Guidelines to NCERT Exercises

2.1. *Fill in the blanks :*

(*i*) *The volume of a cube of side 1 cm is equal to ...... m^3.*

(*ii*) *The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to ...... $(mm)^2$.*

(*iii*) *A vehicle moving with a speed of 18 km/h covers ......m in 1 s.*

(*iv*) *The relative density of lead is 11.3. Its density is ...... gcm^{-3} or ...... kgm^{-3}.*

Ans.

(*i*) $V = l^3 = (1\,\text{cm})^3 = (10^{-2}\,\text{m})^3 = \mathbf{10^{-6}\,m^{-3}}$.

(*ii*) $r = 2\,\text{cm} = 20\,\text{mm},\ h = 10\,\text{cm} = 100\,\text{mm}$

$S = 2\pi r(r+h) = 2 \times 3.14 \times 20\,(20+100)$

$= \mathbf{15072\ mm^2}$.

(*iii*) $v = 18\,\dfrac{\text{km}}{\text{h}} = \dfrac{18 \times 1000\,\text{m}}{60 \times 60\,\text{s}} = \mathbf{5\ ms^{-1}}$.

(*iv*) Density = Relative density × density of water at 4°C

$= 11.3 \times 1\,\text{gcm}^{-3} = \mathbf{11.3\ gcm^{-3}}$

$= 11.3 \times 10^3\ \text{kg m}^{-3} = \mathbf{11300\ kgm^{-3}}$.

2.2. *Fill in the blanks by suitable conversion of units :*

(*i*) $1\,kg\,m^2s^{-2} = ...\,g\,cm^2s^{-2}$

(*ii*) $1\,m =$ ...... *light year*

(*iii*) $3\,ms^{-2} = ...kmh^{-2}$

(*iv*) $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2} = ...\,cm^3s^{-2}g^{-1}$

Ans. (*i*) $1\,\text{kg m}^2\text{s}^{-2} = 1\,(10^3\ \text{g})\,(10^2\ \text{cm})^2\ \text{s}^{-2}$

$= \mathbf{10^7\,g\,cm^2s^{-2}}$.

(*ii*) As 1 light year = 9.46×10^{15} m

$\therefore\ 1\,\text{m} = \dfrac{1}{9.46 \times 10^{15}}$ light year

$= 1.053 \times 10^{-16}$ light year $\simeq \mathbf{10^{-16}}$ **light year.**

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(*iii*) $3\ ms^{-2} = 3(10^{-3}\ km)\left(\frac{1}{60 \times 60}\ h\right)^{-2}$

$= 3 \times 10^{-3} \times 3600 \times 3600\ kmh^{-2}$

$= 3.888 \times 10^4\ kmh^{-2} \simeq \mathbf{3.9 \times 10^4\ kmh^{-2}}$.

(*iv*) $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

$= 6.67 \times 10^{-11}\ kg\ m\ s^{-2}.\ m^2\ kg^{-2}$

$= 6.67 \times 10^{-11} m^3s^{-2}kg^{-1}$

$= 6.67 \times 10^{-11}\ (10^2cm)^3s^{-2}(1000\ g)^{-1}$

$= \mathbf{6.67 \times 10^{-8} cm^3s^{-2}g^{-1}}$.

2.3. *A calorie is a unit of heat energy and it equals about 4.2 J, where $1\ J = 1\ kg\ m^2\ s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in terms of the new units.*

Ans. As 1calorie = 4.2 J, where $1\ J = 1\ kg\ m^2s^{-2}$.

Clearly, $[Energy] = ML^2T^{-2}$

$\therefore\ a = 1,\ b = 2,\ c = -2$

SI	*New System*
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1\ kg$	$M_2 = \alpha\ kg$
$L_1 = 1\ m$	$L_2 = \beta\ m$
$T_1 = 1 s$	$T_2 = \gamma\ s$

$$\therefore\ n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\ kg}{\alpha\ kg}\right]^1\left[\frac{1\ m}{\beta\ m}\right]^2\left[\frac{1\ s}{\gamma\ s}\right]^{-2}$$

$$= 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$$

Hence 1 calorie = 4.2 J = $\mathbf{4.2\alpha^{-1}\beta^{-2}\gamma^2}$ **new units of energy.**

2.4. (*i*) *Explain the statement clearly : To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison.*

(*ii*) *In view of this, reframe the following statements, wherever necessary :*

(*a*) *Atoms are very small objects*

(*b*) *A jet plane moves with great speed*

(*c*) *The mass of jupiter is very large*

(*d*) *The air inside this room contains a large number of molecules*

(*e*) *A proton is much more massive than an electron*

(*f*) *The speed of sound is much smaller than the speed of light.*

Ans. (*i*) The given statement is correct. Measurement is basically a comparison process. Without specifying a standard of comparison, it is not possible to get an exact idea about the magnitude of a dimensional quantity. For example, the statement that the mass of the earth is very large, is meaningless. To correct it, we can say that the mass of the earth is large in comparison to any object lying on its surface.

(*ii*) (*a*) The size of an atom is much smaller than even the sharp tip of a pin.

(*b*) A jet planet moves with a speed greater than that of a superfast train.

(*c*) The mass of jupiter is very large compared to that of the earth.

(*d*) The air inside this room contains more number of molecules than in one mole of air.

(*e*) This is a correct statement.

(*f*) This is a correct statement.

2.5. *A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?*

Ans. Speed of light

= 1 new unit of length / s

Time = 8 min 20 s = 8 × 60 + 20 = 500 s

Distance between the earth and the sun

= Speed of light × time = 1 × 500

= **500 new units of length.**

2.6. *Which of the following is the most precise device for measuring length :*

(*a*) *a vernier calliper with 20 divisions on the sliding scale,*

(*b*) *a screw gauge of pitch 1 mm and 100 divisions on the circular scale*

(*c*) *an optical instrument that can measure length to within a wavelength of visible light ?*

Ans. The device that has minimum least count will be more precise for measuring length.

(*a*) Least count of vernier callipers

$= 1\ MSD - 1\ VSD = 1\ MSD - \frac{19}{20}\ MSD = \frac{1}{20}\ MSD$

$= \frac{1}{20} \times 1\ mm = \frac{1}{200}\ cm = 0.005\ cm.$

(*b*) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$$

$$= \frac{1.0\ mm}{100} = \frac{1}{1000}\ cm = 0.001\ cm$$

(*c*) Least count of optical instrument

= Wavelength of visible (red) light

$= 6000\ Å = 6000 \times 10^{-8}\ cm$

$= 0.00006\ cm.$

Hence the most precise device for measuring length is the given optical instrument.

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2.7. *A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?*

Ans. Average thickness of hair as observed through microscope = 3.5 mm

Magnification produced by the microscope = 100

Actual thickness of hair

$$= \frac{\text{Observed thickness}}{\text{Magnification}} = \frac{3.5}{100} = \mathbf{0.035\ mm.}$$

2.8. *Answer the following :*

(*a*) *You are given a thread and a metre scale. How will you estimate the diameter of the thread ?*

(*b*) *A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?* **[Delhi 13]**

(*c*) *The mean diameter of a thin brass rod is to be measured by vernier calipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?*

Ans. (*a*) The thread is wound on the metre scale so that its turns are as close as possible. Thickness '*l*' of the thread coil is measured and the number of turns '*n*' of the thread coil is counted.

$$\therefore \quad \text{Thickness of thread} = \frac{l}{n}\ \text{cm}.$$

(*b*) Least count of a screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

Theoretically, it appears that the least count can be decreased by increasing the number of divisions on the circular scale. Practically, it may not be possible to take the reading precisely due to low resolution of human eye.

(*c*) Larger the number of readings, closer is the arithmetic mean to the true value and hence smaller the random error. Hence result with a set of 100 measurements is more reliable than that with a set of 5 measurements.

2.9. *The photograph of a house occupies an area of 1.75 cm² on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1.55 m². What is the linear magnification of the projector-screen arrangement ?*

Ans. Size of object $= 1.75\ \text{cm}^2 = 1.75 \times 10^{-4}\ \text{m}^2$

Size of image $= 1.55\ \text{m}^2$

Areal magnification

$$= \frac{\text{Size of image}}{\text{Size of object}} = \frac{1.55}{1.75 \times 10^{-4}} = 8857$$

Linear magnification

$$= \sqrt{\text{Areal magnification}} = \sqrt{8857} = \mathbf{94.1.}$$

2.10. *State the number of significant figures in the following :*

(*i*) $0.007\ m^2$ (*ii*) $2.64 \times 10^{24}\ kg$

(*iii*) $0.2370\ g\ cm^{-3}$ (*iv*) $6.320\ J$

(*v*) $6.032\ Nm^{-2}$ (*vi*) $0.0006032\ m^2$

Ans. (*i*) One : 7 (*ii*) Three : 2, 6, 4

(*iii*) Four : 2, 3, 7, 0 (*iv*) Four : 6, 3, 2, 0

(*v*) Four : 6, 0, 3, 2 (*vi*) Four 6, 0, 3, 2

2.11. *The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.*

Ans. Here $l = 4.234$ m, $b = 1.005$ m,

$$h = 2.01\ \text{cm} = 0.0201\ \text{m}$$

Area of sheet $= 2\,(lb + bh + hl)$

$= 2\,(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ \text{m}^2$

$= 2\,(4.25517 + 0.0202005 + 0.0851034)\ \text{m}^2$

$= 2 \times 4.3604739\ \text{m}^2 = 8.7209478\ \text{m}^2$

$= 8.72\ \text{m}^2$ [Rounded off upto 3 significant figures]

Volume of the sheet

$= lbh = 4.234 \times 1.005 \times 0.0201\ \text{m}^3$

$= 0.0855289\ \text{m}^3 = \mathbf{0.0855\ m^3.}$

[Rounded off upto three significant figures]

2.12. *The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box and (b) the difference in the masses of the pieces to correct significant figures ?*

Ans. (*a*) Total mass of the box

$= 2.3\ \text{kg} + 0.02015\ \text{kg} + 0.02017\ \text{kg}$

$= 2.34032\ \text{kg} = \mathbf{2.3\ kg.}$

The result has been rounded off to first place of decimal because mass (2.3 kg) of box has digits upto this place of decimal.

(*b*) Difference in masses of 2 gold pieces

$= 20.17\ \text{g} - 20.15\ \text{g} = \mathbf{0.02\ g.}$

2.13. *A physical quantity P is related to four observations : a, b, c and d as follows :* $P = a^3 b^2 / \sqrt{c}\,d$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763 , to what value should you round off the result ? **[Central Schools 12 ; Delhi 09]**

Ans. Given : $P = \dfrac{a^3\, b^2}{\sqrt{c}\ d}$

The percentage error in the quantity P is given by

$$100 \times \frac{\Delta P}{P} = 3 \times 100 . \frac{\Delta a}{a} + 2 \times 100 . \frac{\Delta b}{b} + \frac{1}{2} \times 100 . \frac{\Delta c}{c} + 100 \times \frac{\Delta d}{d}$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\% = \mathbf{13\%.}$$

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Since 13% = 0.13, so there are two significant figures in the percentage error. Hence P should also be rounded off to 2 significant figures.

$\therefore \quad P = 3.763 = 3.8.$

2.14. *A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion :*

(i) $y = a \sin \frac{2\pi t}{T}$, (ii) $y = a \sin vt$,

(iii) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$

(iv) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

(a = maximum displacement of the particle, v = speed of the particle, T = time-period of motion). Rule out the wrong formula on dimensional grounds.

Ans. Dimensions of LHS in all the cases (i) to (iv) = L

Dimensions of RHS in different cases are as follows :

(i) $\left[a \sin \frac{2\pi t}{T}\right] = L \cdot \sin \frac{T}{T} = L$

(Angle $\frac{2\pi t}{T}$ is dimensionless)

This relation is dimensionally correct.

(ii) $[a \sin vt] = L \sin (LT^{-1}\, T) = L \sin (L)$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(iii) $\left[\left(\frac{a}{T}\right) \sin \frac{t}{a}\right] = \frac{L}{T} \sin \frac{T}{L}$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(iv) $\left[\frac{a}{\sqrt{2}}\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)\right] = L\left[\sin \frac{T}{T} + \cos \frac{T}{T}\right]$

(Angle is dimensionless)

This relation is dimensionally correct.

Hence formulae (ii) and (iii) are dimensionally wrong.

2.15. *A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relatively due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :*

$$m = \frac{m_0}{(1 - v^2)^{1/2}}$$

Guess where to put the missing c. **[Central Schools 14]**

Ans. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from dimensionless constant 1. It should be divided by c^2 so as to make it dimensionless. Hence the corrected relation is

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}.$$

2.16. *The radius of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?*

Ans. Radius of a hydrogen atom,

$$r = 0.5 Å = 0.5 \times 10^{-10} \text{ m}$$

Volume of one atom $= \frac{4}{3}\pi r^3$

No. of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms $= N \times \frac{4}{3}\pi r^3$

$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$

$= 3.154 \times 10^{-7}\ \text{m}^3 \simeq \mathbf{3 \times 10^{-7}\ m^3}.$

2.17. *One mole of an ideal gas at S.T.P. occupies 22.4 L. What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? Why is this ratio so large ? Take the radius of hydrogen molecule to be 1 Å.*

Ans. Radius of a hydrogen molecule

$= 1 Å = 10^{-10}$ m

Atomic volume of a mole of hydrogen

= Avogadro's no.

× Volume of a hydrogen molecule

$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (10^{-10})^3$

$= 25.2 \times 10^{-7}\ \text{m}^3$

Molar volume $= 22.4 \text{ L} = 22.4 \times 10^{-3}\ \text{m}^3$

$\therefore \quad \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} \simeq 0.89 \times 10^4 \simeq \mathbf{10^4}.$

This ratio is large because the actual size of the gas molecules is negligibly small in comparison with the intermolecular separation.

2.18. *Explain why on looking through the window of a fast moving train, the nearby trees and electric poles etc. appear to run in direction opposite to that of motion of the train, while far off houses, hilltops, Moon, stars etc. appear stationary.*

Ans. The line joining the object and the eye is called the line of slight. The direction of the line of sight of the nearby objects like trees, poles etc. ; changes very rapidly due to fast motion of the train and accordingly they appear to be moving opposite to the direction of motion of the train. But the line of sight of a distant object almost does not change its direction due to its extremely large distance from the eye. Hence the distant objects like hilltops, moon, stars etc. appear stationary.

2.19. *A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the earth to the sun. How much is parsec in terms of metres ?*

Ans. One parsec is the distance at which an arc of length 1 AU makes an angle of 1 second of an arc.

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As $\theta(\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$ $\therefore$ $r = \frac{l}{\theta}$.

Here $l = 1\text{ AU} = 1.496 \times 10^{11}$ m

$$\theta = 1\text{ s of arc} = \frac{\pi}{60 \times 60 \times 180}\text{ rad}$$

$$= 4.85 \times 10^{-6}\text{ rad}$$

$$\therefore \quad 1\text{ parsec} = r = \frac{1496 \times 10^{11}}{4.85 \times 10^{-6}} = \mathbf{3.08 \times 10^{16}\ m.}$$

Order of magnitude of parsec = **16**.

2.20. *The nearest star (Alpha Centauri) to our solar system is 4.29 light years away. How much is this distance in terms of parsec ? How much parallax would this star show when viewed from two locations of the earth six months apart in its orbit around the sun ?*

Ans. As $1\text{ light year} = 9.46 \times 10^{15}$ m,

$$1\text{ parsec} = 3.08 \times 10^{16}\text{ m}$$

$\therefore$ Distance of Alpha Centauri from the earth,

$$S = 4.29\text{ light years} = 4.29 \times 9.46 \times 10^{15}\text{ m}$$

$$= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}\text{ parsec} = 1.32\text{ parsec}$$

In an orbit around the sun, the distance between the two locations of the earth six months apart,

b = Diameter of the earth's orbit = 2 AU

Parallax of the star,

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} = \frac{2\text{ AU}}{1.32\text{ parsec}} = \mathbf{1.515\ s\ of\ arc.}$$

2.21. *Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.*

Ans. Some of the examples of modern science, where precise measurements play an important role, are as follows :

1. Electron microscope uses an electron beam of wavelength 0.2 Å to study very minute objects like viruses, microbes and the crystal structure of solids.
2. The successful launching of artificial satellites has been made possible only due to the precise technique available for accurate measurement of time-intervals.
3. The precision with which the distances are measured in Michelson-Morley Interferometer helped in discarding the idea of hypothetical medium ether and in developing the Theory of Relativity by Einstein.

2.22. *Just as precise measurements are necessary in science, it is equally important to be able to make rough estiamtes of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :*

(*a*) *the total mass of rain-bearing clouds over India during the Monsoon*

(*b*) *the mass of an elephant*

(*c*) *the wind speed during a storm*

(*d*) *the number of strands of hair on your head*

(*e*) *the number of air molecules in your classroom.*

Ans. (*a*) The average rainfall during the Monsoon in India is about 100 cm or 1 m.

Total surface area of India

$$= 3.3 \times 10^6\text{ km}^2 = 3.3 \times 10^6 \times (10^3\text{ m})^2$$

$$= 3.3 \times 10^{12}\text{ m}^2.$$

Volume of rain water,

$$V = Ah = 3.3 \times 10^{12}\text{ m}^2 \times 1\text{ m} = 3.3 \times 10^{12}\text{ m}^3$$

Density of water, $\rho = 10^3\text{ kg m}^{-3}$

Hence total mass of rain-bearing clouds over India,

$$m = V\rho = 3.3 \times 10^{12} \times 10^3 = \mathbf{3.3 \times 10^{15}\ kg.}$$

(*b*) To estimate the mass of an elephant, consider a boat of base area A in a river. Let x_1 be the depth of the boat inside water. Now move the elephant into the boat. Again measure the depth x_2 of the boat inside water.

Volume of water displaced by elephant

$$V = A(x_2 - x_1)$$

According to Archimedes' principle, mass of the elephant is

$$m = \text{Mass of water displaced by the elephant}$$

$$= V\rho = A(x_2 - x_1)\rho$$

Mass of an elephant is about 10^3 kg.

(*c*) The wind speed during a storm can be measured by floating a gas filled balloon in air. When there is no wind storm, suppose the balloon is at vertical height $OA = h$, from the ground. Due to the wind storm, suppose the balloon moves to position B in a small time interval t, as shown in Fig. 2.16.

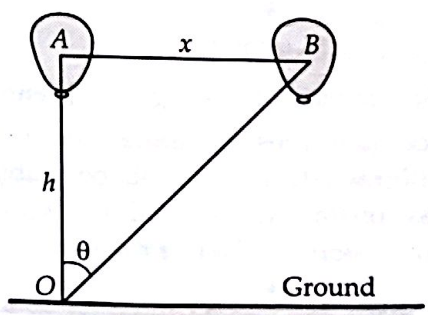

Fig. 2.16

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If θ is the angle of drift of the balloon, then from right angled ΔOAB,

$$\tan\theta = \frac{AB}{OA} = \frac{x}{h} \quad [AB = x, \text{ say}]$$

or

$$x = h\tan\theta$$

Hence the wind speed during the storm,

$$v = \frac{x}{t} = \frac{h\tan\theta}{t}.$$

(*d*) First we count the strands of hair of 1 cm^2 area of the head. Then by multiplying it by the total area of the head, we can estimate the total number of stands of hair on the head. Its order of magnitude may be as large as 10^8.

(*e*) The dimensions of a typical classroom are

$$8 \text{ m} \times 6 \text{ m} \times 4 \text{ m}$$

Volume of the class room $= 8\times 6\times 4 = 192 m^3$

Now one mole of air molecules occupy a volume of 22.4 litres or 22.4×10^{-3} m^3.

∴ Number of molecules in 22.4×10^{-3} m^3

$$= 6.023\times 10^{23}$$

Number of molecules in the classroom

$$= \frac{6.023\times 10^{23}}{22.4\times 10^{-3}}\times 192 = 51.6\times 10^{26} \text{ molecules}$$

$$\approx \mathbf{10^{28}} \textbf{ molecules.}$$

2.23. *The sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be ? In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the sun = 2.0×10^{30} kg, radius of the sun = 7.0×10^8 m.*

Ans. Mass of the sun, $M = 2.0\times 10^{30}$ kg

Radius of the sun, $R = 7.0\times 10^8$ m

Volume of the sun,

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\times(7.0\times 10^8)^3$$

$$= 1.437\times 10^{27} \text{ m}^3$$

Density of the sun,

$$\rho = \frac{M}{V} = \frac{2.0\times 10^{30}}{1.437\times 10^{27}}$$

$$= 1391.8 \text{ kg m}^{-3} \approx \mathbf{1.4\times 10^3 \text{ kg m}^{-3}}.$$

The density of the sun is in the range of the densities of the solids and liquids but not gases. The high density is due to the invard gravitational attraction on the outer layers due to the inward layers of the sun.

2.24. *When the planet Jupiter is at a distance of 824.7 million kilometres from the earth, its angular diameter is measured to be 35.72 s of arc. Calculate the diameter of Jupiter.*

Ans. Distance of Jupiter from the earth,

$$S = 824.7\times 10^6 \text{ km}$$

Angular diameter of Jupiter,

$$\theta = 35.72'' = \left(\frac{35.72}{60\times 60}\right)^\circ = \frac{35.72}{3600}\times\frac{\pi}{180} \text{ rad}$$

Diameter of Jupiter,

$$D = S\times\theta = 824.7\times 10^6\times\frac{35.72}{3600}\times\frac{\pi}{180}$$

$$= \mathbf{1.428\times 10^5 \text{ km}}.$$

2.25. *A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : tan θ = v and checks that the relation has a correct limit : as v → 0, θ → 0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct ? If not, guess at the correct relation.*

Ans. Since trigonometric functions are dimensionless,

∴ $[\tan\theta] = 1$

But $[v] = LT^{-1}$

∴ Dimensions of LHS ≠ Dimensions of RHS

Hence the given relation is dimensionally wrong.

This relation can be corrected by dividing RHS by the speed '*u*' of the rainfall. So the corrected relation is

$$\tan\theta = \frac{v}{u}.$$

2.26. *It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?*

Ans. Here $\Delta t = 0.02$ s,

$$t = 100 \text{ years} = 100\times 365.25\times 86{,}400 \text{ s}$$

Fractional error

$$= \frac{\Delta t}{t} = \frac{0.02}{100\times 365.25\times 86400} = 0.63\times 10^{-11}$$

So there is an accuracy of 10^{-11} part in 1s or 1s in 10^{11} s.

2.27. *Estimate the average density of a sodium atom assuming its radius to be about 2.5 Å. Compare it with the density of sodium in crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ?*

Ans. Radius of a sodium atom,

$$r = 2.5 \text{ Å} = 2.5\times 10^{-10} \text{ m}$$

Volume of a sodium atom,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times(2.5\times 10^{-10})^3$$

$$= 65.42\times 10^{-30} \text{ kg}$$

Mass of a sodium atom

$$= \frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{23}{6.02\times 10^{23}} \text{ g}$$

$$= 3.82\times 10^{-23} \text{ g} = 3.82\times 10^{-26} \text{ kg}$$

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Average density of sodium atom

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{3.82 \times 10^{-26}}{65.42 \times 10^{-30}}$$

$$= 0.58 \times 10^3 \text{ kg m}^{-3}$$

Density of sodium in crystalline phase

$$= 970 \text{ kg m}^{-3} = 0.970 \times 10^3 \text{ kg m}^{-3}$$

Hence the average mass density of sodium atom and the density of crystalline sodium are of the same order of magnitude (10^3). This is because sodium atoms in crystalline phase are closely packed.

2.28. *The unit of length convenient on the nuclear scale is a fermi : 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation $r = r_0 A^{1/3}$; where r is the radius of the nucleus, A its mass number and r_0 is a constant equal to about 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.*

Ans. Radius of a nucleus, $r = r_0 A^{1/3}$

Mass of a nucleus $= \frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{A}{N_A}$

Nuclear mass density $= \frac{\text{Mass of a nucleus}}{\text{Volume of a nucleus}}$

or $$\rho = \frac{A}{N_A \cdot \frac{4}{3}\pi r^3} = \frac{A}{N_A \cdot \frac{4}{3}\pi (r_0 A^{1/3})^3} = \frac{3}{4\pi N_A r_0^3}$$

As ρ is independent of *A*, so nuclear mass density is same for different nuclei.

For a kilomole,

$$N_A = 6.02 \times 10^{26}, \; r_0 = 1.2 \; f = 1.2 \times 10^{-15} \text{ m}$$

$$\therefore \quad \rho = \frac{3}{4\pi \times 6.02 \times 10^{26} \times (1.2 \times 10^{-15})^3}$$

$$= \mathbf{2.3 \times 10^{17} \text{ kg m}^{-3}}$$

Density of sodium nucleus should also be

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

From Exercise 2.27, density of sodium atom

$$= 0.58 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad \frac{\text{Nuclear mass density}}{\text{Atomic mass density}} = \frac{2.3 \times 10^{17}}{0.58 \times 10^3} = 3.96 \times 10^{14}.$$

Nuclear density is typically 10^{15} times atomic density of matter.

2.29. *A laser is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the moon takes 2.56 s to return after reflection at the moon's surface. How much is the radius of the lunar orbit around the earth ?*

Ans. Here $t = 2.56$ s, $\quad c = 3 \times 10^8 \text{ ms}^{-1}$

Radius of the lunar orbit around the earth

= Distance of the moon from the earth

$$= \frac{c \times t}{2} = \frac{3 \times 10^8 \times 2.56}{2} = \mathbf{3.84 \times 10^8 \text{ m.}}$$

2.30. *A SONAR (Sound Navigation and Ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77 s. What is the distance of the enemy submarine ?*

(Speed of sound in water = 1450 ms^{-1}).

Ans. Here $t = 77$ s, $\quad c = 1450 \text{ ms}^{-1}$

Distance of enemy submarine

$$= \frac{c \times t}{2} = \frac{1450 \times 77}{2} = \mathbf{55825 \text{ m.}}$$

2.31. *The farthest objects (known as quasers) in our universe are so distant that light emitted by them takes billion of years to reach the earth. What is the distance in km of a quaser from which light takes 3.0 billion years to reach us ?*

Ans. Here t = 3.0 billion years

$$= 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Speed of light, $c = 3 \times 10^5 \text{ kms}^{-1}$

Distance of quaser

$$= ct = 3 \times 10^5 \times 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60$$

$$= \mathbf{2.84 \times 10^{22} \text{ km.}}$$

2.32. *It is a well known fact that during a solar eclipse the disc of the moon almost completely covers the disc of the sun. From this fact and from the information that sun's angular distance α is measured to be 1920'', determine the approximate diameter of the moon. Given earth-moon distance = 3.8452×10^8 m.*

Ans. During total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameters of both the sun and the moon must be equal.

∴ Angular diameter of the moon,

θ = Angular diameter of the sun

$$= 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad}$$

$$[\because \; 1'' = 4.85 \times 10^{-6} \text{ rad}]$$

Earth-moon distance, $S = 3.8452 \times 10^8$ m

Diameter of the moon,

$$D = \theta \times S = 1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8$$

$$= 3.581 \times 10^6 \text{ m} = \mathbf{3581 \text{ km.}}$$

2.33. *A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (≈ 15 billion years).*

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From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constant ?

Ans. Using basic constants such as speed of light (c), charge on electron (e), mass of electron (m_e), mass of proton (m_p) and gravitational constant (G), we can construct the quantity,

$$\tau = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \times \frac{1}{m_p m_e^2 c^3 G}$$

$$\text{Now } \left[\frac{e^2}{4\pi\varepsilon_0}\right] = \left[\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}r^2\right] = [Fr^2]$$

$$= [MLT^{-2}.L^2] = [ML^3T^{-2}]$$

$$\therefore \quad [\tau] = \frac{[ML^3T^{-2}]^2}{[M][M]^2[LT^{-1}]^3[M^{-1}L^3T^{-2}]} = [T]$$

Clearly, the quantity τ has the dimensions of time.

Put $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, $c = 3 \times 10^8$ ms^{-1}, $e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}

$$\therefore \quad \tau = \frac{[9 \times 10^9 \times (1.6 \times 10^{-19})^2]^2}{1.67 \times 10^{-27} \times (9.1 \times 10^{-31})^2 \times (3 \times 10^8)^3 \times 6.67 \times 10^{-11}}$$

$$= 2.13 \times 10^{16} \text{ s}$$

$$= \frac{2.13 \times 10^{16}}{3.156 \times 10^7} \text{ years} = 0.667 \times 10^9 \text{ years.}$$

$= 0.667$ billion years.

This time is slightly less than the age of the universe ($\approx$ 15 billion years). It implies that the values of the basic constants of physics should change with time because the age of the universe increases with time.

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