Guidelines to NCERT Exercises

3.1. *In which of the following examples of motion can the body be considered approximately a point object :*

(*i*) *a railway carriage moving without jerks between two stations.*

(*ii*) *a monkey sitting on the top of a man cycling smoothly on a circular track.*

(*iii*) *a spinning cricket ball that turns sharply on hitting the ground, and*

(*iv*) *tumbling beaker that has slipped off the edge of a table ?*

Ans.

(*i*) The carriage can be considered as the point object because the distance between two stations is much larger than the size of the carriage.

(*ii*) The monkey can be considered as a point object because its size is much smaller than the distance covered by the cyclist.

(*iii*) The spinning ball cannot be considered as point object because its size is quite appreciable as compared to the distance through which it turns on hitting the ground.

(*iv*) The tumbling beaker slipping off the edge of a table cannot be considered a point object because its size is not negligibly smaller than the height of the table.

3.2. *The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.82. Choose the correct entries in the brackets below :*

(*a*) *A / B lives closer to the school than B / A.*

(*b*) *A / B starts from the school earlier than B / A.*

(*c*) *A / B walks faster than B / A.*

(*d*) *A and B reach home at the (same/different) time.*

(*e*) *A / B overtakes B / A on the road (once/twice.)*

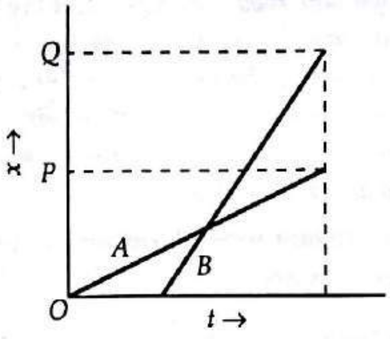


Fig. 3.82

Ans.

(*a*) As $OP < OQ$, *A* **lives closer to the school than** *B*.

(*b*) When $x = 0, t = 0$ for *A*; while *t* has some finite value when *B* starts moving. So *A* **starts from the school earlier than** *B*

(*c*) Speed = Slope of x - t graph.

Slope of x - t graph for *B* > Slope of x - t graph for *A*.

∴ *B* walks faster than *A*

(*d*) Corresponding to the positions *P* and *Q*, time *t* is same on time-axis.

∴ *A* and *B* reach home at the same time.

(*e*) The x - t graphs for *A* and *B* intersect each other only once. Since *B* starts from the school afterwards, so *B* **overtakes** *A* **on the road once.**

3.3. *A woman starts from her home at 9.00 A.M. walks with a speed of 5 kmh^{-1} on a straight road upto her office 2.5 km away, stays at the office upto 5 P.M. and returns home by an auto with a speed of 25 kmh^{-1}. Choose suitable scales and plot the x-t graph of her motion.*

Ans. ***For the journey from home to office :***

The time at which woman leaves for her office is 9 A.M.

As she travels with a speed of 5 kmh^{-1} and the distance of office is 2.5 km, hence time taken by her to reach office,

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2.5 \text{ km}}{5 \text{ kmh}^{-1}} = \frac{1}{2} \text{ h}$$

Hence the time at which she reaches office is 9.30 A.M.

Between 9.30 A.M. to 5.00 P.M., she stays in her office i.e. at a distance of 2.5 km from her home.

For the return journey from office to home :

The time at which she leaves her office = 5 P.M.

Now she travels a distance of 2.5 km with a speed of 25 kmh^{-1}, hence the time taken

$$t' = \frac{2.5 \text{ km}}{25 \text{ kmh}^{-1}} = \frac{1}{10} \text{ h} = 6 \text{ minutes}$$

The time at which she reaches her home = 5.06 P.M.

The *x-t* graph of the woman's motion is shown in Fig. 3.83. Here 9 A.M. is regarded as the origin for time-axis and home as the origin for the position-axis.

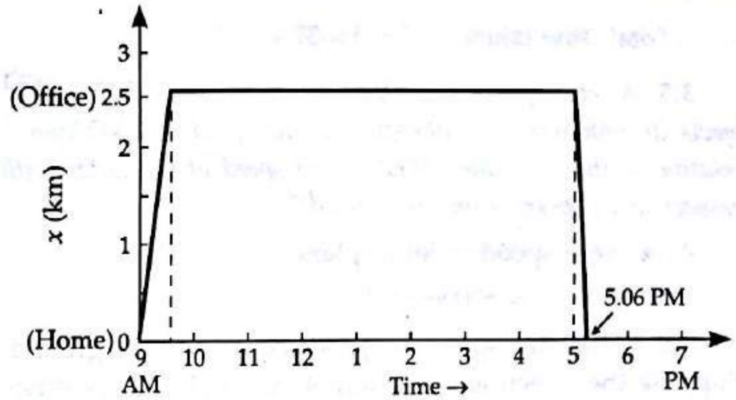


Fig. 3.83

3.4. *A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.*

Ans. (*a*) **Graphical method.** Taking the starting point as origin, the positions of the drunkard at various instants of time are given in the following table.

t (s)	0	5	8	13	16	21	24	29	32	37
x (m)	0	5	2	7	4	9	6	11	8	13

The position-time (x-t) graph for the motion of the drunkard is shown in Fig. 3.84. As is obvious from graph that the drunkard would take 37 s to fall in a pit 13 m away from the starting point.

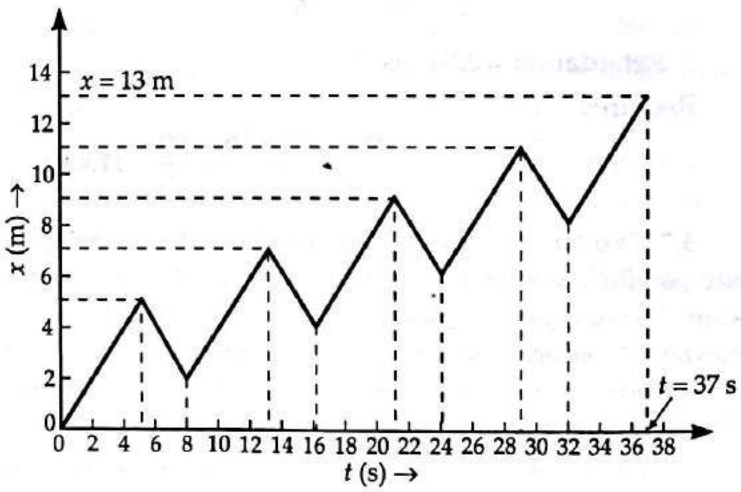


Fig. 3.84

(*b*) **Analytical method.** In each forward motion of 5 steps and backward motion of 3 steps, net distance covered = 5 – 3 = 2 m and time taken = 5 + 3 = 8 s.

∴ Time required to cover a distance of 8 m

$$= \frac{8}{2} \times 8 = 32 \text{ s}$$

Remaining distance of the pit = 13 – 8 = 5 m

In next 5 s, as he moves 5 steps forward, he falls into the pit.

$\therefore$ Total time taken = 32 + 5 = **37 s.**

3.5. *A jet airplane travelling at the speed of 500 kmh^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?*

Ans. Here speed of jet airplane,

$$v_1 = 500\ kmh^{-1}$$

Let v_2 be the speed of products w.r.t. the ground. Suppose the direction of motion of the jet plane is positive. Then the relative velocity of products w.r.t. jet plane is

$$v_2 - v_1 = -1500$$

or $$v_2 = v_1 - 1500 = 500 - 1500 = -1000\ kmh^{-1}$$

Negative sign shows that the direction of products of combustion is opposite to that of the jet plane.

$\therefore$ Speed of products of combustion w.r.t. ground

$$= \mathbf{1000\ kmh^{-1}}.$$

3.6. *A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?*

Ans. Here $u = 126\ kmh^{-1} = 126 \times \frac{5}{18} = 35\ ms^{-1}$,

$$v = 0, \quad s = 200\ m$$

As $$v^2 - u^2 = 2as$$

$\therefore$ $$0^2 - 35^2 = 2a \times 200$$

or $$a = -\frac{35 \times 35}{2 \times 200} = -\frac{49}{16} = -3.06\ ms^{-2}$$

$\therefore$ Retardation = $\mathbf{3.06\ ms^{-2}}$.

Required time,

$$t = \frac{v-u}{a} = \frac{0-35}{-49/16} = \frac{35 \times 16}{49} = \frac{80}{7} = \mathbf{11.43\ s.}$$

3.7. *Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?*

Ans. Let x be the distance between the driver of train A and the guard of train B. Initially, both trains are moving in the same direction with the same speed of 72 km h^{-1}. So relative velocity of B w.r.t. $A = v_B - v_A = 0$. Hence the train B needs to cover a distance with

$$a = 1\ ms^{-2},\ t = 50\ s,\ u = 0$$

As $$s = ut + \frac{1}{2}at^2$$

$\therefore$ $$x = 0 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = \mathbf{1250\ m.}$$

3.8. *On a two-lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident ?*

Ans. At the instant when B decides to overtake A, the speeds of three cars are

$$v_A = 36\ kmh^{-1} = 36 \times \frac{5}{18} = +10\ ms^{-1}$$

$$v_B = +54\ kmh^{-1} = +54 \times \frac{5}{18} = +15\ ms^{-1}$$

$$v_C = -54\ kmh^{-1} = -15\ ms^{-1}$$

Relative velocity of C w.r.t. A,

$$v_{CA} = v_C - v_A = -15 - 10 = -25\ ms^{-1}$$

$\therefore$ Time that C requires to just cross A

$$= \frac{1\ km}{v_{CA}} = \frac{1000\ m}{25\ ms^{-1}} = 40\ s$$

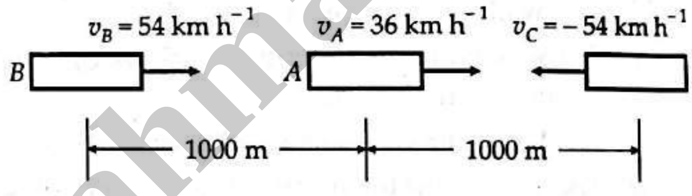


Fig. 3.85

In order to avoid the accident, B must overtake A in a time less than 40 s. So, for car B we have

Relative velocity of car B w.r.t. A,

$$v_{BA} = v_B - v_A = 15 - 10 = 5\ ms^{-1}$$

$\therefore$ $$s = 1\ km = 1000\ m,\ u = 5\ ms^{-1}, t = 40\ s$$

As $$s = ut + \frac{1}{2}at^2$$

$\therefore$ $$1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$

or $$1000 = 200 + 800\ a$$

or $$a = \mathbf{1\ ms^{-2}}$$

Thus 1 ms^{-2} is the minimum acceleration that car B requires to avoid an accident.

3.9. *Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?*

Ans. Let speed of each bus = v kmh^{-1}

For buses going from town A to B :

Relative speed of a bus in the direction of motion of the man $= (v - 20)\ kmh^{-1}$

Buses plying in this direction go past the cyclist after every 18 min.

$\therefore$ Distance covered

$$= (v - 20)\frac{18}{60} \text{ km}$$

Since a bus leaves the town after every T min, so the above distance covered

$$= v \times \frac{T}{60} \text{ km}$$

$$\therefore \quad (v - 20)\frac{18}{60} = v \times \frac{T}{60} \quad ...(i)$$

For buses going from town B to A :

Relative speed of bus in the direction opposite to the motion of the man

$$= (v + 20) \text{ kmh}^{-1}$$

Buses going in this direction go past the cyclist after every 6 min, therefore

$$(v + 20)\frac{6}{60} = v \times \frac{T}{60} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{(v - 20)\,18}{(v + 20)\,6} = 1$$

or $\quad 3v - 60 = v + 20$

or $\quad v = \mathbf{40 \text{ kmh}^{-1}}.$

From equation (*ii*),

$$(40 + 20)\frac{6}{60} = \frac{40 \times T}{60}$$

or $\quad T = \frac{60 \times 6}{40} = \mathbf{9 \text{ min.}}$

3.10. *A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.*

(*i*) *What is the direction of acceleration during the upward motion of the ball ?*

(*ii*) *What are the velocity and acceleration of the ball at the highest point of its motion ?*

(*iii*) *Choose the $x = 0$ and $t = 0$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of X-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.*

(*iv*) *To what height does the ball rise and after how long does the ball return to the player's hands ?*

(Take $g = 9.8$ ms^{-2}, and neglect air resistance).

Solution. (*i*) The ball moves under the effect of gravity. The direction of acceleration due to gravity is always *vertically downwards.*

(*ii*) At the highest point, velocity of the ball = **0.**

Acceleration = Acceleration due to gravity 'g'.

$$= \mathbf{9.8 \text{ ms}^{-2}} \quad \text{(vertically downwards)}$$

(*iii*) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of X-axis :

During upward motion. Position is positive, velocity is negative and acceleration is positive.

During downward motion. Position is positive, velocity is positive and acceleration is positive.

(*iv*) *For upward motion.*

$$u = -29.4 \text{ ms}^{-1}, \; g = +9.8 \text{ ms}^{-2}, \; v = 0$$

If s is the height to which the ball rises, then

$$v^2 - u^2 = 2as$$

or $\quad 0^2 - (-29.4)^2 = 2 \times 9.8 \times s$

or $\quad s = -\frac{(29.4)^2}{2 \times 9.8} = \mathbf{-44.1 \text{ m}}$

Negative sign shows that the distance is covered in upward direction.

If the ball reaches the highest point in time t, then

$$v = u + at$$

or $\quad 0 = -29.4 + 9.8t$

or $\quad t = \frac{29.4}{9.8} = 3 \text{ s}$

As $\quad$ time of ascent = time of descent

$\therefore \quad$ Total time taken = 3 + 3 = **6 s.**

3.11. *Read each statement below carefully and state with reasons and examples, if it is true or false. A particle in one-dimensional motion :*

(*a*) *with zero speed at an instant may have non-zero acceleration at the instant,*

(*b*) *with zero speed may have non-zero velocity,*

(*c*) *with constant speed must have zero acceleration,*

(*d*) *with positive value of acceleration must be speeding up.*

Ans.

(*a*) **True.** When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 ms^{-2}.

(*b*) **False.** Speed is the magnitude of velocity and the magnitude of non-zero velocity cannot be zero.

(*c*) **True.** When a particle moves with a constant speed in the same direction, neither the magnitude nor the direction of velocity changes and so acceleration is zero. In case a particle rebounds instantly with the same speed, its acceleration will be infinite which is physically not possible.

(*d*) **False.** If the initial velocity of a body is negative, then even in case of positive acceleration, the body speeds down. A body speeds up when the acceleration acts in the direction of motion.

3.12. *A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.*

Ans. (*i*) Time taken by the ball to fall through a height of 90 m is obtained as follows :

$$x = v(0)t + \frac{1}{2}gt^2$$

$$90 = 0 + \frac{1}{2} \times 9.8 t^2$$

or $$t = \sqrt{\frac{2 \times 90}{9.8}} = \frac{30}{7} \text{ s} \simeq 4.3 \text{ s}$$

Now $\quad v(t) = v(0) + gt$

$\therefore \quad v(4.3) = 0 + 9.8 \times \frac{30}{7} = 42 \text{ ms}^{-1}$

From time $t = 0$ to $t = 4.3$ s,

$$v(t) = gt = 9.8t \quad \text{or} \quad v(t) \propto t$$

In this duration speed increases linearly with time t from 0 to 42 ms^{-1} during the downward motion of the ball and this speed–time variation has been shown by straight line *OA* in Fig. 3.86.

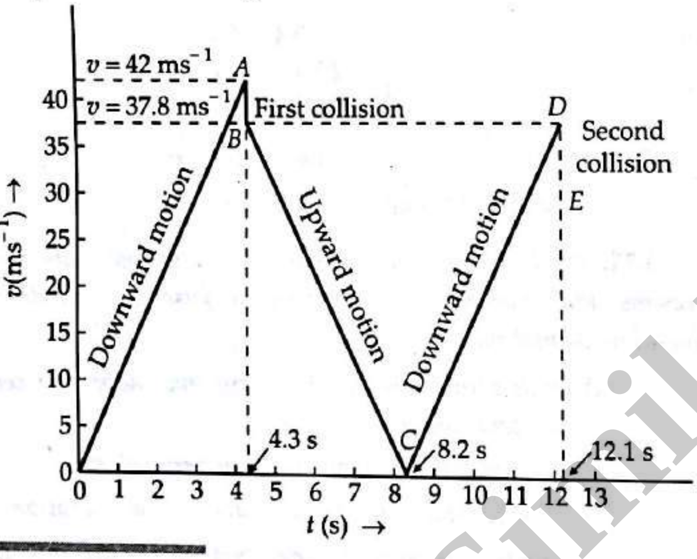


Fig. 3.86

(*ii*) At first collision with the floor, speed lost by ball

$$= \frac{1}{10} \times 42 = 4.2 \text{ ms}^{-1}.$$

Thus, the ball rebounds with a speed of $42 - 4.2 = 37.8 \text{ ms}^{-1}$. For the further upward motion, the speed at any instant t is given by

$$v(t) = v(0) - gt = 37.8 - 9.8 \times t$$

Now the speed decreases linearly with time and becomes zero after time

$$t = \frac{37.8}{9.8} \simeq 3.9 \text{ s}$$

Thus, the ball reaches the highest point again after time $t = 4.3 + 3.9 = 8.2$ s from the start. Straight line *BC* represents the speed-time graph for this upward motion.

(*iii*) At highest point, speed of ball is zero. It again starts falling. At any instant *t*, its speed is given by

$$v(t) = 0 + 9.8t$$

Again the speed of the ball increases linearly with time *t* from 0 to 37.8 ms^{-1} (initial speed of the previous upward motion) in the next time-interval of 3.9 s. Total time taken from the start = 4.3 + 3.9 + 3.9 = 12.1 s. This part of motion has been shown by straight line CD.

Here, we have assumed a negligible time of collision between the ball and the floor.

3.13. *Explain clearly, with examples, the distinction between :*

(*a*) *magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval ;*

(*b*) *magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].*

(*c*) *Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].*

Solution.

(*a*) Suppose a body moves from point *A* and to point *B* along a straight path and then returns back to the point *A* along the same path.

As the body returns back to its initial position *A*, so magnitude of displacement = 0.

Distance covered

= Total length of the path covered

= *AB* + *BA* = *AB* + *AB* = 2*AB*.

(*b*) In the above example, suppose the body takes time *t* to complete the whole journey. Then

Magnitude of average velocity

$$= \frac{\text{Magnitude of displacement}}{\text{Time taken}} = \frac{0}{t} = 0.$$

$$\text{Average speed} = \frac{2AB}{t}.$$

(*c*) In example (*a*), distance covered > magnitude of displacement.

In example (*b*), average speed > magnitude of average velocity.

The sign of equality will hold when the body moves along a straight line path in a fixed direction.

3.14. *A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the*

(*a*) *magnitude of average velocity, and*

(*b*) *average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min ?*

Ans. Case (*i*) : *0 to 30 min*

Speed = $5\ \text{kmh}^{-1}$

Distance covered in 30 min

$$= 5\ \text{kmh}^{-1} \times \frac{30}{60}\ \text{h} = 2.5\ \text{km}$$

Displacement covered = 2.5 km

(*a*) Average velocity

$$= \frac{\text{Displacement}}{\text{Time}} = \frac{2.5\ \text{km}}{30/60\ \text{h}} = \mathbf{5\ kmh^{-1}}.$$

(*b*) Average speed

$$= \frac{\text{Distance}}{\text{Time}} = \frac{2.5\ \text{km}}{30/60\ \text{h}} = \mathbf{5\ kmh^{-1}}.$$

Case (*ii*) : *0 to 50 min*

Displacement covered in first 30 min in going to market

$$= 5\ \text{kmh}^{-1} \times \frac{30}{60}\ \text{h} = 2.5\ \text{km}$$

Displacement covered in next 20 min in coming back to home

$$= 7.5\ \text{kmh}^{-1} \times \frac{20}{60}\ \text{h} = 2.5\ \text{km}$$

Net displacement = 2.5 − 2.5 = 0

Total distance covered = 2.5 + 2.5 = 5 km

(*a*) Average velocity

$$= \frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{50/60\ \text{h}} = \mathbf{0}.$$

(*b*) Average speed

$$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{5\ \text{km}}{50/60\ \text{h}} = \mathbf{6\ kmh^{-1}}.$$

Case (*iii*) : *0 to 40 min*

Displacement covered in first 30 min in going to market = 2.5 km

Displacement covered in next 10 min in coming back to home

$$= 7.5\ \text{kmh}^{-1} \times \frac{10}{60}\ \text{h} = 1.25\ \text{km}$$

Net displacement = 2.5 − 1.25 = 1.25 km

Total distance travelled = 2.5 + 1.25 = 3.75 km

(*a*) Average velocity

$$= \frac{\text{Net displacement}}{\text{Time taken}}$$

$$= \frac{1.25\ \text{km}}{40/60\ \text{h}} = \mathbf{1.875\ kmh^{-1}}.$$

(*b*) Average speed

$$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{3.75\ \text{km}}{40/60\ \text{h}} = \mathbf{5.625\ kmh^{-1}}.$$

3.15. *The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?*

Ans. Instantaneous speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous velocity,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

For an arbitrary small interval of time (Δt), the magnitude of displacement $|\Delta \vec{x}|$ is equal to the length of the path Δx. So instantaneous speed is always equal to the magnitude of instantaneous velocity.

3.16. *Look at the graphs (a) to (d) [Fig. 3.87] carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.*

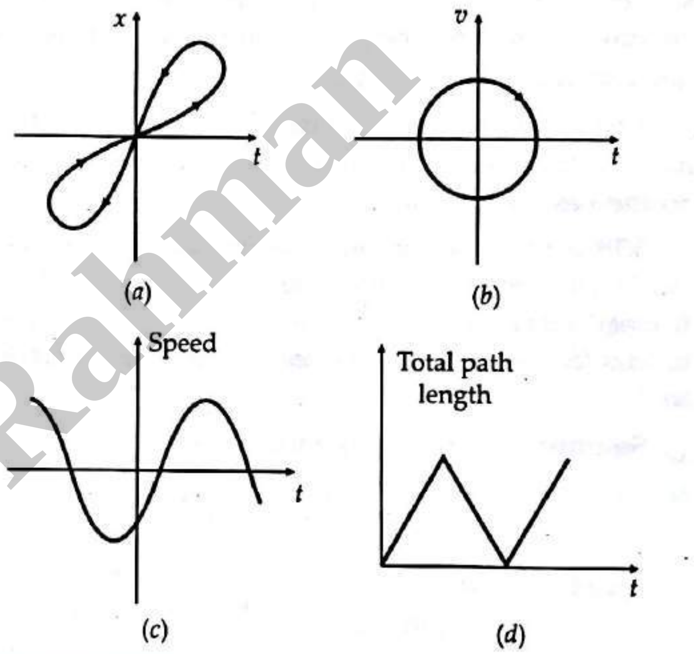


Fig. 3.87

Ans. All the four graphs are impossible.

(*a*) If we draw a line parallel to the position-axis, it intersects the *x-t* graph at two points. This means that the particle occupies two different positions at the same time which is not possible.

(*b*) If we draw a line parallel to the velocity-axis, it meets the circle in two points. This means the particle has two velocities (positive and negative) in opposite directions at the same time. This is not possible.

(*c*) The graph indicates that speed is negative in some time intervals. But speed cannot be negative.

(*d*) The graph indicates that the total path length decreases after a certain time. But total path length can never decrease with time.

3.17. *Fig. 3.88 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.*

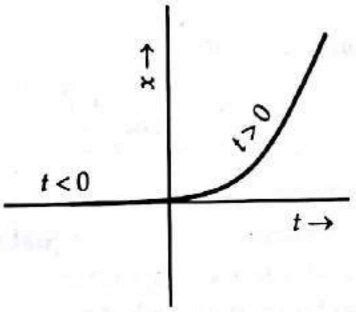


Fig. 3.88

Ans. No, it is wrong to say that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$, because a position-time (x-t) graph does not represent the trajectory of a moving particle.

This graph can represent the motion of a freely falling particle dropped from a tower when we take its initial position as $x = 0$, at $t = 0$.

3.18. *A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?*

Solution. Speed of police van,

$$v_p = 30 \text{ kmh}^{-1} = \frac{25}{3} \text{ ms}^{-1}$$

Speed of bullet,

$$v_b = 150 \text{ ms}^{-1}$$

Speed of the police van is shared by the bullet.

∴ Relative speed of bullet w.r.t. ground

$$= v_b + v_p$$

$$= 150 + \frac{25}{3}$$

$$= \frac{475}{3} \text{ ms}^{-1}$$

Speed of thief's car,

$$v_t = 192 \text{ kmh}^{-1} = \frac{160}{3} \text{ ms}^{-1}$$

Relative speed of bullet w.r.t. thief's car

$$= (v_b + v_p) - v_t$$

$$= = \frac{475}{3} - \frac{160}{3}$$

$$= 105 \text{ ms}^{-1}.$$

Hence the speed of the bullet with which it hits the thief's car = **105 ms^{-1}**.

3.19. *Suggest a suitable physical situation for each of the following graphs [Fig. 3.89]:*

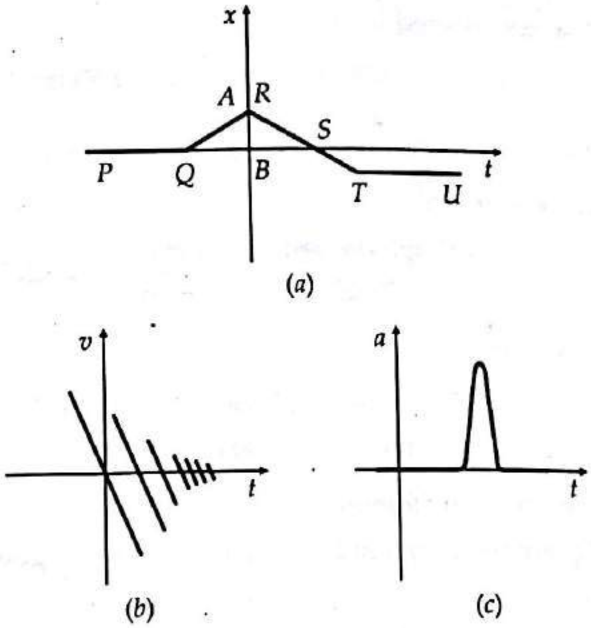


Fig. 3.89

Ans.

(a) A ball lies at rest on a smooth floor, as indicated by straight line PQ of the x-t graph. It is kicked towards a wall. Slope of line QR gives speed of kicking. The ball rebounds from the wall with a reduced speed (given by the slope of line RS). At S, the sign of position coordinate changes sign, it indicates that the ball moves to the opposite wall which stops it. The line TU indicates the final rest position of the ball.

(b) The velocity-time graph represents the motion of a ball thrown up with some initial velocity, it hits the ground and gets rebounded with a reduced speed. It goes on hitting the ground and after each hit its speed decreases unit it becomes zero and the ball comes to rest finally.

(c) The acceleration-time graph represents the motion of a uniformly moving cricket ball turned back by hitting it with a bat for a very short time interval.

3.20. *Figure 3.90 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2s, − 1.2s.*

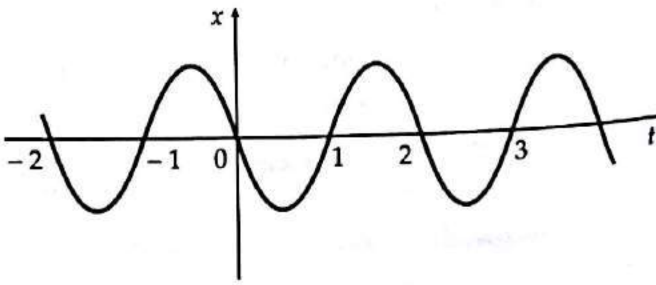


Fig. 3.90

Ans. The acceleration of a particle executing S.H.M. is given by

$$a = -\omega^2 x$$

where ω (angular frequency) is a constant.

At time $t = 0.3$ s

As is obvious from the graph, $x < 0$

As slope of x-t graph is negative, so $v < 0$

As $a = -\omega^2 x$, so $a > 0$.

At time $t = 1.2$ s

As is obvious from the graph, $x > 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a < 0$.

At time $t = -1.2$ s

As is obvious from the graph, $x < 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a > 0$.

3.21. *Figure 3.91 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.*

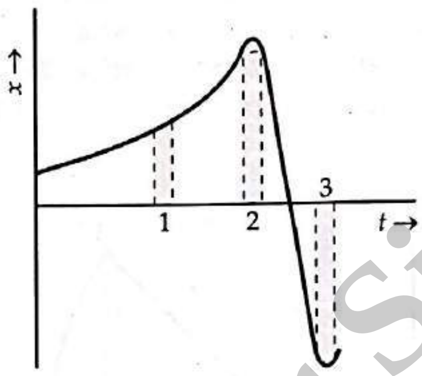


Fig. 3.91

Solution. Slope of x-t graph in a small time interval
= Average speed in that interval

As the slope of x-t is greatest in interval 3 and least in interval 1, so **the average speed is greatest in interval 3 and least in interval 1.**

As the slope of x-t is positive in intervals 1 and 2 and negative in interval 3, **so average velocity is positive in intervals 1 and 2 and negative in interval 3.**

3.22. *Figure 3.92 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. (a) In which interval is the average acceleration greatest in magnitude ? (b) In which interval is the average speed greatest ? (c) Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. (d) What are the accelerations at the points A, B, C and D ?*

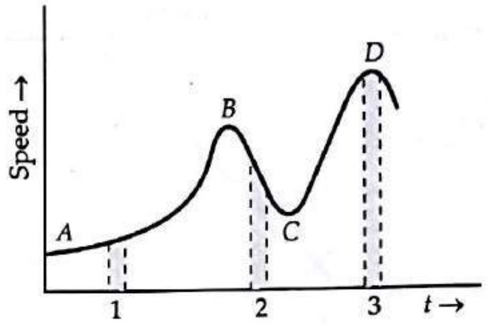


Fig. 3.92

Ans.

(*a*) As the change in speed is greatest in interval 2, so **magnitude of average acceleration is greatest in interval 2.**

(*b*) Obviously from the graph, **average speed is greatest in interval 3.**

(*c*) v is positive in all three intervals. a is positive in the intervals 1 and 3 as speed is increasing in these intervals. But a is negative in interval 2 as speed is decreasing in this interval.

(*d*) At the points A, B, C and D, the v-t graph is parallel to the time-axis or has a zero slope, so $a = 0$ at all these points.

3.23. *A three-wheeler starts from rest, accelerates uniformly with $1\ \text{ms}^{-2}$ on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second ($n = 1, 2, 3\ldots$) versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?*

Ans. Distance travelled in nth second,

$$s_{n\,th} = u + \frac{a}{2}(2n - 1)$$

As $\quad u = 0,\ a = 1\ \text{ms}^{-2}$

$$\therefore \quad s_{n\,th} = 0 + \frac{1}{2}(2n - 1) = \frac{1}{2}(2n - 1)\ \text{m}$$

Thus the distances travelled by the three-wheeler at the end of each second are given by

n(s)	1	2	3	4	5	6	7	8	9	10
$s_{n\,th}$ (m)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

Now, velocity of the three-wheeler at the end of 10th s is given by

$$v = u + at = 0 + 1 \times 10 = 10\ \text{ms}^{-1}.$$

Upto $n = 10$ s, the motion is accelerated and the graph between s_{nth} and n is a straight line AB inclined to time-axis as shown in Fig. 3.93. After 10th second, the three-wheeler moves with uniform velocity of 10 ms^{-1}, so graph is straight line *BC* parallel to time-axis.

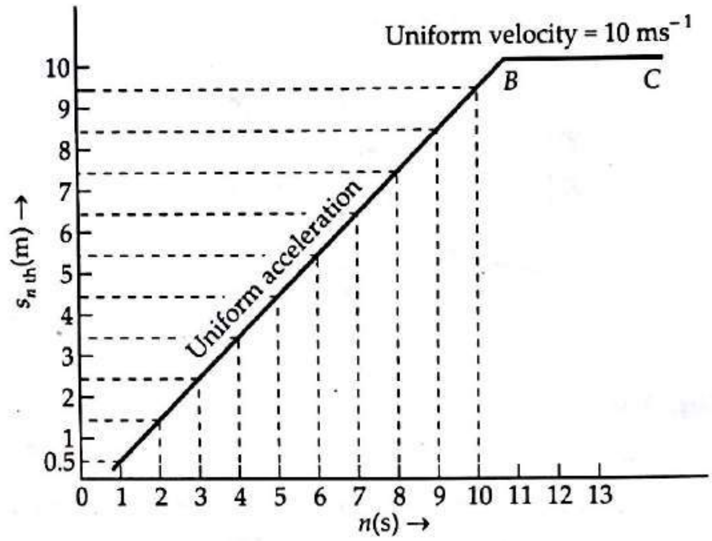


Fig. 3.93

3.24. *A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms^{-1}. (i) How much time does the ball take to return to his hands ? (ii) If the lift starts moving up with a uniform speed of 5 ms^{-1}, and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?*

Ans. (*i*) *When the lift is stationary* : For upward motion of the ball, we have

$$u = 49\ ms^{-1},\quad g = -9.8\ ms^{-2},\quad v = 0,\quad t = ?$$

As $\qquad v = u + at$

$\therefore \qquad 0 = 49 - 9.8t \quad$ or $\quad t = \frac{49}{9.8} = 5$ s.

As time of ascent = time of descent

$\therefore$ Total time taken = 5 + 5 = **10 s.**

(*ii*) *When the lift moves up with uniform speed* : The uniform speed of the lift does not change the relative velocity of the ball w.r.t. the boy *i.e.* it still remains 49 ms^{-1}.

Hence total time after which the ball returns = **10 s.**

3.25. *On a long horizontally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the*

(*i*) *speed of the child running in the direction of motion of the belt,*

(*ii*) *speed of the child running opposite to the direction of motion of the belt, and*

(*iii*) *time taken by the child in (i) and (ii) ?*

Which of the answers alter if motion is viewed by one of the parents ?

Ans. (*i*) Speed of the child running in the direction of motion of the belt

$$= (9 + 4)\ kmh^{-1} = \mathbf{13\ kmh^{-1}}.$$

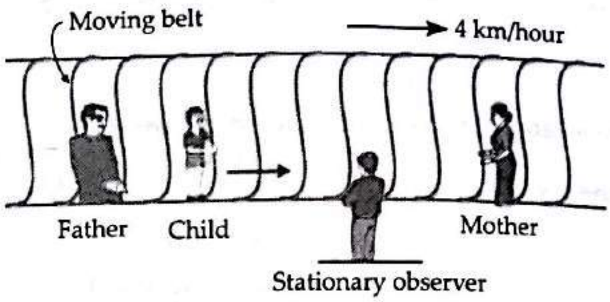


Fig. 3.94

(*ii*) Speed of the child running opposite to the direction of motion of the belt

$$= (9 - 4)\ kmh^{-1} = \mathbf{5\ kmh^{-1}}.$$

(*iii*) Speed of the child w.r.t. either parent

$$= 9\ kmh^{-1} = 2.5\ ms^{-1}$$

Distance to be covered = 50 m

$$\text{Time taken} = \frac{50}{2.5} = \mathbf{20\ s.}$$

If the motion is viewed by one of the parents, answers to (*i*) and (*ii*) are altered but answer to (*iii*) remains unchanged.

3.26. *Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the following graph correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10\ ms^{-2}$. Give the equations for the linear and curved parts of the plot.*

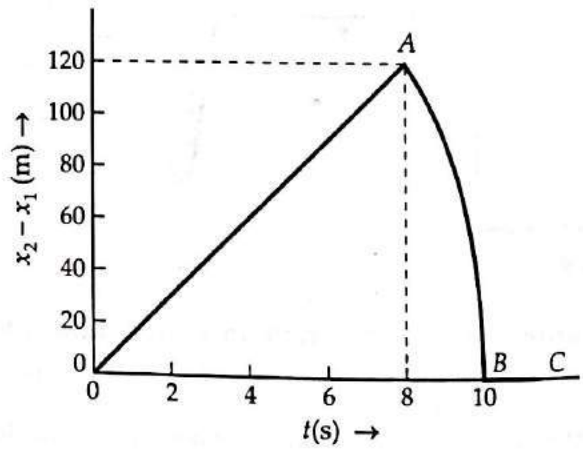


Fig. 3.95

Ans. $\quad x(t) = x(0) + v(0)t + \frac{1}{2}gt^2$

If we take origin for position measurement on the ground, then the positions of the two stones at any instant *t* will be

$$x_1 = 200 + 15t - \frac{1}{2} \times 10t^2 \qquad ...(1)$$

$$x_2 = 200 + 30t - \frac{1}{2} \times 10t^2 \qquad ...(2)$$

When the first stone hits the ground,

$$x_1 = 0$$

$$200 + 15t - 5t^2 = 0$$

or $$5t^2 - 15t - 200 = 0$$

or $$t^2 - 3t + 40 = 0$$

or $$\therefore \quad t = \frac{3 \pm \sqrt{9+160}}{2} = \frac{3 \pm 13}{2} = 8\,\text{s} \text{ or } -5\,\text{s}$$

As time cannot be negative, so $t = 8$ s

i.e. the first stone hits the ground after 8 s.

From (1) and (2), the relative position of second stone *w.r.t.* first is given by

$$x_2 - x_1 = 15t$$

As there is a linear relationship between $x_2 - x_1$ and t, so the graph is straight line OA upto $t = 8$ s. After $t = 8$ s, only the second stone is in motion. So the graph is parabolic (AB) in accordance with quadratic equation,

$$x_2 = 200 + 30t - 5t^2$$

The second stone will hit the ground, when $x_2 = 0$

or $$200 + 30t - 5t^2 = 0$$

On solving, $t = 10$ s

After $t = 10$ s, the separation between the balls is zero, which explains the part BC of the graph.

3.27. *The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.96. Obtain the distance travelled by the particle between (i) $t = 0$ to 10 s (ii) $t = 2$ to 6 s. What is the average speed of the particle in intervals in (i) and (ii)?*

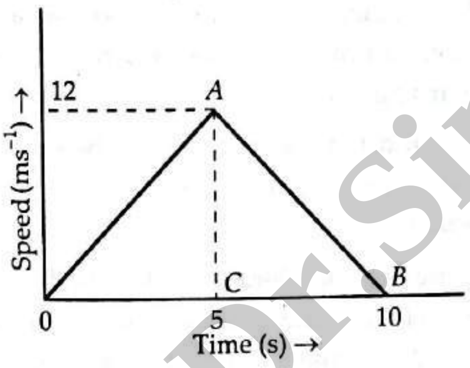


Fig. 3.96

Ans. (*i*) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \Delta\, OAB = \frac{1}{2}\, OB \times AC$$

$$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}$$

Average speed

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{60}{10} = \mathbf{6\ ms^{-1}}.$$

(*ii*) Acceleration of the particle during journey OA is given by

$$v = u + at \quad \text{or} \quad 12 = 0 + a \times 5$$

or $$a = +2.4 \text{ ms}^{-2}$$

Similarly, acceleration of the particle during journey AB is given by

$$v = u + at \quad \text{or} \quad 0 = 12 + a \times 5$$

or $$a = -2.4 \text{ ms}^{-2}$$

Velocity of the particle after 2 s from start will be

$$v = u + at = 0 + 2.4 \times 2 = 4.8 \text{ ms}^{-1}$$

$\therefore$ Distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2}at^2$$

$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times 3^2 = 25.2 \text{ m}$$

Distance covered by the particle in $t = 5$ to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2}at^2$$

$$= 12 \times 1 + \frac{1}{2} \times (-2.4) \times 1^2 = 10.8 \text{ m}$$

Total distance travelled in $t = 2$ to 6 s,

$$s = s_1 + s_2 = 25.2 + 10.8 = 36 \text{ m}$$

Average speed in the interval $t = 2$ to 6 s

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{36}{4} = \mathbf{9\ ms^{-1}}.$$

3.28. *The velocity-time graph of particle in one-dimensional motion is shown in Fig. 3.97.*

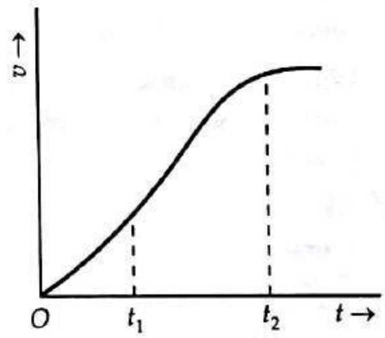


Fig. 3.97

Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2:

(*a*) $x(t_2) = x(t_1) + v(t_1) \times (t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

(*b*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*c*) $v_{av} = \{x(t_2) - x(t_1)\}/(t_2 - t_1)$

(*d*) $a_{av} = \{v(t_2) - v(t_1)\}/(t_2 - t_1)$

(*e*) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + (\frac{1}{2})a_{av}(t_2 - t_1)^2$.

(*f*) $x(t_2) - x(t_1)$ = *area under the v-t curve bounded by the t-axis and the dotted line shown.*

Ans. (*a*) It is not correct because in the time interval between t_1 and t_2, a is not constant.

(*b*) This relation is also not correct for the same reason as in (*a*).

(*c*) This relation is correct.

(*d*) This relation is also correct.

(*e*) This relation is not correct because average acceleration cannot be used in this relation.

(*f*) This relation is correct.

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. Are rest and motion absolute or relative terms ?
2. Can an object be at rest as well as in motion at the same time ?
3. Is it true that a body is at rest in a frame within which it has been fixed ?
4. Under what condition can an object in motion be considered a point object ?
5. Give an example of a physical phenomenon in which earth cannot be regarded as a point mass.
6. Under what condition will the distance and displacement of moving object have the same magnitude ? **[Chandigarh 08]**
7. A bullet fired vertically upwards falls at the same place after some time. What is the displacement of the bullet ?
8. A particle is moving along a circular track of radius r. What is the distance traversed by particle in half revolution ? What is its displacement ?
9. Will the displacement of an object change on shifting the position of origin of the coordinate system ? **[Himachal 06C]**
10. What does the speedometer of a car measure-average speed or instantaneous speed ?
11. What is the numerical ratio of velocity to speed of an object ?
12. A ball hits a wall with a velocity of 30 ms^{-1} and rebounces with the same velocity. What is the change in its velocity ?
13. Why does time occur twice in the unit of acceleration ?
14. Give an example which shows that a positive acceleration can be associated with a slowing down object.
15. Give an example which shows that a negative accele ration can be associated with a speeding up object.
16. Is the acceleration of a car greater than when the accelerator is pushed to the floor or when brake pedal is pushed hard ?
17. The v-t graphs of two objects make angles of 30° and 60° with the time-axis. Find the ratio of their accelerations.
18. Is it possible that your cycle has a northward velocity but southward acceleration ? If yes, how ?
19. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero ?
20. A woman standing on the edge of a cliff throws a ball straight up with a speed of 8 kmh^{-1} and then throws another ball straight down with a speed of 8 kmh^{-1} from the same position. What is the ratio of the speeds with which the balls hit the ground ?
21. A body travels, with uniform acceleration a_1 for time t_1 and with uniform acceleration a_2 for time t_2. What is the average acceleration ?
22. What is the nature of position-time graph for a uniform motion ? **[Chandigarh 03]**
23. What does the slope of position-time graph indicate ? **[Himachal 07]**
24. What is the nature of velocity-time graph for uniform motion ?
25. If the displacement-time graph for a particle is parallel to displacement axis, what should be the velocity of the particle ?
26. If the displacement-time graph for a particle is parallel to time-axis, how much is the velocity of the particle ?
27. How can the distance travelled be calculated from the velocity-time graph in a uniform one-dimensional motion ?
28. Suppose the acceleration of a body varies with time. Then what does the area under its acceleration-time graph for any time interval represent ?
29. What is the area under the velocity-time curve in the case of a body projected vertically upwards from the ground after reaching the ground ?
30. Can a particle with zero acceleration speed up ?
31. Is the formula : $s = vt - \frac{1}{2}at^2$ correct, when the body is moving with uniform acceleration ?
32. A body projected up reaches a point P of its path at the end of 4 seconds and the highest point at the end of 12 seconds. After how many seconds from the start will it reach P again ?
33. Can a body subjected to a uniform acceleration always move in a straight line ?

34. Suggest a suitable physical situation for the graph shown in Fig. 3.89(*b*) on page 3.50.

35. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time-interval. Suggest acceleration-time graph for the situation.

36. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$ (*x* in metres and *t* in seconds). What is its velocity at $t = 2$ sec. ? **[Central Schools 14]**

37. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}. What are the velocity and acceleration of the ball at the highest point of its motion ? **[Delhi 05 ; Central Schools 13, 14]**

38. Under what condition will the distance and displacement of a moving object have the same magnitude ? **[Chandigarh 08]**

39. State the condition when the magnitude of velocity and speed of an object are equal. **[Delhi 08]**

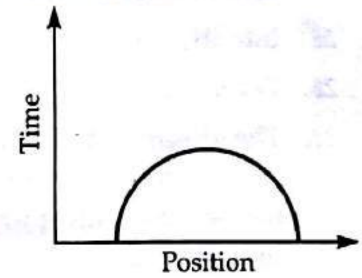
40. What does the slope of velocity-time graph represent ? **[Delhi 10 ; Central Schools 13]**

41. What does the area under velocity-time graph represent ? **[Central Schools 13]**

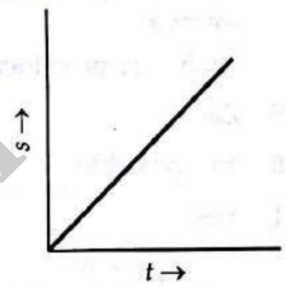
42. What does the area under acceleration-time graph represent ?

43. The displacement-time graphs for the two particles *A* and *B* are straight lines inclined at angles of 30° and 45° with the time-axis. What is the ratio of the velocities $v_A : v_B$? **[Delhi 1]2]**

44. Is the time variation of position, shown in the adjacent figure, observed in nature ? **[Central Schools 12]**



45. Displacement-time graph of any object is shown in the adjacent figure. Draw velocity-time graph for this motion. **[Delhi 14 ; Central Schools 14]**



Answers

1. Yes, both rest and motion are relative terms.
2. Yes. A body may be at rest relative to one object and at the same time it may be in motion relative to another object.
3. Yes.
4. An object can be considered a point object if its size is much smaller than the distance travelled by it.
5. Solar or Lunar eclipse.
6. When the object moves along a straight line in the same fixed direction.
7. Zero.
8. Distance travelled = πr.
   Displacement covered = $2r$.
9. No, the displacement of the object will remain unaltered even on shifting the position of the origin.
10. The speedometer measures the instantaneous speed of the car.
11. Less than or equal to one.
12. Change in velocity $= v - u = -30 - 30 = \mathbf{-60\ ms^{-1}}$.
13. Acceleration

$$= \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\text{Displacement / Time taken}}{\text{Time taken}}$$

$$= \frac{\text{Displacement}}{\text{Time taken}^2}$$

Hence time occurs twice in the unit of acceleration.

14. An object with positive acceleration is slowing down if its initial velocity is negative.
15. An object in simple harmonic motion speeds up while moving from an extreme position to the mean position but its acceleration is negative.
16. Acceleration is greater in the second case, because car suddenly comes to halt, the rate of change of velocity is large.
17. $$\frac{a_1}{a_2} = \frac{\text{Slope of } v\text{–}t \text{ graph of first object}}{\text{Slope of } v\text{–}t \text{ graph of second object}}$$

$$= \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = \mathbf{1 : 3.}$$

18. Suppose brakes are applied to a cycle moving northward. At that instant, it has a northward velocity and a southward acceleration.
19. No. When a stone is thrown vertically upwards, its velocity is zero at the highest point but it has a non-zero acceleration of 9.8 ms^{-2} at the same instant.
20. Both the balls will hit the ground with the same speed, so ratio of their speeds = 1 : 1. This is because the first fall will cross the point of projection with the same speed of 8 kmh^{-1} while moving downward.
21. Average acceleration $= \dfrac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$.

22. For uniform motion, the position-time graph is a straight line inclined to time-axis.

23. The slope of position-time graph gives velocity of the object.

24. For uniform motion, velocity-time graph is a straight line parallel to time-axis.

25. Infinity.

26. Zero.

27. The distance travelled by an object in any time interval can be determined by finding area between the velocity-time graph and time-axis for the given time interval.

28. Area under acceleration time graph for any time interval
= Change of velocity of the body in that interval.

29. Zero.

30. Not possible.

31. Yes.

$$s = \left(\frac{u+v}{2}\right) \times t = \left(\frac{v - at + v}{2}\right) \times t = vt - \frac{1}{2}at^2.$$

32. $12 + (12 - 4) = 20$ seconds.

33. No. The path of a projectile is a parabola even when it has a uniform acceleration.

34. Refer to the answer of Exercise 3.19(*b*) on page 3.50.

35. The given velocity-time graph shown in Fig. 3.89(*b*) represents the motion of a ball thrown up with some initial velocity and rebounding from the floor with reduced speed after each hit.

36. Given $x = 6 + 18t + 9t^2$

$$v = \frac{dx}{dt} = 18 + 18t$$

At $t = 2$ s,

$$v = 18 + 18 \times 2 = \mathbf{54\ ms^{-1}}.$$

37. At the highest point, the velocity of the ball is zero and its acceleration is equal to acceleration due to gravity acting in the downward direction.

38. When the body moves along a straight line path.

39. When the body moves along a straight line path.

40. Acceleration.

41. Displacement.

42. Change in velocity in the given time interval.

43. $\frac{v_A}{v_B} = \frac{\tan 30°}{\tan 45°} = \frac{1/\sqrt{3}}{1} = \mathbf{1 : \sqrt{3}}$

44. No, a body cannot occupy two different positions at the same instant of time.

45. The v - t graph for the given uniform motion is shown below :