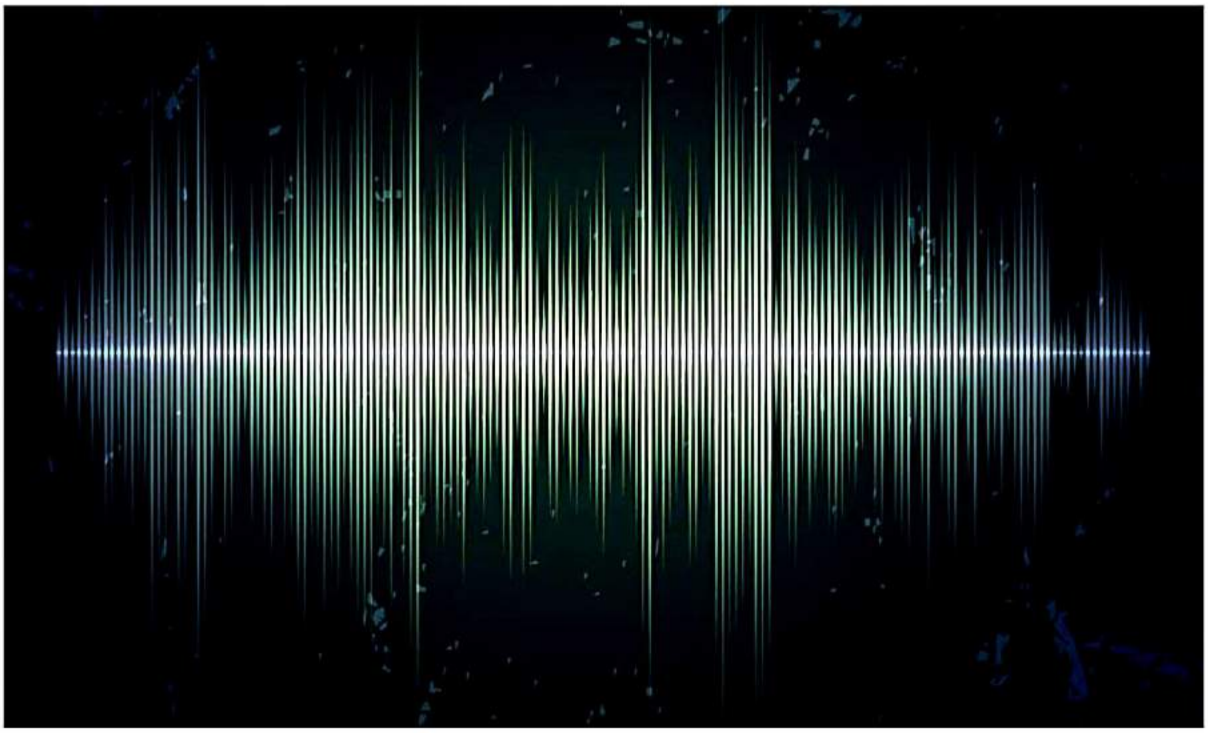
Chapter–15:

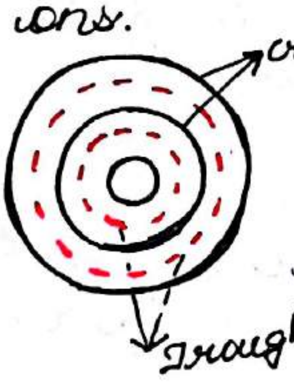
Waves

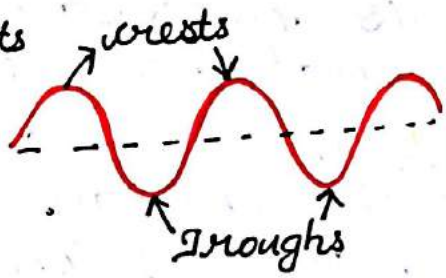


CBSE CLASS XI NOTES

Dr. SIMIL RAHMAN

wave motion is a kind of disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions.





waves
(1) mechanical waves
(2) Electromagnetic waves
(3) matter waves.

* Differentiate transverse and longitudinal waves.

Transverse waves	longitudinal waves.
1. particles of medium vibrate perpendicular to the direction of propogation of wave.	1. particles of medium vibrate parallel to the direction of propogation of wave.
2. Wave travels in the form of crests and troughs	2. wave travels in the form of compressions and rarefractions.
3. one crest and one trough constitute one wave.	3. one compression and rarefraction constitute one wave.

* Define wave length, frequency and velocity ?

wave length (λ)

It is the distance travelled by wave during one complete vibration of a particle in the medium.
SI unit – m.

frequency (ν)

It is the number of vibration executed by a particle of medium in one second.

$\nu = \frac{1}{T}$ SI unit : Hertz.

Velocity (v)

Distance travelled by the wave in one second.
SI unit m/s

$v = \nu\lambda$

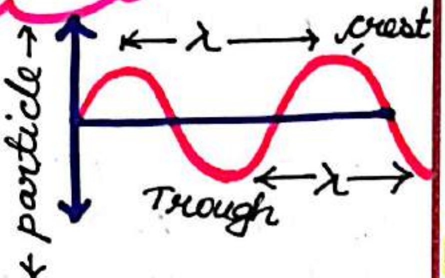
* Derive the expression $v = \nu\lambda$?

velocity of wave = $\dfrac{\text{Distance travelled by the wave during one complete vibration of particle}}{\text{Time taken}}$

$$V = \frac{\lambda}{T}$$

$$V = \left(\frac{1}{T}\right)\lambda = \nu\lambda$$

$$V = \nu\lambda$$



when wave travels from one medium to another medium, waves frequency remains same, but its speed and wave length change.

* write the factors which are changing and constant when a wave travels from one medium to another medium.

(i) frequency remains the same

(ii) speed and wavelength changes.

* what are the characteristics of wave motion?

(a) Wave motion is a form of disturbance which travels through a medium, it moves from one particle to another particle without the actual movement of the particle.

(b) A material medium is necessary to propagate a wave.

(c) In a wave motion the energy is transferred from one particle to another particle.

(d) Particle velocity is not constant but wave velocity is constant.

(e) Displacement of a vibrating particle of the medium is zero over one complete vibration.

* write the expression for the speed of transverse wave in a streched string.

$$V = \sqrt{T/m}$$

where T – Tension in the string.

m – linear density

* write the general expression for the velocity of longitudinal wave in a medium?

$$V = \sqrt{\frac{E}{\rho}}$$

where E – modulus of elasticity
ρ – density of medium

* Write expressions for velocity of longitudinal wave (i) in a solid (ii) in a liquid

(i) solid.

$$V = \sqrt{\frac{Y}{\rho}}$$

where Y – young's modulus
ρ – density.

(ii) In a liquid

$$V = \sqrt{\frac{B}{\rho}}$$

where B – bulk modulus of liquid
ρ – density.

* Derive Newton's formula for velocity of longitudinal wave in an elastic medium. what is laplace's correction to the Newton's formula.

Newton's formula.

Newton assumed that when sound travels in a gaseous medium, the change takes place in the medium is isothermal in nature. ie, when sound travels, the temperature remains constant. ③

speed of sound $V = \sqrt{\frac{P}{\rho}}$

P → pressure
ρ → density

at S.T.P $P = 1.013 \times 10^5 \, N/m^2$

$\rho = 1.293 \, kg m^{-3}$

$$V = \sqrt{\frac{1.013 \times 10^5}{1.293}} = 280 \, m/s$$

$V = 280 \, m/s$

The experimental value of velocity of sound in air at S.T.P is 332 m/s.

The result from Newton's formula is 16% less than the experimental value.

Laplace's correction

Laplace pointed out that it was wrong to assume that when sound travels in a gaseous medium, the changes are isothermal.

He assumed condensations and rarefractions take place quickly and no exchange of heat energy as gas medium is poor conductor of heat. Laplace said that changes are not isothermal, they are adiabatic in nature

velocity of sound

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

where P → pressure

ρ – density

γ – Ratio of specific heats of gas

$$\gamma = \frac{C_p}{C_v}$$ for air $\gamma = 1.4$

* velocity of sound at STP

$$V = \sqrt{\frac{\gamma P}{\rho}} = 331.5 \text{ m/s}$$

This value agrees with the experimental value.

* **Write the various factors affecting the velocity of sound in air?**

The factors like density of a gas, temperature, presence of moisture, wind speed affect the velocity of sound.

* **Discuss the effect of pressure and temperature of air on velocity of sound in air?**

(a) Effect of pressure

velocity of sound

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

P → pressure, ρ – density

At constant temperature, P/ρ remains constant.

∴ The change in pressure has no effect on the speed of sound.

velocity of sound does not depend upon change in pressure.

(b) Effect of temperature

velocity of sound $V = \sqrt{\frac{\gamma P}{\rho}}$

as $\rho = \frac{M}{V}$

$$V = \sqrt{\frac{\gamma P V}{M}}$$

but $PV = RT$

$$\therefore V = \sqrt{\frac{\gamma R T}{M}}$$

$$\therefore V \propto \sqrt{T}$$

∴ velocity of sound varies directly as the square root of absolute temperature.

If V_1 and V_2 are speeds of sounds at temperatures T_1 and T_2 respectively then

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

Let V_0 – velocity of sound at 0°c

V_t – velocity of sound at t°c

Then

$$\frac{V_0}{V_t} = \sqrt{\frac{273}{273+t}}$$

$$\frac{V_t}{V_0} = \sqrt{\frac{273+t}{273}}$$

$$V_t = V_0\sqrt{\frac{273+t}{273}}$$

If $t = 0°c$, $V_0 = 331.1$ m/s

If $t = 1°c$, $V_t = V_0\sqrt{\frac{273+t}{273}} \Rightarrow$

$t = 1$ $\quad V_1 = 331.1\sqrt{\frac{274}{273}}$

$$V_1 = 331.7 \text{ m/s}$$

Increase in velocity for 1°c rise of temperature

$V_1 - V_0 = 331.7 - 331.1$
$= 0.6$ m/s

$$\therefore V_t = V_0 + 0.6t$$

For every 1°c rise of temperature velocity of sound increases by 0.6 m/s.

* How does humidity affect the velocity of sound in air.

velocity of sound $V = \sqrt{\frac{\gamma P}{\rho}}$

$$V \propto \frac{1}{\sqrt{\rho}}$$

density of moist air is less than the density of dry air.

velocity of sound is inversely proportional to square root of density.

∴ velocity of sound in moist air is greater than that in dry air.

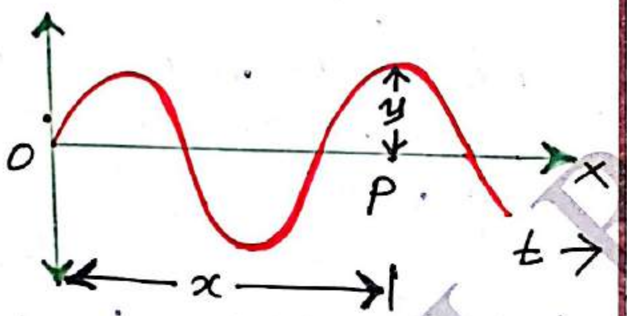
* what is meant by progressive wave?

A wave which travels continuously in a medium in the same direction without any change in its amplitude is called progressive wave or travelling wave.

It can be transverse or longitudinal.

* Derive the displacement relation for a progressive wave?

suppose that a plane simple harmonic wave travels from origin O along the positive direction of x-axis



The displacement of a particle at 'O' at any time 't' is given by

$$y = A\sin\omega t = A\sin\frac{2\pi}{T}t$$

where A is the amplitude of SHM, executed by the particle and ω is its angular frequency.

Let us find the displacement of particle at P, at a distance of x from the origin at time t.

The wave starting from 'O' would reach point P after a time $\left(\frac{x}{v}\right)$. so particles at 'P' starts vibrating at a time $\left(\frac{x}{v}\right)$, little later than the particle at 'O'.

∴ There is a time lag of $\frac{x}{v}$.

∴ Displacement of particle 'P' at 't' is

$$y = A\sin\frac{2\pi}{T}\left(t - \frac{x}{v}\right)$$

$$y = A\sin\frac{2\pi}{T}\left(\frac{vt - x}{v}\right)$$

$$y = A\sin\frac{2\pi}{vT}(vt - x)$$

$$\therefore y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

$vT = \lambda$

If φ is the epoch

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right] \quad \ldots (1)$$

[Displacement equation of progressive wave if it travels in +x axis-direction]

If the wave travels along '−x' axis

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt+x)+\phi\right] \quad \ldots (2)$$

equations (1) and (2) are displacement equations of progressive waves, they are also called as wave functions.

Note:

* since $\omega = \frac{2\pi}{T} = \frac{2\pi v}{\lambda} = \omega$

* propagation constant $k = \frac{2\pi}{\lambda}$

displacement equations can also be written as

$y = A\sin(\omega t - kx + \phi)$ → '+x' axis

or

$y = A\sin(\omega t + kx + \phi)$ → '-x' axis

* wave speed $v = \frac{\omega}{k}$

* **what are the characteristics of a progressive wave?**

1. The disturbance always travels forward and transferred from one particle to another particle.
2. The wave velocity is different from particle velocity.
3. Transverse progressive waves can be characterised by crests and troughs. Longitudinal progressive waves can be characterised by condensation and rarefractions.
4. Each particle in the medium vibrates to and fro with constant amplitude and frequency simple harmonically.
5. The vibration of each particle begins a little later than that of its preceding particle.
6. No particles permanently at rest.

* **state the principle of superposition?**

It states that when two or more waves travel in a medium in such a way that each wave represents its seperate motion individually, then the resultant displacement of particle of the medium at any time is equal to the vector sum of the individual displacements.

Resultant displacement

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \dots \bar{Y}_n$$

where y_1, y_2, y_3 are Individual displacements.

*** What is meant by interference?**

The phenomenon of superposition of two waves, travelling continously, having same amplitude, same frequency, same wavelength, same phase or constant phase difference, producing maximum intensity at some points and minimum intensity at some other points, is called interference of two waves.

*** What are the types of interference? Explain them.**

There are two types of interference

(i) constructive interference.

(ii) Distructive Interference

Constructive Interference.

When a crest of a wave meets a crest of another wave (or) trough of a wave meets trough of another wave, it results its maximum amplitude and maximum intensity. It is called constructive interference.

*** condition for constructive Interference.**

path difference

$$\delta = n\lambda$$

where $n = 0, 1, 2, 3 \dots$

Destructive Interference.

When a crest of one wave meets a trough of another wave, it results in minimum amplitude and minimum intensity. It is called destructive Interference.

*** What are called stationary waves?**

Whenever two progressive waves of the same wavelength and amplitude travel with same speed through a medium in opposite direction and superpose each other, they give rise to what is called standing waves or stationary waves.

* what are nodes and anti nodes?

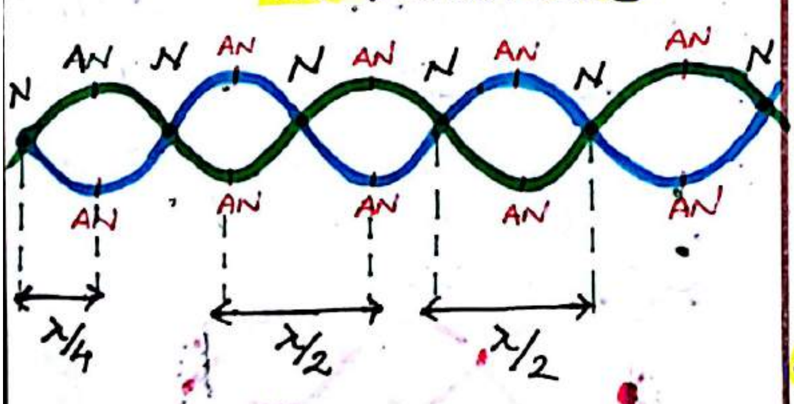
Nodes:-

In stationary waves, there are some points at which displacement is zero, amplitude is zero and strain is maximum and maximum change is pressure and density, called nodes.

Antinodes:-

In stationary waves there are some points at which displacement is maximum, strain is less and no change in pressure and density, called antinodes.

N → Node
AN → Anti Node



* what are characteristics of stationary waves?

1. stationary waves are not progressive crests or trough, compressions or rarefactions do not travel forward or backward
2. Energy is not transferred from one particle to another particle
3. Every particle, except the particles at nodes executes SHM with same period.
4. Amplitude of vibration of different particles is different. It is zero at nodes and maximum at antinodes
5. Distance between two consecutive nodes or antinodes is λ/2
6. Distance between two consecutive node or antinode is λ/4
7. change in pressure and density is maximum at nodes and minimum at antinodes.
8. The direction of motion of particles in

one segment is opposite to that of particles in preceding or succeeding segments.

* **What is meant by fundamental frequency, overtone and harmonics?**

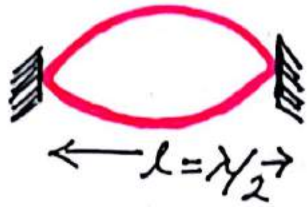
The lowest frequency is called fundamental frequency.

All the other frequencies other than fundamental frequencies are called overtones. The integral multiple of fundamental frequency is called harmonic.

* **Find the ratio of frequencies for different modes of vibrations in streched string?**

consider a string which is streched between two points (fixed). when it is plucked, the wave produced gets reflected back results in stationary wave, forming nodes at fixed ends.

(a) when the string vibrates with one segment.



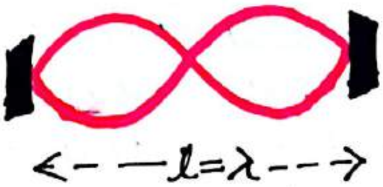
$l = \lambda/2$

$\lambda = 2l$

$\nu = \frac{V}{\lambda} = \frac{V}{2l}$

$\therefore \nu_1 = \frac{V}{2l}$ It is called as fundamental frequency or first harmonic.

(b) when the string vibrates with two segments.



$l = \lambda$

$\nu_2 = \frac{V}{\lambda}$

$\nu_2 = \frac{V}{l} = \frac{2V}{2l} = 2\left(\frac{V}{2l}\right)$

$\therefore \nu_2 = 2\nu_1$

ν_2 is called 2nd harmonic or Ist overtone

(c) when the string vibrates with three segments.

$l = \frac{3\lambda}{2}$

$\therefore \lambda = \frac{2l}{3}$

$$\nu_3 = \frac{V}{\lambda} = \frac{V}{\frac{2l}{3}}$$

$$\nu_3 = \frac{3V}{2l} = 3\left(\frac{V}{2l}\right)$$

$$\therefore \nu_3 = 3\nu_1$$

ν_3 is called third harmonic or 2nd overtone

∴ The frequencies are in the ratio

$$\nu_1 : \nu_2 : \nu_3 = \frac{V}{2l} : 2\left(\frac{V}{2l}\right) : 3\left(\frac{V}{2l}\right)$$

$$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$$

Hence all harmonics (even or odd) are present.

velocity $V = \sqrt{\frac{T}{m}}$

fundemetal frequency

$$\nu = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

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* state the laws of transverse vibrations of streched string?

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

1. Fundamental frequency (ν) is inversely proportional to length (l) of string when tension (T) and linear density (m) are constants.

$\nu \propto \frac{1}{l}$ when T and m are constants

2. Fundamental frequency (ν) is directly proportional to the square root of tension T when length (l) and linear density (m) are constants.

$\nu \propto \sqrt{T}$ when 'l' and T are constants.

3. Fundamental frequency (ν) is inversely proportional to the square root of linear density ($\sqrt{m}$) when l and T are constants.

$\nu \propto \frac{1}{\sqrt{m}}$ when l & T are constants

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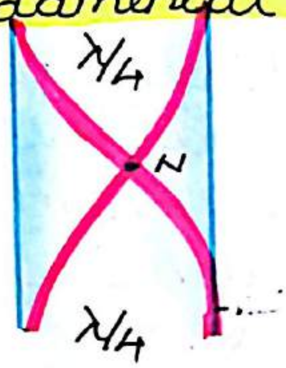
* <u>Describe the various modes of vibrations</u>

of air column in open organ pipe and show that open organ pipe can produce all harmonics ?

consider a tube of length 'l' open at both ends. when we place an excited tuning fork over one of the ends stationary waves are produced.

(a) Fundamental frequency

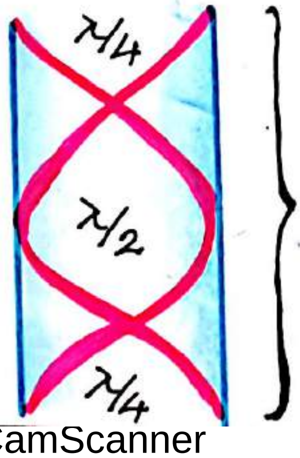


$l = \frac{\lambda}{2}$

$\lambda = 2l$

$\nu_1 = \frac{V}{\lambda}$ $\therefore \nu_1 = \frac{V}{2l}$ – fundamental frequency or first harmonic

(b) First overtone



$l = \lambda$

$\nu_2 = \frac{V}{\lambda}$

$\nu_2 = \frac{V}{l} = \frac{2V}{2l}$

$\nu_2 = 2\left(\frac{V}{2l}\right) = 2\nu_1$

$\nu_2 = 2\nu_1$ 2nd harmonic, 1st overtone.

(c) second overtone

$l = \frac{3}{2}\lambda$

$l = \frac{3}{2}\lambda$ $\lambda = \frac{2l}{3}$

$\lambda = \frac{2l}{3}$ $\nu_3 = \frac{V}{\lambda} = \frac{V}{\frac{2l}{3}}$.

$\therefore \nu_3 = \frac{3V}{2l}$

$\nu_3 = 3\left(\frac{V}{2l}\right) = 3\nu_1$

$\nu_3 = 3\nu_1$ → 3rd harmonic, 2nd overtone

ratio of frequencies

$\nu_1 : \nu_2 : \nu_3 = \frac{V}{2l} : 2\left(\frac{V}{2l}\right) : 3\left(\frac{V}{2l}\right)$

$\therefore \nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$

Thus in open pipe (organ), all harmonics are present. There fore it is prefered in all musical Instruments.

The general equation for frequency in open pipe $\nu = n\left(\frac{V}{2l}\right)$

where $n = 1, 2, 3 \ldots\ldots$

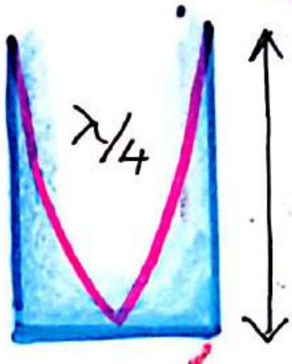
* Discribe the various Modes of vibration in air column in closed pipe. (organ). Show that a closed pipe can produce only odd harmonics?

Consider a tube of length 'l' closed at one end and opened at the other end. Placed an excited tuning fork over the open end. A stationary wave is produced; a node is formed at the closed end and an antinode is formed at the open end.

(a) Fundamental frequency

$$l = \lambda/4$$

$$\lambda = 4l$$

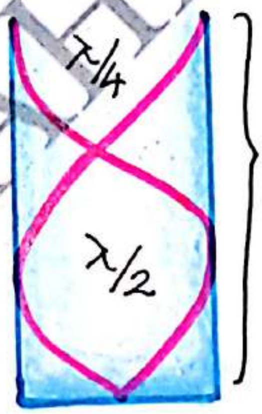


$$l = \frac{\lambda}{4}$$

$$\nu = \frac{V}{\lambda} = \frac{V}{4l}$$

$$\nu_1 = \frac{V}{4l}$$ → First harmonic or fundamental frequency.

(b) First overtone.



$$l = \frac{3\lambda}{4}$$

$$l = 3\lambda/4$$

$$\therefore \lambda = \frac{4l}{3}$$

$$\nu = \frac{V}{\lambda}$$

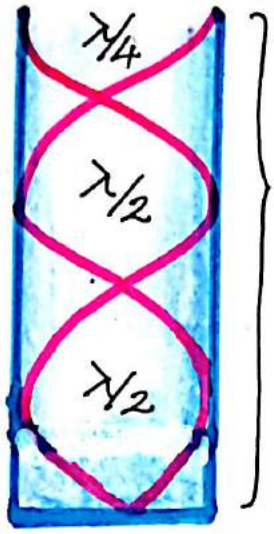
$$\nu_2 = \frac{V}{\frac{4l}{3}} \Rightarrow \nu_2 = \frac{3V}{4l}$$

$$\nu_2 = 3\left(\frac{V}{4l}\right) = 3\nu_1$$

$$\nu_2 = 3\nu_1$$ → First overtone or Third harmonic

(c) second overtone.

$$l = \frac{5\lambda}{4}, \quad \lambda = \frac{4l}{5}$$



$$l = \frac{5\lambda}{4}$$

$$l = \frac{5\lambda}{4} \qquad \lambda = \frac{4l}{5}$$

$$\nu = \frac{V}{\lambda} \qquad \nu_3 = \frac{V}{\frac{4l}{5}} = \frac{5V}{4l}$$

$$\nu_3 = \frac{5V}{4l} = 5\left(\frac{V}{4l}\right) = 5\nu_1$$

$$\therefore \nu_3 = 5\nu_1$$

→ Fifth harmonic or second overtone

Ratio of frequencies

$$\nu_1 : \nu_2 : \nu_3 = \frac{V}{4l} : \frac{3V}{4l} : \frac{5V}{4l}$$

$$\nu_1 : \nu_2 : \nu_3 = 1 : 3 : 5$$

Only odd harmonics are present in closed organ pipe.

Expression for frequency is

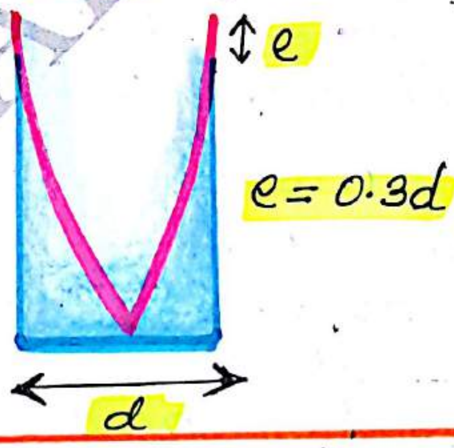
$$\nu = (2n+1)\frac{V}{4l}$$

where $n = 0, 1, 2, 3 \ldots$

* What is meant by end correction? give the expression for the same?

The antinode formed at the open end will never coincide with the end of the tube. It will project outside by an amount 'e' which is called end correction. If 'd' is the diameter of the tube end correction

$$e = 0.3d$$

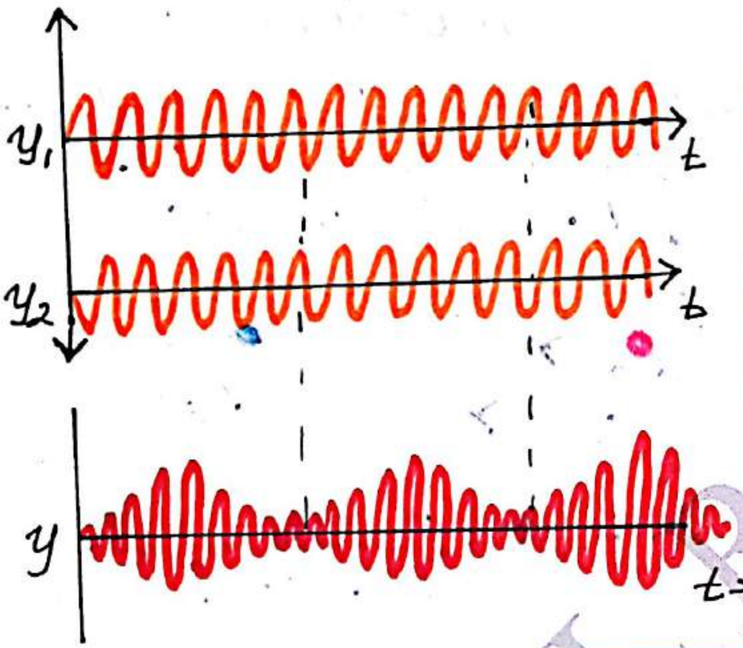


* What are beats?

The superposition of two sound waves of nearly equal frequencies travelling in the same direction in a medium produces regular variation in the intensity of sound with time. This phenomenon is called as beat. The no of beats heard per second is called **beat frequency**. It is equal to the difference in frequency.

Beat frequency = $\nu_1 \sim \nu_2$

Graphical representation



**Imp

Explain Doppler effect. obtain the general expression for apparent frequency of Sound?

Doppler Effect:-

The phenomenon of the apparent change in the frequency of sound produced by the source heard by the listener when there is a relative motion between the source and the listener is called **doppler effect**.

Derivation for apparent frequency.

consider a source (S) producing sound waves of frequency (ν).

Let V be the velocity of sound in the medium

V_S → velocity of sound moving towards listener L.

V_L → velocity of listener moving away from the source.

(a) when listener and source are at rest

Let 'V' be the velocity of sound and λ be the wave length of sound wave

frequency $\nu = \frac{V}{\lambda}$

$$\therefore \lambda = \frac{V}{\nu}$$

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(b) When source moves towards listener with velocity V_s

There is an apparent change in wavelength when S moves towards L. Relative velocity of sound w.r.t source is

$V - V_s$

Apparent wavelength $\lambda' = \dfrac{V - V_s}{\nu}$

(c) Listener moves away from source with velocity V_L

There is an apparent change in frequency of sound heard by listner.

Relative velocity of sound w.r.t listner is $V - V_L$.

$\therefore$ Apparent frequency $\nu' = \dfrac{V - V_L}{\lambda'}$

$$\nu' = \frac{V - V_L}{\frac{V - V_s}{\nu}}$$

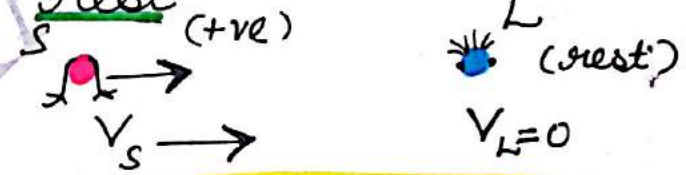
$$\therefore \nu' = \left(\frac{V - V_L}{V - V_s}\right)\nu$$

This is the expression for the apparent frequency of sound heard by the listener.

Special Cases

Case 1: Listener at rest and source in motion

(a) When source moves towards the listener and listener is at rest



$$\therefore \nu' = \left(\frac{V}{V - V_s}\right)\nu$$

(b) Source moves away from the listener, Listener at rest

$$\therefore \nu' = \left(\frac{V}{V - -V_s}\right)\nu$$

$$\nu' = \left(\frac{V}{V + V_s}\right)\nu$$

Case 2: Listener is motion and source at rest.

(a) Listener moves towards source and source is at rest.

$$\nu' = \left(\frac{V+V_L}{V+0}\right)\nu$$

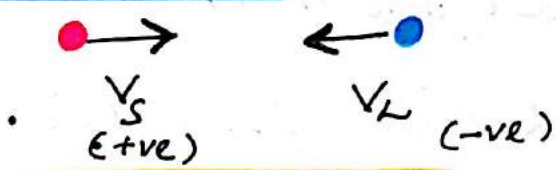
$$\nu' = \left(\frac{V+V_L}{V}\right)\nu$$

(b) Listener moves away from the source and source is at rest.

$$\therefore \nu' = \left(\frac{V-V_L}{V}\right)\nu$$

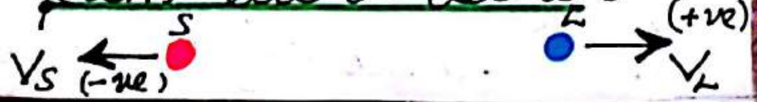
Case 3: when both the source and the listener are in motion.

(a) source and listener are moving towards each other



$$\nu' = \left(\frac{V+V_L}{V-V_S}\right)\nu$$

(b) source and listener are moving away from each other



$$\nu' = \left(\frac{V-V_L}{V+V_S}\right)\nu$$

(c) source moves away from listener and listener moves towards the source.

$$\therefore \nu' = \left(\frac{V+V_L}{V+V_S}\right)\nu$$

(d) Listener moves away from the source and source moves towards the listener

$$\nu' = \left(\frac{V-V_L}{V-V_S}\right)\nu$$

* If wind blows with a velocity 'w' in the direction of velocity of sound (V) resultant velocity is $V+w$

$$\nu' = \left(\frac{V+w-V_L}{V+w-V_S}\right)\nu$$

* If wind blows with a velocity 'w' in a

direction opposite to that of V.

$$V' = V - w$$

$$\therefore \nu' = \left(\frac{V - w - V_L}{V - w - V_S} \right) \nu$$

* Write some applications of Doppler effect.

(1) To estimate the speed of submarine (SONAR)

The ultrasonic waves transmitted from a ship gets reflected from submarine. There is a shift in the frequency b/w the transmitted and reflected wave from which velocity of submarine can be measured.

(2) To estimate the speed of automobile, aeroplane etc.

(3) To estimate the velocity and rotation of the sun

(4) To Track artificial satellites.

note:

(1) Doppler effect is asymmetric in sound

(2) Doppler effect is symmetric in light

5 MARK QUESTIONS & imp Qustns

(1) Explain Doppler effect. Obtain the general expression for the apparent frequency of sound? [repeated]

(2) Describe various modes of vibration of air column in open and closed pipes. show that a closed pipe can produce only odd harmonics whereas an open pipe can produce all harmonics?

(3) Derive an expression for a harmonic wave?

(4) Derive Newton's formula for the velocity of longitudinal wave in an elastic medium? what is Laplace's correction?

(5) Discuss the effect of pressure & temperature — velocity of sound