D. Essays

1. Derive Newton's formula for the velocity of longitudinal wave in an elastic medium. What is Laplace's correction to the Newton's formula?
   Discuss the effect of pressure and temperature of air on the velocity of sound in air.
2. Describe various modes of vibration of air column in open and closed pipes. Show that a closed pipe can produce only odd harmonics whereas an open pipe can produce all harmonics.
3. Derive an expression for a harmonic wave.
4. What are beats? Discuss the formation of beats analytically as well as graphically.
5. Explain Doppler effect. Obtain the general expression for the apparent frequency of sound.

E. Problems

1. A uniform wire is stretched under a tension 2 kg f. The length of the wire is 0.5 m and its mass is 3.735×10^{-5} kg. Calculate (i) the speed of transverse vibrations of the string (ii) and the fundamental frequency of vibrations of the string.
   [Ans: 512 ms^{-1}; 512 Hz.]
2. A steel wire of length 1 m and diameter 0.5 mm is stretched by a load 10 kg. Calculate the fundamental frequency of the wire. ($g = 9.8\ ms^{-2}$, density of steel = $7.8 \times 10^3\ kg\ m^{-3}$)
   [Ans: 126.51 Hz.]
3. The velocity of sound in air is 348 ms^{-1}. Find the wavelength of the sound emitted by a string of frequency 464 Hz.
   [Ans: 0.75 m]
4. A person drops a stone into a well and hears the sound of the splash after 5.35 s. If the depth of the well is 122.5 m, calculate the velocity of sound in air.
   [Ans: 350 ms^{-1}]
5. Calculate the velocity of sound in air at 27° C($\gamma = 1.40$; Density of air at STP = $1.29\ kg\ m^{-3}$)
   [Ans: 347.57 ms^{-1}]
6. Calculate the ratio of specific heats for hydrogen. Density of hydrogen at STP = $1.255\ kg\ m^{-3}$ (velocity of sound in hydrogen at STP = $336\ m\ s^{-1}$)
   [Ans: 1.4]
7. At what temperature is the velocity of sound in air double the velocity at 0°C?
   [Ans: 819°C]
8. Calculate the speed of sound in oxygen at 0°C. $\gamma = 1.4$; $M = 32\ g\ mol^{-1}$; $R = 8.31\ J\ mol^{-1}K^{-1}$
   [Ans: 315 ms^{-1}]
9. The sound of a gun is heard by an observer 6 s after the flash of the gun is seen. Calculate the distance of the gun from the observer. The atmospheric temperature is 20°C. (Velocity of sound at 0°C is 331 ms^{-1})
   [Ans: 2057 m]

10. The equation of a transverse wave travelling along a coil spring is, $y = 4.0 \sin \pi (0.010\, x - 2.0\, t)$ cm. Find (i) amplitude (ii) wavelength (iii) initial phase at the origin (iv) speed and (v) frequency of the wave.
[Ans: (i) 4.0×10^{-2} m; (ii) 2 m; (iii) 0; (iv) 2.0 ms^{-1}; (v) 1 Hz.]

11. The transverse displacement of a string is given by,

$$y(x, t) = 0.060 \sin \frac{2\pi x}{3} \cos(120\pi t) \text{ m.}$$

The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. (i) Does the function represent a travelling wave or a stationary wave? (ii) Interpret the wave superposition of two travelling waves in opposite direction. What are the wavelength, frequency and speed of propagation of each wave (iii) determine the tension in the string.
[Ans: (i) stationary wave (ii) $\lambda = 3$ m, $\nu = 60$ Hz, V=180 ms^{-1} (iii) 648 N.]

12. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe, if both its ends are open? (Speed of sound = 340 ms^{-1}) [NCERT]
[Ans: First harmonic; No]

13. Two sitar strings A and B playing the note *Ga* are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B? [NCERT]
[Ans: 318 Hz.]

14. A stretched string produces 5 beats per second with a tuning fork of frequency 512 Hz. With another tuning fork of frequency 510 Hz, it makes 3 beats. Calculate the frequency of vibration of the string.
[Ans: 507 Hz.]

15. Two notes of wavelengths 2.04 m and 2.08 m produce 200 beats per minute in a gas. Find the velocity of sound in the gas.
Hint: No. of beats per second = $\nu_1 - \nu_2 = v/\lambda_1 - v/\lambda_2 = v[1/\lambda_1 - 1/\lambda_2]$
[Ans: 353.6 ms^{-1}]

16. Estimate the speed of sound in air at STP. The mass of 1 mol of the gas is given to be 29.0×10^{-3} kg. Volume of 1 mole of air at STP = 22.4 litres.
[Ans: 331.6 ms^{-1}]

17. When the tension of a sonometer wire is increased by 4.5 kg., the pitch of the note emitted by a given length of the wire increases in the ratio 4 : 5. Calculate the original tension of the wire.
[Ans: 8 kg wt]

18. Calculate the fundamental frequency of the note emitted by a tube of length 27 cm if the tube is (1) closed at one end (2) open at both ends. $V = 345.6$ ms^{-1}
[Ans: 320 Hz; 640 Hz.]

19. The shortest length of an air column in a pipe closed at one end resonating with a fork of frequency 320 Hz, is 0.26 m. Calculate the velocity of sound in air.
[Ans: 332.80 ms^{-1}]

20. A tuning fork is found to resonate with an air column closed at one end for lengths 27 cm and 82 cm. Calculate the frequency of the tuning fork. (Velocity of sound $= 330\ \text{ms}^{-1}$)

[Ans: 300 Hz.]

21. Find the fundamental frequency of the note emitted by a tube of length 0.34 m open at both ends. (Velocity of sound in air $= 348.16\ \text{ms}^{-1}$)

[Ans: 512 Hz.]

22. A man standing near a railway line hears the whistle of an engine, which has a velocity of 72 km/hr. What frequencies does the man hear the note when the engine is (a) coming towards him (b) going away from him; if the true frequency of whistle is 512 Hz? (Velocity of sound $= 340\ \text{ms}^{-1}$)

[Ans: 544 Hz; 484 Hz]

23. A man on the railway platform observes that as the train passed him at 90 km/hr., the frequency of the whistle of the train appeared to drop to 490 Hz. Calculate the frequency of the whistle. (Velocity of sound $= 350\ \text{ms}^{-1}$)

[Ans: 525 Hz.]

24. The apparent frequency of the whistle of an engine changes in the ratio 6 : 5 as the engine passes a stationary observer. Calculate the velocity of the engine. (Velocity of sound $= 352\ \text{ms}^{-1}$)

[Ans: 32 ms^{-1}]

Hint: $\nu' = \left(\dfrac{V - V_l}{V - V_s}\right)\nu = \left(\dfrac{V}{V - V_s}\right)\nu; \quad \therefore V_l = 0$

As the engine approaches the observer, V_s is positive

$$\nu_1' = \left(\frac{352}{352 - V_s}\right)\nu \qquad \text{(i)}$$

As the engine moves away from the observer, V_s is negative

$$\nu_2' = \left(\frac{352}{352 + V_s}\right)\nu \qquad \text{(ii)}$$

Eqns: (i)/(ii) $\dfrac{\nu_1'}{\nu_2'} = \dfrac{352 + V_s}{352 - V_s}$; *i.e.*, $\dfrac{6}{5} = \dfrac{352 + V_s}{352 - V_s}$

$\therefore V_s = \mathbf{32\ ms^{-1}}$

25. A railway engine and a car are moving on parallel tracks in opposite directions each with speed 90 km/h, the engine is continuously sounding the horn of frequency 512 Hz. Calculate the frequency of sound of the horn heard by the driver of the car when (a) the engine and the car approach each other and (b) the two are moving away from each other (Velocity of sound $= 340\ \text{ms}^{-1}$)

[Ans: 585 Hz; 449 Hz.]

26. A bat is flitting about in a cave, navigating via ultrasonic sound. Assume that the sound emission frequency of the bat is 40 kHz. During one fast sweep directly towards a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear the sound reflected off the wall? [NCERT]

Hint: *Take the bat as the source and wall as the listener*:

$v_l = 0; \quad v_s = 0.03\ v; \quad \nu = 40\ \text{kHz}; \quad \nu' = ?$

Now, take the wall as the source and bat as the listener:

$$v_s = 0; \quad v_l = -0.03\, v; \quad \nu = \nu' = 41.24 \text{ kHz}; \quad \nu'' = ?$$

$$\nu'' = \left(\frac{v - v_l}{v - v_s}\right)\nu' = \left(\frac{v + 0.03\, v}{v}\right) \times 41.24 = \mathbf{42.47\ kHz}$$