduated in mm is 13.6 cm, the number of significant figures is three. In this case the first and the second digits *i.e.*, 1 and 3, are reliably known and the last digit 6 is uncertain. The maximum error possible in this measurement is ±0.1 cm.

$$\therefore \text{ Accuracy } = \frac{0.1}{13.6} = \frac{1}{136}$$

If the same measurement is expressed in metres, we get 0.136 m. In this case, the measurement is known to us with an accuracy of 0.001 m in the total of 0.136 m.

$$\therefore \text{ Accuracy } = \frac{0.001}{0.136} = \frac{1}{136}$$

If the length is expressed in km, we get 0.000136.

$$\therefore \text{ Accuracy } = \frac{0.000,001}{0.000136} = \frac{1}{136}$$

Thus we see that the uncertainty in the measurement does not change, if we change the unit of measurement. Hence, in the above cases, the number of significant figures remains the same as three.

Rules to determine significant figures

1. All the non-zero digits are significant.
2. The number of significant figures in a number is equal to the number of digits counted from the first non-zero digit on the left to the last digit on the right. In the number 18.45, there are four significant figures.
3. All the zeros occurring between two non-zero digits are significant. In the number 180.045, there are six significant figures.
4. If a number starts with decimal all zeros on the left side of the first non-zero digit are not significant; but all zeros on the right side of the first non-zero digit are significant.

   Ex. (i) The number of significant figures in 0.0001845 is four.

   (ii) The number of significant figures in 0.004030 is four.
5. If a number has an integral part and a decimal part, all zeros in the number are significant.

   Ex. (i) The number 30.10 has four significant figures

   (ii) The number 30.00 has four significant figures.
6. Where there is no decimal part, the last zeros are not significant.

Ex. The number of significant figures in the measurement 102000 is three.

However, the last zeros are also significant if they represent the accuracy of the instrument. In a measurement $x = 300$ m, it does not indicate whether there are one, two or three significant figures; we do not know whether the zeros are carrying information or merely serving as place holders. Instead, we would write $x = 3 \times 10^2$ (one significant figure), or 3.0×10^2 (two significant figures) or 3.00×10^2 (three significant figures) to specify the precision more clearly.

Ex. The number 10200 has three significant figures and 1.200×10^3 has four.

Number	Significant Digits
403	3
40.3	3
4.03	3
0.403	3
0.0403	3
0.04030	4
403.0	4
40.30	4
1.02×10^3	3

Significant figures in calculations

In an experiment different physical quantities may be measured. These quantities may be added, subtracted, multiplied or divided to get the required experimental result. If all the observations have been made with great accuracy except one observation, then the inaccuracy in the single observation is going to affect the accuracy of the final result.

Examples:-

1. Significant figures in multiplication and division

The result of multiplying or dividing two or more numbers can have no more significant figures than are present in the number having the least significant figures.

Ex. (i)

$a = 10.43$ (Number of significant figures is four)

$b = 2.8612$ (Number of significant figures is five)

$a \times b = 29.842316 = 29.84$ (Number of significant figures is four)

(ii)

If $a = 16000$; $b = 4.58$, then,

$$\frac{a}{b} = \frac{16000}{4.58} = 3493.449782 = \mathbf{3500}$$

2. Significant figures in addition and subtraction

In adding or subtracting, the least significant digit of the sum or difference occupies the same relative position as the least significant digit of the quantities being added or subtracted. In this case the number of significant figures is not important; it is the position that matters.

Ex. (i)

204.9 + (9 is the least significant digit. Its position is the first decimal place.)

2.10 + (0 is the least significant digit. Its position is the second decimal place.)

0.319 (9 is the least significant digit. It is in the third decimal place.)

207.319 = **207.3**

(In the sum, the least significant digit should come in the first decimal place.)

(ii)

$$\text{If } a = 10.43 \text{ and } b = 2.8612, \text{ then}$$
$$a - b = 10.43 - 2.8612 = 7.5688 = \mathbf{7.57}$$

Order of Magnitude

To estimate how big or small a given physical quantity is, we make use of the concept of the order of magnitude. To determine the order of magnitude of a quantity we first express the quantity in terms of the nearest power of ten. The power of ten thus obtained is called order of magnitude.

Illustration:

Quantity	*Expressed in nearest power of 10*	*Order of magnitude*
8	0.8×10^1	1
16	1.6×10^1	1
47	4.7×10^1	1
53	0.53×10^2	2
97	0.97×10^2	2
999	0.999×10^3	3

Note that the order of magnitude of 47 is not the same as that of 97. The number 97 is one order of magnitude higher than the number 47. We can express these numbers as 4.7×10^1 and 9.7×10^1. Since 9.7 is greater than 5, the convention is to regard it closer to ten. Hence it contributes one more power to ten. So, 97 may be expressed as 0.97×10^2. Thus, we see that the order of magnitude of 47 is *one* and that of 97 is *two*.

Examples

I.8. The length, breadth and thickness of a rectangular sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures. [NCERT]

$$\text{Area of the sheet} = (4.234 \times 1.005) \times 2 = 8.51034 = 8.510\,\text{m}^2$$

(We have considered only two surfaces of the plate).

$$\text{Volume of the sheet} = 4.234 \times 1.005 \times 0.0201$$
$$= 0.085528917 = \mathbf{0.0855\ m^3}$$

I.9. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box (b) the difference in the masses of the pieces to correct significant figures? [NCERT]

(a) Total mass of the box $= 2.3 + 0.02015 + 0.02017 = 2.34032 = \mathbf{2.3\ kg}$

(b) Difference in the masses of the pieces $= 20.17 - 20.15 = 0.02$ **g**

I.10. Calculate the area enclosed by a circle of radius 0.16 m

$$\pi = 3.14; \quad r = 0.16 \text{ m}; \quad A = ?$$

$$A = \pi r^2 = 3.14 \times 0.16 \times 0.16 = 0.080384 = \mathbf{0.080\ m^2}$$

ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT

When we make a measurement with an instrument the measured value is usually different from the true value. There is always an uncertainty in the measurement. This uncertainity is called error.

The *accuracy* of a measurement is a measure of how close a measured value is to the true value of the quantity.

Precision indicates the limit upto which the measurement can be taken accurately. It is determined by the least count of the instrument. Smaller the least count, greater is the precision.

The accuracy of a measurement can be increased by minimising errors and using precision instruments.

When we take measurement using various measuring instruments various types of errors may creep into the observations. Given below are the various types of errors which are likely to affect the result of a measurement.

1. Systematic errors

These errors are due to known causes. This type of error can be minimised by detecting the source of error and the rule governing the error. Some of the sources of systematic errors are:

(a) Instrumental error

This is due to the imperfection of the design or calibration of the measuring instrument. Often there may be zero error in the instrument. The instrumental error can be reduced by using more accurate instruments or by applying zero correction when required.

(b) Imperfection in experimental technique or procedure

This is due to imperfect experimental arrangement. In this case we know the cause for the error; but we cannot eliminate it. The error due to radiation in the calorimetric experiments in heat is an error of this type. Proper correction can be applied for such errors.

(c) Error due to external causes

This error is caused by external conditions such as pressure, temperature etc. Here also we know the cause for the error; but it cannot be eliminated. For example, if we use a steel tape calibrated at 0°C, to measure the length of a body at a different temperature, there is an error in the measurement due to the thermal expansion of the tape. These types of error can be taken care of by applying suitable corrections.

2. Random (or accidental) error

Errors which occur in a random manner and cannot be associated with a systematic cause are called random errors.

These errors are due to irregular causes. For example, while measuring diameter of a wire with a screw gauge, one may get different readings in different observations. In such cases it is not possible to indicate which observation is most accurate. Here, if we repeat the experiment a number of times, the average of all these readings is found to be the most accurate observation. If $a_1, a_2, a_3 \ldots a_n$ are the values of a physical quantity obtained in several measurements, the most accurate observation (*i.e.*, the true value) $\bar{a}$ is given by,

$$\bar{a} = \frac{a_1 + a_2 + \cdots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{i=n} a_i$$

3. Gross error

These errors are due to the carelessness on the part of the observer. No correction can be applied for them.

Gross error arises because of

(i) neglecting the source of error

(ii) reading the instrument incorrectly

(iii) the improper recording of the reading.

Absolute error

If $\bar{a}$ is the mean value *i.e.*, the true values of a number of observations, some observations will be greater than $\bar{a}$ while some others will be less than $\bar{a}$. The *magnitude of the difference between the true value and a measured value is called the absolute error in the measurement.*

If $\bar{a}$ is the true value, the absolute error in the i^{th} measurement is given by,

$$\Delta a_i = |\bar{a} - a_i|$$

It is expressed in the unit of the measured values.

Mean absolute error

The arithmetic mean of the absolute errors in various measurements is the mean absolute error.

If $\Delta a_1, \Delta a_2, \ldots, \Delta a_n$ are the absolute errors in the measurements $a_1, a_2, a_3 \cdots a_n$, then, the mean absolute error in the measurements,

$$\overline{\Delta a} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \cdots + \Delta a_n}{n}$$

The measurements can have any value lying between $\bar{a} + \Delta a$ and $\bar{a} - \Delta a$.

Relative error and percentage error

The relative error is defined as the ratio of the mean absolute error to the true value.

$$\text{Relative error} = \overline{\Delta a}/\bar{a}$$

$$\text{Percentage error} = (\overline{\Delta a}/\bar{a}) \times 100\ \%$$

Combination of errors

The final result of an experiment is arrived at from a number of observations taken with the help of different instruments. These observations are allowed to undergo a number of mathematical operations such as addition, subtraction, multiplication, division etc. There may be some error in each of these observations. Thus the final result depends on the way how these errors combine together during different mathematical operations.

The following rules are followed in combining errors to get the maximum permissible error:-

(i) Error in a sum

Let z be the sum of two observed quantities x and y.

$$z = x + y$$

Let Δz = absolute error in z, *i.e.* the sum of x and y

Δx = absolute error in x and

Δy = absolute error in y

$$\therefore z \pm \Delta z = (x \pm \Delta x) + (y \pm \Delta y) = x + y \pm \Delta x \pm \Delta y$$

$$i.e., z \pm \Delta z = z \pm \Delta x \pm \Delta y$$

Hence maximum absolute error in z is,

$$\Delta z = \pm(\Delta x + \Delta y)$$

$\therefore$ Relative error in z is,

$$\frac{\Delta z}{z} = \pm\frac{(\Delta x + \Delta y)}{x + y}$$

$\therefore$ Percentage error in z is,

$$\frac{\Delta z}{z} \times 100 = \pm\frac{(\Delta x + \Delta y)}{x + y} \times 100\ \%$$

(ii) Error in a difference

Let z be the difference between two measured quantities x and y.

$$z = x - y$$

Let $\pm\Delta x$ and $\pm\Delta y$ be the absolute errors in the measurement of x and y. If $\pm\Delta z$ is the error in the difference $x - y$, then,

$$z \pm \Delta z = (x \pm \Delta x) - (y \pm \Delta y) = x - y \pm \Delta x \pm \Delta y$$
$$= z \pm \Delta x \pm \Delta y$$
$$\therefore \pm\Delta z = \pm\Delta x \pm \Delta y$$

Hence maximum absolute error in z is

$$\Delta z = \pm(\Delta x + \Delta y)$$

$\therefore$ Relative error in z is

$$\frac{\Delta z}{z} = \pm\frac{(\Delta x + \Delta y)}{z} = \pm\frac{(\Delta x + \Delta y)}{x - y}$$

Percentage error in z is,

$$\frac{\Delta z}{z} \times 100 = \pm\frac{(\Delta x + \Delta y)}{x - y} \times 100$$

(iii) Error in a product

Let z be the product of two measured quantities x and y.

$$\therefore z = xy$$

If Δx and Δy are the errors in the measurement of the quantities x and y, then the error Δz in the product is given by the equation

$$(z \pm \Delta z) = (x \pm \Delta x)(y \pm \Delta y)$$
$$z[1 \pm (\Delta z/z)] = x[1 \pm (\Delta x/x)] \times y[1 \pm (\Delta y/y)]$$
$$= xy[1 \pm (\Delta x/x)] \times [1 \pm (\Delta y/y)]$$
$$= z[1 \pm (\Delta y/y) \pm (\Delta x/x) \pm (\Delta x/x)(\Delta y/y)]$$
$$i.e.\ 1 \pm \Delta z/z = 1 \pm (\Delta y/y) \pm (\Delta x/x)$$

[Since $\Delta x/x$ and $\Delta y/y$ are very small, $(\Delta x/x)(\Delta y/y)$ is neglected].

$$\therefore \pm\Delta z/z = \pm(\Delta x/x) \pm (\Delta y/y)$$

$\therefore$ Maximum relative error in z is,

$$\Delta z/z = \pm[(\Delta x/x) + (\Delta y/y)]$$

$\therefore$ Percentage error in z is,

$$(\Delta z/z) \times 100 = \pm[(\Delta x/x) + (\Delta y/y)] \times 100\ \%$$

(iv) Error in a quotient

As explained above,

$$z \pm \Delta z = (x \pm \Delta x)/(y \pm \Delta y)$$

$$z[1 \pm (\Delta z/z)] = x[1 \pm (\Delta x/x)] \div y[1 + (\Delta y/y)]$$

$$= \frac{x}{y}\left[\frac{1 \pm \Delta x/x}{1 \pm \Delta y/y}\right] = z\left[\frac{1 \pm \Delta x/x}{1 \pm \Delta y/y}\right]$$

$$1 \pm (\Delta z/z) = (1 \pm \Delta x/x)(1 \pm \Delta y/y)^{-1}$$

$$= (1 \pm \Delta x/x)(1 \mp \Delta y/y) \quad [\text{since } (\Delta y/y) \ll 1]$$

$$= 1 \pm \Delta x/x \mp \Delta y/y \pm (\Delta x/x)(\Delta y/y)$$

$$= 1 \pm \Delta x/x \mp \Delta y/y$$

[$(\Delta x/x)(\Delta y/y)$ is neglected being very small]

$$\therefore \pm(\Delta z/z) = \pm(\Delta x/x) \mp (\Delta y/y)$$

$\therefore$ Maximum relative error in z is,

$$\Delta z/z = \pm\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$$

Percentage error in z is,

$$(\Delta z/z) \times 100 = \pm\left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right] \times 100\ \%$$

(v) Error when a quantity is raised to a power

Let $z = \dfrac{x^a}{y^b}$ (1)

Let Δx and Δy be the absolute errors in the measurement of x and y; and Δz the absolute error in z.

Taking log on both sides of eqn. (1), we get,

$$\log z = a \log x - b \log y$$

Differentiating both sides, we get

$$\Delta z/z = a(\Delta x/x) - b(\Delta y/y)$$

$\therefore$ Relative error in z is,

$$\pm(\Delta z/z) = \pm a(\Delta x/x) \mp b(\Delta y/y)$$

$\therefore$ Maximum relative error is,

$$\Delta z/z = \pm[a(\Delta x/x) + b(\Delta y/y)]$$

Percentage error in z is,

$$(\Delta z/z) \times 100 = \pm[a(\Delta x/x) + b(\Delta y/y)] \times 100\ \%$$

Examples

I.11. The sides of a rectangular lamina are (8.5 ± 0.2) cm and (5.6 ± 0.1) cm. Calculate the perimeter of the lamina with error limits.

$$l = 8.5 \text{ cm}; \ \Delta l = 0.2 \text{ cm}; \ b = 5.6 \text{ cm}; \ \Delta b = 0.1 \text{ cm}$$

$$\text{Perimeter, } P = 2(l + b) = 2(8.5 + 5.6) = 28.2 \text{ cm}$$

$$\Delta P = \pm 2(\Delta l + \Delta b) = \pm 2(0.2 + 0.1)$$

$$= \pm 0.6 \text{ cm}$$

$\therefore$ Perimeter with error limits, $P = \mathbf{(28.2 \pm 0.6)\ cm}$

I.12. The period of oscillation of a simple pendulum measured turns out to be 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s. Find (a) true period of oscillation of the pendulum (b) absolute error in each measurement (c) mean absolute error (d) fractional error (e) percentage error and (f) period of the pendulum with error limits. **[NCERT]**

(a) The true period, $\bar{T} = \dfrac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$

$$= 2.624 = \mathbf{2.62\ s}$$

(b) Absolute error in the measurement, 2.63 s, $\Delta T = |2.62 - 2.63| = 0.01$ s

Absolute error in the measurement, 2.56 s, $\Delta T = |2.62 - 2.56| = 0.06$ s

Absolute error in the measurement, 2.42 s, $\Delta T = |2.62 - 2.42| = 0.20$ s

Absolute error in the measurement, 2.71 s, $\Delta T = |2.62 - 2.71| = 0.09$ s

Absolute error in the measurement, 2.80 s, $\Delta T = |2.62 - 2.80| = 0.18$ s

(c) Mean absolute error, $\overline{\Delta T} = \dfrac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$

$$= 0.108 = \mathbf{0.11\ s}$$

(d) Fractional error $= \overline{\Delta T}/\overline{T} = 0.11/2.62$

$= 0.0419847 = \mathbf{0.04}$

(e) Percentage error $= 0.04 \times 100 = \mathbf{4\%}$

(f) Period of the simple pendulum, $= T = \overline{T} \pm \overline{\Delta T}$

$= \mathbf{(2.62 \pm 0.11)\ s}$

I.13. The maximum and minimum temperatures at a place are (40.6 ± 0.3)°C and (32.4 ± 0.2)°C. Calculate the difference in the temperatures with error limits.

$$\theta_1 = 40.6°\text{C}; \quad \Delta\theta_1 = 0.3°\text{C}; \quad \theta_2 = 32.4°\text{C};$$

$$\Delta\theta_2 = 0.2°\text{ C}; \quad \theta = ?$$

$$\theta = \theta_1 - \theta_2 = 40.6 - 32.4 = 8.2°\text{ C}$$

$$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.3 + 0.2) = \pm 0.5°C$$

$\therefore$ Difference in temperature with error limits $= (8.2 \pm 0.5)°C$

I.14. The error in the measurement of radius of a sphere is 0.4%. Find the permissible error in the surface area.

$$(\Delta r/r) \times 100 = 0.4\%; \; A = 4\pi r^2$$

$$\frac{\Delta A}{A} = 2\left(\frac{\Delta r}{r}\right).$$

$\therefore$ Percentage error in area $= 2(\Delta r/r) \times 100 =$ **0.8%**

I.15. A physical quantity P is related to four observables a, b, c and d as $P = a^3b^2/\sqrt{cd}$. The percentage errors of the measurement in a, b, c, and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result? **[NCERT]**

Percentage error in $a^3 \quad = 3\left(\frac{\Delta a}{a} \times 100\right) = 3 \times 1 = 3\%$

Percentage error in $b^2 \quad = 2\left(\frac{\Delta b}{b} \times 100\right) = 2 \times 3 = 6\%$

Percentage error in $\sqrt{c} \quad = \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\right) = \frac{1}{2} \times 4 = 2\%$

Percentage error in $\sqrt{d} \quad = \frac{1}{2}\left(\frac{\Delta d}{d} \times 100\right) = \frac{1}{2} \times 2 = 1\%$

Percentage error in $P \quad = 3 + 6 + 2 + 1 \quad = 12\%$

Since the relative error in P has only two significant figures, the value of P should be rounded to two significant figures. $\therefore P = 3.8$

I.16. In the determination of g by a simple pendulum, 100 oscillations are taken and total time measured is 200 s. The least count of the stop watch is 0.1 s. The length of the pendulum measured with a metre scale of least count 1 mm is 1 m. Find the percentage error in the value of g

$$l = 1 \text{ m}; \; \Delta l = 1 \text{ mm} = (1/1000) \text{ m};$$

$$(\Delta l/l) \times 100 = (1/1000) \times 100 = 0.1\%$$

$$t = 200 \text{ s}; \; \Delta t = 0.1 \text{ s};$$

$$\therefore (\Delta T/T) \times 100 = (\Delta t/t) \times 100 = (0.1/200) \times 100 = 0.05\%$$

$$g = 4\pi^2 l/T^2;$$

$$\therefore (\Delta g/g) \times 100 = (\Delta l/l) \times 100 + 2(\Delta T/T) \times 100$$

$$= 0.1 + 2 \times 0.05 = \mathbf{0.2\%}$$

I.17. Two resistances $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\ \Omega$ are connected in series. Find the equivalent resistance of the series combination **[NCERT]**

$$R_1 = 100\ \Omega; \quad \Delta R_1 = \pm 3\ \Omega; \quad R_2 = 200\ \Omega; \; \Delta R_2 = \pm 4\ \Omega$$

$$R = R_1 + R_2 = 100 + 200 = 300\ \Omega$$

$$\Delta R = \pm(\Delta R_1 + \Delta R_2) = \pm(3 + 4) = \pm 7\ \Omega$$

Equivalent resistance with error limit, $\mathbf{R = (300 \pm 7)\ \Omega}$.

IMPORTANT POINTS

S.I. Base Quantities and Units

Length	*metre*	*m*
Mass	*kilogram*	*kg*
Time	*second*	*s*
Electric current	*ampere*	*A*
Temperature	*kevin*	*K*
Luminous intensity	*Candela*	*cd*
Amount of substance	*mole*	*mol*

Some practical units

Light year (ly) $= 9.46 \times 10^{15}\ m$
Astronomical unit (AU) $= 1.496 \times 10^{11}$
Parsec (pc) $= 3.08 \times 10^{16}\ m$
Fermi (fm) $= 10^{-15}\ m$
Angstrom (Å) $= 10^{-10}\ m$

Dimensions

Fundamental quantities

Length – *L*
Mass – *M*
Time – *T*
Temperature – *K*
Electric current – *A*

Some pairs of physical quantities whose dimensions are the same

1. *Impulse and momentum*
2. *Work and energy*
3. *Pressure or stress and modulus of elasticity*
4. *Surface tension and force constant*
5. *Angular momentum and Planck's constant*

EXERCISES

A. Multiple choice questions (Choose the best alternative)

1. Parsec is the unit of
   A. speed B. time C. distance D. velocity E. none of the above
2. Which of the following is not a unit of time?
   A. Leap year B. Micro second C. Lunar month D. Light year
   E. Nano second
3. Which of the following ratios is dimensionless
   A. work/power B. work/energy C. force/power D. momentum/energy
   E. impulse/force

4. Which one has the significant figures 3?
A. $0.007\ m^2$ B. 2.64×10^{24} kg C. $0.2370\ kg\ m^{-3}$ D. 6.320 J
E. $6.032\ Nm^{-2}$

5. A book with many printing errors contains four different formulae for displacement y of a particle undergoing a certain periodic motion. Here a = maximum displacement, v = speed, T = time period
(1) $y = a\sin(2\pi t/T)$ (2) $y = a\sin vt$.
(3) $y = (a/T)\sin(t/a)$ (4) $y = a\sqrt{2}(\sin(2\pi t/T) + \cos(2\pi t/T))$
A. (1) is wrong B. (4) is wrong C. (2) and (3) are wrong
D. only (2) is wrong E. all are wrong

6. The velocity of a particle v is given in terms of time t as $v = at + b/(t + c)$. The dimensions of a, b and c are
A. L^2, T, LT^{-2} B. LT^2, LT, L C. LT^{-2}, L, T D. $L : LT, T^2$
E. L^2, T, LT

7. The dimensions of gravitational constant G are
A. $ML^{-1}T^{-1}$ B. MLT^2 C. $M^{-1}L^3T^{-2}$ D. $M^2L^{-1}T^2$ E. M^2LT^{-3}

8. From the following pairs choose the pair that does not have identical dimensions
A. Impulse and momentum B. work and torque
C. moment of inertia and moment of a force
D. Angular momentum and Planck's constant E. Energy and work

9. The equation of state of a gas can be expressed as $(P + a/V^2)(V - b) = RT$. The dimensions of a are
A. ML^5T^{-2} B. $ML^{-5}T^2$ C. $ML^{-1}T^{-2}$ D. $ML^{-1}L^5T^{-2}$
E. $M^{-2}L^5T^{-2}$

10. The unit of impulse is the same as that of
A. energy B. force C. angular momentum D. linear momentum
E. power

11. A sphere of radius a moving down through a viscous medium with terminal velocity v experiences a force $F = 6\pi\eta av$. The dimensions of η are
A. MT^{-1} B. MLT^{-1} C. $ML^{-1}T^{-1}$ D. MLT E. $ML^{-1}T^{-2}$

12. The SI unit of length is metre. Suppose we adopt a new unit of length equal to x metre. The area $1\ m^2$ expressed in terms of the new unit has a magnitude
A. x B. x^2 C. $1/x$ D. $1/x^2$ E. none of the above

13. If $x = a + bt + ct^2$, when x is in metres and t is seconds then dimensions of c is
A. L B. LT^2 C. LT^{-2} D. T^2 E. LT

14. The equation of a wave having angular velocity ω and linear velocity v is given by $y = a\sin(\omega t - x/v)$. The dimensions of x is
A. L B. LT^{-1} C. $L^{-1}T$ D. LT^{-2} E. LT

15. An experiment measures the quantities a, b and c. The physical quantity x is calculated from the formula $x = ab^2/c^3$. The percentage error in a, b and c are $+1\%$, $+2\%$ and $+3\%$ respectively. The percentage error in x can be
A. $\pm 14\%$ B. $\pm 7\%$ C. $\pm 4\%$ D. $\pm 1\%$ E. $\pm 3\%$

16. If force F, velocity V and time T are chosen as base units then the dimensions of mass in terms of these are
A. FVT^{-2} B. $FV^{-1}T$ C. $FV^{-1}T^{-1}$ D. FVT E. FVT^{-1}

Key to multiple choice questions

(1) C (2) D (3) B (4) B (5) C (6) C (7) C (8) C (9) A (10) D (11) C (12) D (13) C (14) B (15) A (16) B

B. Very short answer questions

1. Which of the following length measurements is most accurate and why?
   5.0 cm; 0.005 cm; 0.0005 cm; 5.000 cm
2. What are the number of significant figures in the following?
   (i) 4.00 m (ii) 3.12×10^4 kg (iii) 0.00006
   (iv) 0.0000600 m (v) 60.32 J (vi) 0.006032 J
3. The definition of a metre is related to the speed of light in vacuum; why?
4. What are the base units in S.I.?
5. Mention the advantages of S.I. over the other systems of unit.
6. What do you understand by the dimensions of a physical quantity ?
7. Explain the principle of homogeneity of dimensions.
8. Write down the dimensional formulae of the following physical quantities:
   (i) Force (ii) Energy (iii) Surface tension (iv) Viscosity
   (v) Torque (vi) Angle (vii) Frequency.
9. Can a quantity have units but still be dimensionless ?
10. Can a quantity have dimensions but still have no units ?
11. Are all constants dimensionless ?
12. What is an astronomical unit (AU) ?
13. Distinguish between angstrom unit (Å) and astronomical unit (AU).
14. What is a light-year (ly) ?
15. Define parsec (pc).
16. Explain which of the following instruments is the most precise device for measuring length.
    (a) Vernier calipers with 10 divisions on the vernier.
    (b) Screw gauge of pitch 1 mm and 100 head scale divisions
    (c) An optical instrument that can measure length within a wavelength of light.
17. Give the order of magnitude of 988.
18. Explain relative error and percentage error.

C. Short answer questions

1. In the gas equation $(P + a/V^2)(V - b) = RT$, what are the dimensions of constants a and b?
2. If the units of length and force be increased four times each, show that the unit of energy is increased sixteen times.
3. If the units of length, mass and force are chosen as the fundamental units, what will be the dimension of time in terms of the dimensions of these units ?
4. What are the limitations of dimensional analysis?

5. Given below is an equation of a wave.

$$y = r \sin \omega \left(\frac{x}{v} - k\pi \right);$$

where ω is the angular velocity and v the linear velocity. What are the dimensions of x and k ?

Hint: since $\omega\left(\frac{x}{v} - k\pi\right)$ *is an angle, it is a dimensionless quantity.*

6. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 2.01 m and 1.005 m respectively. Find the total surface area and volume of the sheet to correct significant figures.

7. Check the accuracy of the physical relation:
$T = 2\pi\sqrt{l/g}$, for the period of a simple pendulum.

8. Using dimensions, check the correctness of the equation, kinetic energy = (1/2) mv^2.

9. Check the correctness of the equation $S_n = u + a(n - \frac{1}{2})$

D. Essays

1. What is meant by dimensions of a physical quantity? What are the uses of dimensional equations? Check the correctness of the following equations:
(i) $v^2 = u^2 + 2aS$; (ii) Kinetic energy $= 1/2mv^2$; (iii) $P = hdg$.

2. By the method of dimensions, derive an expression for the viscous force acting on a small spherical ball moving through a viscous medium.

E. Problems

1. Fill in the blanks
   (a) The volume of a cube of side 1 cm is equal to . . . m^3
   (b) The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to . . . $(mm)^2$
   (c) 1 kg m^2s^{-2} = . . . g cm^2s^{-2}
   (d) 3m s^{-2} = . . . km h^{-2} [NCERT]

2. 5.74 g of a substance occupies a volume 1.2 cm^3. Give the density of the substance correct to significant figures.
[Ans: 4.8 g/cm^3]

3. A 'laser' light beamed at moon takes 2.56 seconds to return after reflection from moon. What is the radius of the lunar orbit around the earth? [NCERT]
[Ans: 3.84 × 10^8 m]

4. In a submarine equipped with a 'sonar', the time delay between generation of probe wave and reception of its echo after reflection from an enemy submarine is 77 seconds. What is the distance of the enemy submarine? Speed of sound in water = 1450 ms^{-1}.
[NCERT]
[Ans: 55.8 km]

5. The velocity of water waves may depend on their wave length λ, the density of water ρ and the acceleration due to gravity g. Obtain the form of the relationship by dimensional method.
[Ans: $v \propto \sqrt{\lambda g}$]

6. The diameter of a wire is measured as 0.236 cm with the help of a screw gauge. The least count of the screw gauge is 0.001 cm. Calculate the possible percentage error in the measurement.

[Ans: 0.4%]

7. In the estimation of Young's modulus $Y = MgL/(\pi r^2 l)$ for a specimen the following observations are recorded. $L = 2.890$ m, $M = 3.00$ kg, $r = 0.041$ cm, $g = 9.81\ ms^{-2}$ and $l = 0.087$ cm. Calculate the possible percentage error.

[Ans: 6.36%]

8. The velocity of a particle varies with time according to the relation $v = at^2 + bt + c$, where v is in ms^{-1} and t in seconds. Find the dimensions and units of a, b and c.

[Ans: LT^{-3}, LT^{-2}, LT^{-1}, ms^{-3}, ms^{-2}, ms^{-1}]

9. The resistance $R = V/I$; where $V = (100 \pm 5)$ volt and $I = (10 \pm 0.2)$ ampere. Find the percentage error in R. [NCERT]

Hint: $V = 100$ volt; $\Delta V = 5$ volt; $I = 10$ ampere; $\Delta I = 0.2$ ampere; $(\Delta R/R) \times 100 = ?$

$R = V/I \quad \therefore (\Delta R/R) \times 100 = (\Delta V/V) \times 100 + (\Delta I/I) \times 100$ [Ans: 7%]

10. The moon is observed from two diameterically opposite points A and B on earth. The angle θ subtended at the moon by the two directions of observation is $1°54'$. Given the diameter of the earth to be about 1.276×10^7 m, compute the distance of the moon from the earth. [NCERT]

Hint: $x = 1.276 \times 10^7$ m; $\theta = 1°54' = 1.9° = 1.9 \times (\pi/180)$ rad.

$r = ?$; $\quad \theta = x/r$ [Ans: 3.85×10^8 m]

11. The sun's angular diameter is measured to be $1920''$. The distance of the sun from the earth is 1.496×10^{11} m. What is the diameter of the sun? [NCERT]

$\theta = 1920'' = (1920/60 \times 60)° = 0.533° = 0.533 \times (\pi/180)$ rad;

$x = ?$ $\quad r = 1.496 \times 10^{11}$ m. [Ans: 1.39×10^9 m]