WAVES

A wave is a mode of transference of energy from point to point in the direction of propagation of the wave. The waves are periodic disturbances transmitted through a medium from a point of origin called the source. For sound waves, a vibrating body is the source of disturbance and for light waves, the source of light is the source of disturbance. These disturbances from the source are handed over to the particles in the medium. The particles in the medium behave as if they are connected by some elastic forces. When a particle in the medium is set into vibration, the adjacent particles acquire a similar kind of vibration.

A wave motion is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their equilibrium positions, the disturbance being handed over from particle to particle.

***A wave** is a disturbance which propagates energy from one place to another without transport of matter.*

A familiar example is the ripples formed on the surface of water when a stone is thrown on water in a pond. The ripples travel in concentric circles of ever increasing radius till they strike the boundary of the pond.

A wave involves propagation of a physical condition in space and time. The fundamental differentiation among waves of different types is the nature of the quantity which is being transported by the waves and the way in which the vibrations are related to the direction of propagation.

Mechanical and electromagnetic waves

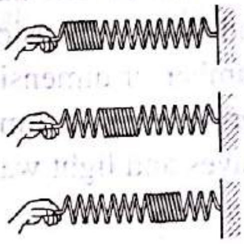
A wave may or may not require a medium for its propagation. *The waves which do not require a medium for their propagation are called* ***non-mechanical wave***. E.g. light waves, heat waves, radio waves etc. Non-mechanical waves can travel through vacuum. In fact all electromagnetic waves are non-mechanical. On the other hand waves which require a medium for propagation are called **mechanical waves**. In the propagation of mechanical waves elasticity and density of the medium play an important role. Waves on strings, springs, sound waves etc are familiar examples of mechanical waves.

TYPES OF MECHANICAL WAVES

(i) Longitudinal wave

If the vibrations of the particles of the medium conveying a wave are to and fro and parallel to the direction of propagation of the wave, then the wave is called a longitudinal wave.

For example, when a spring under tension is set up oscillating to and fro at one end, a longitudinal wave travels along the spring; the coils vibrate back and forth in the direction in which the disturbance travels along the spring.



A longitudinal pulse along a stretched spring. The disturbance of the medium (the displacement of the coils) is in the direction of the wave motion.

Fig. 10

Sound waves in a gas are longitudinal waves. When a sound wave propagates through a gas, particles in the medium vibrate to and fro and parallel to the direction of propagation of the wave.

Owing to the longitudinal motion of the particles, sound waves consist of a series of compressions followed by rarefactions.

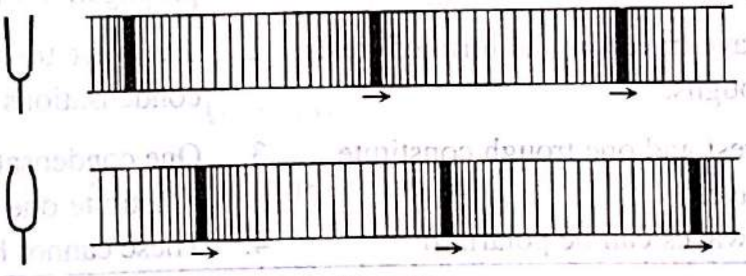


Fig. 11

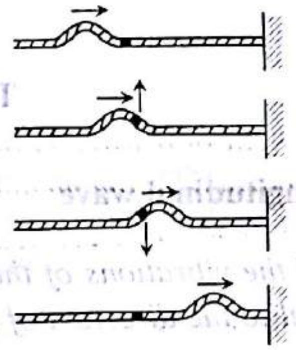
Figure 2 shows how a vibrating tuning fork sends out a sound wave. When the prong moves out, it compresses the air particles. This region of higher pressure is called **condensation**. Thus a pulse of compression moves outwards. Similarly a reverse movement of the prong gives rise to a region of low pressure called **rarefaction**. This pulse of rarefaction moves outwards. The particles at the centre of compression move from their equilibrium position in the direction of the wave; whereas the particles at the centre of the rarefaction move in the opposite direction.

(ii) Transverse wave

If the vibrations of the particles of the medium conveying a wave are perpendicular to the direction of propagation of the wave, then the wave is called a transverse wave.

For example, consider a coil spring or a light rope tied at one end. Hold its other end. It is kept under tension by stretching. Give a sudden flipping motion by a jerk of the hand perpendicular to the length of the spring. A disturbance in the shape of a pulse can be seen travelling along the spring towards the fixed end. The waves travelling along the spring (or rope) are transverse.

Some waves are neither purely longitudinal nor purely transverse. For example, in waves on the surface of water, the particle of water move both up and down and back and forth. In some media such as steel bars both transverse and longitudinal wave motion can take place. It is found that generally transverse and longitudinal waves travel with different speeds in the same medium.



A pulse travelling on a stretched rope is a transverse wave. That is, any element P on the rope moves in a direction perpendicular to the wave motion.

Fig. 12

Waves can also be classified as one, two and three dimensional waves, according to the number of dimensions in which they propagate energy. Waves moving along a string or a spring are one dimensional. Ripples on the surface of water are two dimensional. Sound waves and light waves are three dimensional.

Difference between Transverse and Longitudinal waves

	Transverse waves		*Longitudinal waves*
1.	The particles of the medium vibrate perpendicular to the direction of propagation of the wave.	1.	Particles in the medium vibrate parallel to the direction of propagation of the wave.
2.	The wave travels in the form of crests and troughs.	2.	The wave travels in the form of condensations and rarefactions.
3.	One crest and one trough constitute one wave.	3.	One condensation and one rarefaction constitute one wave.
4.	These waves can be polarised	4.	These cannot be polarised.

Some important terms connected with wave motion

Amplitude (A)

Amplitude of a wave is the maximum displacement of the wave. In the case of mechanical waves it is the maximum displacement of the particles of the medium from their equilibrium position

Unit : m

Period (T)

Period of a wave is the time taken by the particles of the medium to execute one complete to and fro motion.

Unit : s

Frequency (ν)

Frequency of a wave is the number of vibrations executed by a particle of the medium *in one second.*

$\nu = 1/T$ Unit: $hertz (Hz)$ or s^{-1}

Wavelength (λ)

Wave length of a wave is the distance travelled by the wave during one complete vibration of a particle in the medium.

It is also defined as the distance between two consecutive particles which are in the same phase of vibration.

Unit : m

So the wavelength is the distance between two successive crests or troughs of the wave.

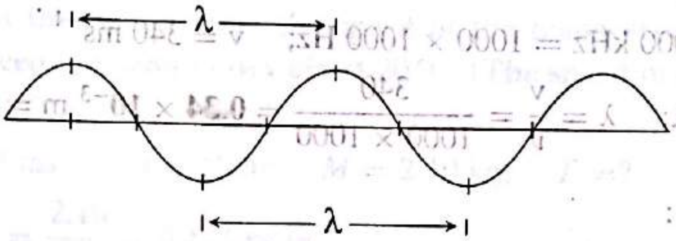


Fig. 13

Velocity (v)

Velocity of a wave is the distance travelled by the wave in one second.

Unit : ms^{-1}

Relation between velocity (v), frequency (ν) and wavelength (λ)

Distance travelled by the wave during one complete vibration of the particle $= \lambda$

No. of vibrations of the particle in one second $= \nu$

$\therefore$ Distance travelled by the wave in one second $= \nu\lambda$

i.e. Velocity of the wave $= \nu\lambda$

$$v = \nu\lambda$$

When a wave passes from one medium to another, the velocity v and wavelength λ change; but the frequency ν remains unchanged.

Examples

XIII.1. The audible frequency range of a human ear is 20 Hz to 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperature to be 340 ms^{-1}.

$\nu_1 = 20$ Hz; $\nu_2 = 20$ kHz $= 20,000$ Hz; $v = 340\ ms^{-1}$

$\lambda_1 = ?$ $\lambda_2 = ?$

$$v = \nu\lambda; \quad \lambda = \frac{v}{\nu}; \quad \lambda_1 = \frac{340}{20} = 17\ m; \quad \lambda_2 = \frac{340}{20,000} = \mathbf{17 \times 10^{-3}\ m}$$

XIII.2. A bat emits ultrasonics of frequency 1000 kHz **in air. If the sound meets the water surface, what is the wavelength of (a) the reflected sound (b) the transmitted sound? (speed of sound in air =** $340\ ms^{-1}$ **and in water =** $1486\ ms^{-1}$**) [NCERT]**

(a) *In air* :

$$\nu = 1000\ kHz = 1000 \times 1000\ Hz; \quad v = 340\ ms^{-1}$$

$$v = \nu\lambda; \quad \lambda = \frac{v}{\nu} = \frac{340}{1000 \times 1000} = \mathbf{0.34 \times 10^{-3}\ m = 0.34\ mm}$$

(b) *In water* :

$\nu = 1000 \times 1000$ Hz (frequency remains constant)

$v = 1486\ ms^{-1}$

$$\therefore \lambda = \frac{v}{\nu} = \frac{1486}{1000 \times 1000} = \mathbf{1.486 \times 10^{-3}\ m = 1.486\ mm}$$

SPEED OF WAVE MOTION

Wave motion can be transverse or longitudinal. Generally transverse and longitudinal waves travel with different speeds in the same medium.

1. Speed of transverse wave in a stretched string

A string is tied to a rigid support. It is stretched under a tension by pulling at the free end. A transverse wave pulse is produced at the end. The wave travels along the string with a velocity v given by

$v = \sqrt{T/m}$, where T is the tension in the string and m is linear density or mass per unit length of the string.

The linear density m is related to volume density 'ρ' as $m = a\rho$ where a = cross sectional area of the string.

Examples

XIII.3. A steel wire 0.72 m long has a mass of 5.0 × 10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse wave on the wire? [NCERT]

$l = 0.72$ m; $M = 5.0 \times 10^{-3}$ kg; $T = 60$ N; v =?

$$m = \frac{M}{l} = \frac{5.0 \times 10^{-3}}{0.72} = 6.9 \times 10^{-3}\ \text{kgm}^{-1}$$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{60}{6.9 \times 10^{-3}}} = \mathbf{93\ ms^{-1}}$$

XIII.4. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of the transverse wave on the wire equals the speed of sound in dry air at 20 °C. (The speed of sound in dry air at 20 °C = 343 ms^{-1})

v = 343 ms^{-1}; $l = 12$ m; $M = 2.10$ kg; T =?

$$m = \frac{M}{l} = \frac{2.10}{12} = 0.175\ \text{kg m}^{-1}$$

$$v = \sqrt{\frac{T}{m}};\quad v^2 = T/m \quad \therefore T = mv^2$$

$$T = 0.175 \times 343^2 = \mathbf{2.06 \times 10^4\ N}$$

Velocity of longitudinal wave

The general expression for velocity of longitudinal wave in a medium is given by,

$$v = \sqrt{E/\rho};$$

where E is the modulus of elasticity and ρ the density of the medium.

(a) **In a solid** $E = Y$, the Young's modulus

$$\therefore v = \sqrt{Y/\rho}$$

For example in iron $Y = 20 \times 10^{10}$ Nm^{-2} and $\rho = 7.7 \times 10^3$ kg m^{-3}

$$v = \sqrt{\frac{20 \times 10^{10}}{7.7 \times 10^3}} = \mathbf{5025\ ms^{-1}}$$

(b) **In a liquid**

$E = B$, the bulk modulus of the liquid

$$\therefore v = \sqrt{B/\rho}$$

For example, in water $B = 2.1 \times 10^9$ Nm^{-2}

$$v = \sqrt{\frac{2.1 \times 10^9}{10^3}} = \mathbf{1500\ ms^{-1}}$$

(c) In a gas

Newton's formula

Newton assumed that when sound wave is propagated through a gaseous medium, the change in pressure and volume of the gas due to condensations and rarefactions does not produce a change in temperature. The bulk modulus of the gas under isothermal change is equal to the pressure P exerted by the gas. Hence,

$$v = \sqrt{P/\rho}$$

At S.T.P, for air, $P = 1.013 \times 10^5\ \text{Nm}^{-2}$; $\rho = 1.293\ \text{kg m}^{-3}$

$$\therefore\ v = \sqrt{1.013 \times 10^5/1.293} = 280\ \text{m s}^{-1}$$

However, the experimental value of the velocity of sound in air at S.T.P. is about 332 ms^{-1}. This shows that the result obtained by Newton's formula is 16% lower than the accepted experimental value.

Laplace's correction

The discrepancy in the Newton's formula was explained by Laplace.

Laplace assumed that the condensations and rarefactions are taking place so quickly that there is no chance for the medium to exchange the heat with the surrounding. So the temperature of the gas changes. Hence the change of pressure and volume of the gas is an adiabatic change. The bulk modulus E of the medium is the adiabatic elasticity γP. Taking this into consideration, the Newton's equation is modified as,

$$v = \sqrt{\frac{\gamma P}{\rho}};$$

where $\gamma = \frac{C_p}{C_v}$, the ratio of the specific heats of the gas. The equation is called Newton–Laplace equation. For air, $\gamma = 1.4$

Hence, velocity of sound at S.T.P. is calculated as,

$$v = \sqrt{\frac{\gamma P}{\rho}} = 331.5\ \text{ms}^{-1}$$

This value agrees remarkably with the experimental result.

Factors affecting the velocity of sound in air

Effect of pressure

Velocity of sound in a gas is given by Newton–Laplace equation,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At constant temperature P/ρ is constant. Hence v is independent of pressure changes provided the temperature remains constant.

Effect of temperature

The formula for velocity of sound in a gas is,

$$v = \sqrt{\frac{\gamma P}{\rho}},$$

But, $\rho = \frac{M}{V}$; where M is the mass of a gas occupying the volume V.

$$v = \sqrt{\frac{\gamma PV}{M}}$$

But $PV = RT$ $\quad \therefore v = \sqrt{\frac{\gamma RT}{M}}$ where, M is the mass of 1 mole of the gas and R is universal gas constant.

$\therefore v \propto \sqrt{T}$. The *velocity of sound in a gas is directly proportional to the square root of absolute temperature.*

If v_1 and v_2 are the velocities of sound in a gas at temperatures T_1 K and T_2 K. Then,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

If v_0 is the velocity of sound in a gas at 0 °C and v_t the velocity at t °C, then,

$$\frac{v_0}{v_t} = \sqrt{\frac{273}{273+t}}; \quad v_t = v_0\sqrt{\frac{273+t}{273}}$$

Velocity of sound in air at 0°C $= 331.1 \text{ m s}^{-1}$

Velocity of sound in ait at 1°C $= v_0\sqrt{(273+t)/273}$

$= 331.1\sqrt{274/273} = 331.7 \text{ m s}^{-1}$

$\therefore$ Increase in velocity for 1°C rise of temperature $= 331.7 - 331.1 = 0.6 \text{ m s}^{-1}$

In general velocity at any temperature t°C is given by;

$$v_t = (v_0 + 0.6t) \text{ ms}^{-1}$$

Effect of Humidity

$$v = \sqrt{\gamma P/\rho} \quad \therefore v \propto \frac{1}{\sqrt{\rho}}$$

The humidity in air depends upon the water vapour present in atmosphere. The density of water vapour is less than the density of dry air.

Since the density of moist air is less than that of dry air, sound travels faster in moist air than in dry air.

Speed of sound waves through different media

Medium	*Temperature* (°C)	*Speed* (ms^{-1})
Air	0	333.3
Hydrogen	0	1286
Water	15	1450
Copper	20	3560
Iron	20	5130
Granite	20	6000
Vulcanized rubber	0	54

Examples

XIII.5. A piezo-electric quartz plate of thickness 0.005 m is vibrating in resonant condition. Calculate the fundamental frequency, if for quartz $Y = 8 \times 10^{10}\ Nm^{-2}$ and $\rho = 2.65 \times 10^3\ kg\,m^{-3}$.

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{8 \times 10^{10}}{2.65 \times 10^3}} = 5.5 \times 10^3\ ms^{-1}.$$

For the fundamental mode, $\lambda/2 = L$; $\lambda = 2L = 2 \times 0.005 = 10^{-2}$ m

$$\text{Frequency } f = \frac{v}{\lambda} = 5.5 \times 10^3/10^{-2} = \mathbf{5.5 \times 10^5\ Hz}$$

XIII.6. Calculate the speed of sound in air at STP. The mass of 1 mole of air is 29.0×10^{-3} kg [For air, $\gamma = 1.40$] [NCERT]

1 mole of a gas occupies a volume 22.4 litres. Density of air at S.T.P,

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{29.0 \times 10^{-3}}{22.4 \times 10^{-3}} = 1.29\ kg\ m^{-3}$$

$\gamma = 1.40$; $P = 1.013 \times 10^5\ Nm^{-2}$; $v = ?$

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.29}} = \mathbf{331.6\ ms^{-1}}$$

XIII.7. Velocity of sound in air at 289 K is 340 ms^{-1}. What will be the velocity if the pressure is doubled and the temperature is raised to 324 K?

Since the velocity of sound is independent of the pressure of air, the change in pressure does not affect it.

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}; \quad \frac{340}{v_2} = \sqrt{\frac{289}{324}}$$

$$v_2 = 340 \times \sqrt{\frac{324}{289}} = \mathbf{360\ ms^{-1}}$$

XIII.8. Find the temperature at which the velocity of sound in oxygen is the same as that in hydrogen at 0°C. (1mol of oxygen = 32 g & 1mol of hydrogen = 2 g)

(iii)
$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{M}} = \sqrt{\frac{\gamma RT}{M}}$$

Let T K be the temperature of oxygen at which its velocity becomes equal to that of hydrogen at 0°C.

For oxygen: $T = ?$ $M = 0.032$ kg

$$\therefore v = \sqrt{\gamma RT/0.032} \quad (1)$$

For hydrogen: $T = 273$ K; $M = 0.002$ kg

$$\therefore v = \sqrt{\frac{\gamma R \times 273}{0.002}} \quad (2)$$

Equating RHS of equations (1) and (2)

$$\sqrt{\frac{\gamma RT}{0.032}} = \sqrt{\frac{\gamma R \times 273}{0.002}}$$

$$\therefore T = \frac{273 \times 0.032}{0.002} = 4368 \text{ K} = \mathbf{4095°C}$$

DISPLACEMENT RELATION IN A PROGRESSIVE WAVE – Expression for a progressive wave – Wave functions – propagation constant

A harmonic wave is one in which the particles in the medium vibrate simple harmonically when the disturbance propagates through the medium.

A harmonic wave is represented by simple harmonic function such as sine or cosine function.

Consider a plane harmonic wave starting from a point 'O' and travelling along the positive direction of X-axis.

Every particle in the medium executes simple harmonic motion with amplitude A and period T. The displacement y of the particle at O from its mean position at any time t is given by,

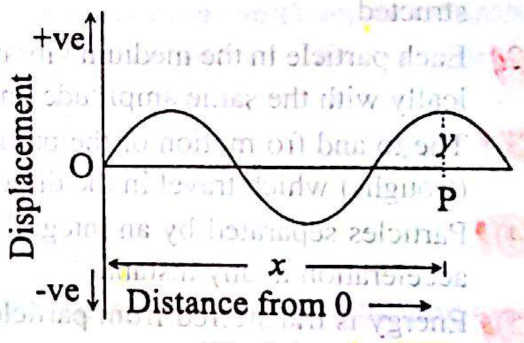


Fig. 14

$$y(x,t) = y(x=0,t) = A\sin\omega t = A\sin\frac{2\pi t}{T} \quad \text{(i)}$$

Let v be the velocity of the wave. If we consider a particle of the medium at P distant x from O, the wave starting from O would reach P after a time (x/v). So the particle at P lags in phase by a time x/v than the particle at O.

Hence the displacement of the particle P at the time t is given by,

$$y = A\sin\frac{2\pi}{T}(t - x/v) = A\sin\frac{2\pi}{vT}(vt - x)$$

Since $v = \nu\lambda$, $v/\nu = \lambda$; *i.e.,* $vT = \lambda$,

$$\therefore \mathbf{y = A\sin\frac{2\pi}{\lambda}(vt - x)}, \quad \text{(ii)}$$

If ϕ is the epoch, then,

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right] \quad \text{(iii)}$$

A wave travelling in the negative direction of the X-axis is given by,

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + \phi\right] \quad \text{(iv)}$$

The function represented in equation (iii) is periodic in position coordinate x and time t. It represents a transverse wave propagating in the positive direction of the x-axis. A wave travelling in the negative direction of the x-axis is represented by the equation (iv). Displacement functions, such as given in eqn: (iii) and (iv), are called **wave functions.**

The wave function can also be written as

$$y = A\sin(\omega t - kx + \phi) \quad \text{(v)}$$

$$\text{where } \omega = \frac{2\pi}{\lambda}v = 2\pi/T \ [\because v/\lambda = f = \frac{1}{T}] \quad \text{(vi)}$$

$$\text{and } k = \frac{2\pi}{\lambda} \quad \text{(vii)}$$

The constant $k = 2\pi/\lambda$ is called the **propagation constant** or the **angular wave number.** Its unit is radian per metre (rad m^{-1}). $\omega = (2\pi/\lambda) \times v = (2\pi/T)$ is called the angular frequency of the wave. Its unit is rad s^{-1}.

$\omega/k = v$, the wave speed

$(\omega t - kx + \phi)$ is in radians.

Characteristics of a progressive wave

(1) A progressive wave is one which travels through the medium undamped and unobstructed.

(2) Each particle in the medium vibrates to and fro about its mean position simple harmonically with the same amplitude and frequency.

(3) The to and fro motion of the particles produce condensations (crests) and rarefactions (troughs) which travel in the direction of propagation of the wave.

(4) Particles separated by an integral multiple of λ have same displacement, velocity and acceleration at any instant.

(5) Energy is transferred from particle to particle and there is no transfer of matter, whatsoever.

(6) The vibration of each particle begins a little later than that of its predecessor.

Reflection of sound–Echo

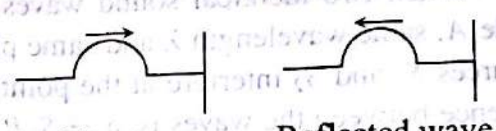
When a sound wave travelling in a medium strikes the surface separating two media a part of the incident wave is reflected back into the first medium obeying ordinary laws of reflection; while the remaining part is partly absorbed and partly refracted into the second medium.

When sound wave is reflected from a **rarer medium** or **free boundary** there is **no phase change** but the nature of the sound wave is changed *i.e.*, on reflection, condensation is reflected back as rarefaction and vice versa.

If the incident wave is $y = A\sin(\omega t - kx)$, then the equation for the reflected wave is of the form,

$$y' = A'\sin(\omega t + kx)$$

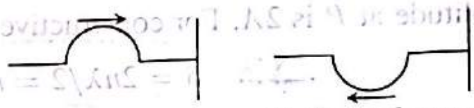
where A' is new amplitude of the reflected wave.



Incident wave Reflected wave

Fig. 15

When sound wave is reflected from a **rigid boundary** or a denser medium the **phase** of the wave is **reversed** but the nature does not change. The compression is returned as compression and a rarefaction as rarefaction.



Incident wave Reflected wave

Fig. 16

The equation of the incident wave is

$$y = A\sin(\omega t - kx)$$

and the reflected wave will be

$$y' = -A'\sin(\omega t + kx)$$

Refraction of waves

If the boundary on which a wave falls is not completely rigid or if a wave pulse is incident obliquely on a boundary between two different media, a part of the incident wave is reflected and a part is transmitted. The transmitted wave is called the *refracted wave*. The refracted wave obeys Snell's law of refraction.

PRINCIPLE OF SUPERPOSITION

If two or more waves arrive simultaneously, the particles of the medium are subjected to two or more simultaneous displacements and a new wave is produced. This phenomenon of intermixing of two or more waves to produce a new wave is called superposition of waves.

In case of superposition of waves the resultant wave function at any point is the algebraic sum of the wave functions of individual waves.

$$y = y_1 + y_2 + y_3 + \cdots$$

This principle is called principle of **superposition** and holds good as long as the amplitude of the wave is not too large.

Important applications of superposition principle are (1) Interference of waves (2) Stationary waves and (3) Beats.

Interference of two waves

Consider two waves of the same amplitude and frequency travelling in the same direction. If the two waves arrive in such a way that crests meet crests or troughs meet troughs the displacement of the two waves add and the resultant wave has twice the amplitude. These two waves are said to show *constructive interference*. On the other hand, if the crest of one wave meets the trough of the other, then the two waves cancel each other. *The two waves are said to show destructive interference.*

When two identical sound waves of same amplitude A, same wavelength λ and same phase ϕ, from the sources S_1 and S_2 interfere at the point P, the path difference between the waves is, $\delta = S_2P - S_1P$.

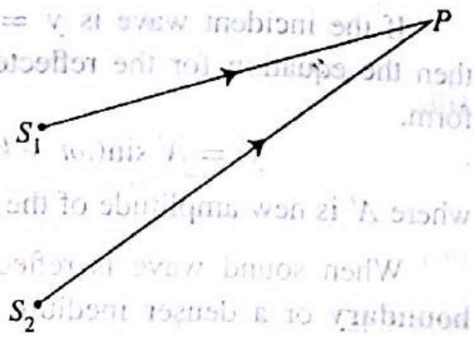


Fig. 17

If this path difference is an even multiple of $\lambda/2$ the waves interfere constructively at the point and the intensity of sound will be maximum at P. The resultant amplitude at P is $2A$. For constructive interference,

$$\delta = 2n\lambda/2 = n\lambda, \text{ where } n = 0, 1, 2, 3, \ldots$$

If the path difference is an odd multiple of $\lambda/2$, the waves interfere destructively at the point and the intensity of sound will be zero at the point. The resultant amplitude at P is zero. For destructive interference,

$$\delta = (2n+1)\lambda/2, \text{ where } n = 0, 1, 2, 3 \ldots$$

Examples

XIII.9. A transverse harmonic wave on a string is described by, $y(x,t) = 3.0\sin(36t + 0.018x + \pi/4)$; where x and y are in cm and t in seconds. (i) Is this a travelling or a stationary wave? If it is a travelling wave, what are its speed and direction of propagation? (ii) What are its amplitude and frequency? (iii) What is its initial phase at the origin? (iv) What is the distance between two successive crests in the wave? [NCERT]

General equation of a harmonic wave moving in the negative direction of X-axis is given by,

$$y = A\sin(\omega t + kx + \phi) = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + \phi\right]$$

Comparing the given equation with the general equation, we get,

(i) It is a travelling wave and moves in the negative direction of X-axis.

$$\text{velocity of the wave} = \omega/k = 36/0.018 = 2000 \text{ cm s}^{-1} = 20 \text{ ms}^{-1}$$

(ii) Amplitude of the wave = 3 cm = $\mathbf{3 \times 10^{-2}}$ **m**

$$\left(\frac{2\pi}{\lambda}\right)x = 0.018x$$

$$\therefore \lambda = \frac{2\pi}{0.018} = 348.9 \text{ cm} = \mathbf{3.489 \text{ m}}$$

$$\text{Frequency} = \nu = v/\lambda = 20/3.489 = \mathbf{5.7 \text{ Hz}}$$

(iii) Initial phase at the origin, $\phi = \pi/4$ rad.

(iv) Distance between successive crests; $\lambda = \mathbf{3.489 \text{ m}}$

XIII.10. A wave travelling along a string is described by $y(x,t) = 0.005\sin(80.0x - 3.0t)$m. Calculate (a) the amplitude (b) the wavelength, (c) the period and (d) frequency of the wave. [NCERT]

Standard equation for the displacement of a particle at a distance x at a time t is,

$$y = A\sin[(2\pi/\lambda)(vt - x)] \text{ if } \phi = 0$$

$$= A\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

The given equation is

$$y = 0.005\sin(80.0x - 3.0t)$$

Comparing these equations we get:

(a)Amplitude, $A = \mathbf{0.005}$ m

(b)$2\pi/\lambda = 80.0$; $\lambda = 2\pi/80 = \mathbf{0.0785}$ m

(c)$2\pi v/\lambda = 2\pi/T = 3.0$; $T = 2\pi/3 = \mathbf{2.09}$ s

(d)$f = 1/T = 1/2.09 = \mathbf{0.478}$ Hz

XIII.11. Two loudspeakers S_1 and S_2 as shown in the figure separated by a distance 1.5 m are in phase. Assume that the amplitude of sound from the speakers is approximately same at the position of the listener P who is at a distance 4 m in front of one of the speakers. For what frequencies in the audio range (20 Hz to 20 kHz) does the listener hear the minimum signal? (velocity of sound in air = 330 m/s) [NCERT]

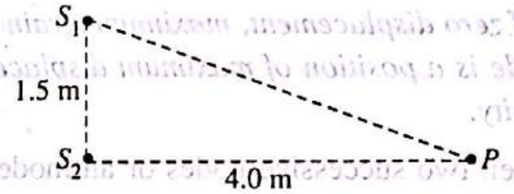


Fig. 18

For minimum signal, the path difference,

$$\delta = (2n+1)\lambda/2$$

$$\therefore \lambda = \frac{2\delta}{2n+1}; \text{ But } v = f\lambda \therefore \lambda = \frac{v}{f}$$

$$\therefore \frac{v}{f} = \frac{2\delta}{(2n+1)}; \quad f = \frac{v(2n+1)}{2\delta}, n = 0, 1, 2, 3, \ldots$$

$$v = 330\,\text{ms}^{-1};\ \delta = S_1P - S_2P = (\sqrt{1.5^2 + 4^2}) - 4 = 0.27\,\text{m}$$

$$\therefore f = \frac{330}{2 \times 0.27}(2n+1) = 611 \times (2n+1)$$

Hence the minimum frequencies of the audible range for which the listener would hear minimum signal are $611 \times 1 = 611$ Hz or 0.611 kHz, $0.611 \times 3 = 1.833$ kHz, $0.611 \times 5 = 3.055$ kHz, …, $0.611 \times 33 \approx 20.0$ kHz.

STATIONARY WAVES

When two waves of the same frequency and amplitude, travel in opposite directions in a straight line at the same speed their superposition gives rise to a new type of wave called stationary wave or standing wave.

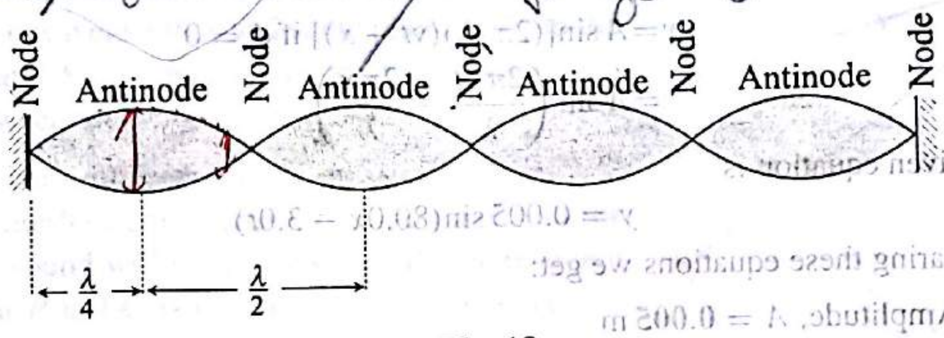


Fig. 19

Stationary waves are produced when a progressive wave and its reflected wave are super-imposed. The displacement of any particle in the medium is the algebraic sum of the displacements of the individual waves. When stationary wave is produced, certain particles of the medium always remain at their mean position and at certain other positions, particles vibrate simple harmonically about the mean position with double the amplitude of each wave. The positions of the particles which always remain at their mean position are called *nodes* and the positions of the particles which vibrate with maximum amplitude are called *antinodes.*

Node is a position of zero displacement, maximum strain and maximum change in pressure and density. Antinode is a position of maximum displacement, least strain and no change in pressure and density.

Distance between two successive nodes or antinodes is $\lambda/2$ and that between a node and the next antinode is $\lambda/4$.

Standing waves can be produced in stretched strings or in air columns. Because of the boundary conditions, the vibrating system can vibrate only in certain special patterns which are called *the normal modes.* This means that the vibrating system can be vibrated only with certain frequencies. The lowest frequency is called the **fundamental frequency** or the *pitch of the tone* and the higher frequencies are called **overtones**. Integral multiples of the fundamental are called **harmonics**; the fundamental frequency being the first *harmonic*.

Characteristics of stationary waves

(1) The stationary waves are not progressive *i.e.*, condensations (crests) or rarefactions (troughs) do not travel forward or backward.

(2) Since the stationary waves do not advance through the medium there is no transfer of energy from one particle to another.

(3) Every particle, except those at nodes, executes SHM with same period.

(4) Particles at different points of the medium vibrate with different amplitudes.

(5) The amplitude changes gradually from zero at nodes to maximum at antinodes.

(6) The distance between two consecutive nodes or antinodes is $\lambda/2$

(7) At any instant the direction of motion of the particles in one segment is opposite to that of the particles in the preceding or succeeding segment.

Fundamental, Harmonics and Overtones

A vibrating system transmits waves through air. This is the basic principle of production of sound by voice or by musical instruments. The system may be capable of vibrating at a number of frequencies $\nu, \nu_1, \nu_2, \nu_3, \ldots$ such at $\nu < \nu_1 < \nu_2 < \nu_3 < \ldots$. The lowest frequency ν is called the *fundamental frequency*, and the corresponding mode of vibration is called the fundamental mode. The higher frequencies $\nu_1, \nu_2, \nu_3, \ldots$ are called the *overtones*; with ν_1 being the first overtone, ν_2 the second overtone, and so on.

In certain systems, the overtones are all integer multiples of the fundamental frequency. These integer multiples of the fundamental frequency, i.e., $1\nu, 2\nu, 3\nu, \ldots$ are called the *harmonics*. The first, second, third,... harmonics are $1\nu, 2\nu, 3\nu, \ldots$.

Transverse vibrations in stretched string–standing waves

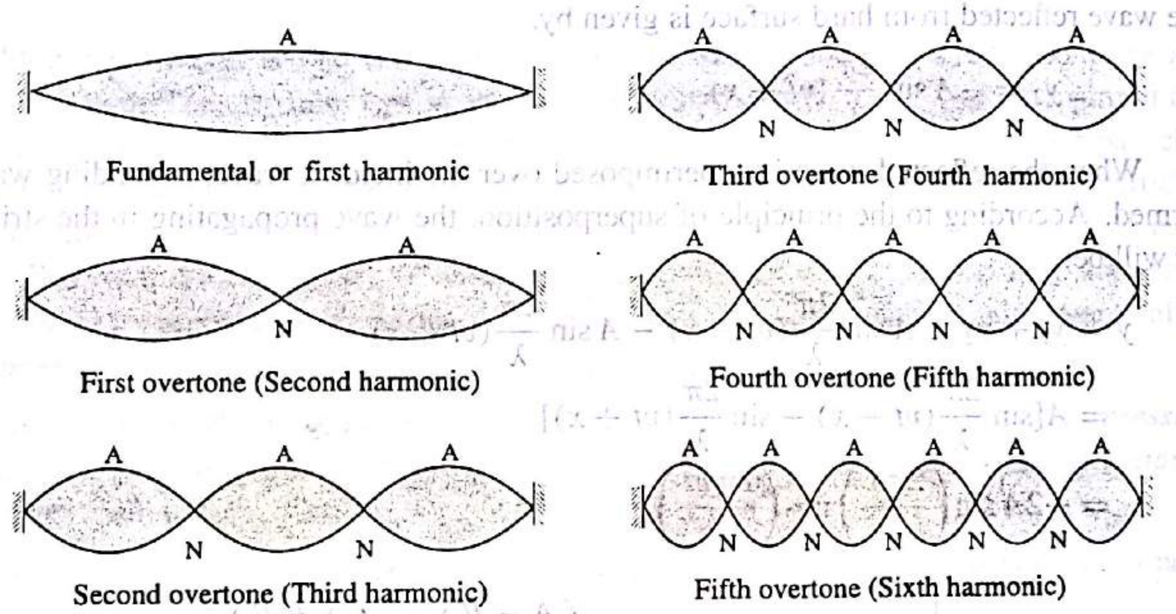


Fig. 20

Consider a string stretched between two fixed points. When it is plucked, a wave train travels along the string which is reflected back and forth from end to end. Superposition of the wave trains travelling in opposite directions results in the formation of stationary waves with nodes at the two ends.

When the string vibrates in one segment the length of the string $l = \lambda/2$ or $\lambda = 2l$. The *fundamental frequency*, $\nu = \dfrac{v}{2l}$, where v is the speed of the wave.

The string can vibrate in 2, 3, . . . segments, they are called overtones. The overtones can have frequency $\dfrac{2v}{2l}, \dfrac{3v}{2l}, \dfrac{4v}{2l}$ etc.

The frequencies are in the ratio 1 : 2 : 3 : .... Hence all *harmonics* are present.

Since the velocity of transverse wave is given by $v = \sqrt{T/m}$

The fundamental frequency is given by, $\nu = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$.

Laws of transverse vibrations of a stretched string

1. The fundamental frequency is inversely proportional to the length l of the string, when tension T and linear density m are constants.

$$\nu \propto \frac{1}{l}; \qquad \nu l = \text{constant, when } T \text{ and } m \text{ are constants}$$

2. The fundamental frequency is directly proportional to the square root of tension T when length l and linear density m are constants

$$\nu \propto \sqrt{T}; \qquad \frac{\nu}{\sqrt{T}} = \text{constant, when } l \text{ and } m \text{ are constants}$$

3. The fundamental frequency is inversely proportional to square root of linear density m of the string when length l and tension T are constants.

$$\nu \propto \frac{1}{\sqrt{m}}; \qquad \nu\sqrt{m} = \text{constant, when } l \text{ and } T \text{ are constants}$$

These laws can be verified using a sonometer.

Sonometer

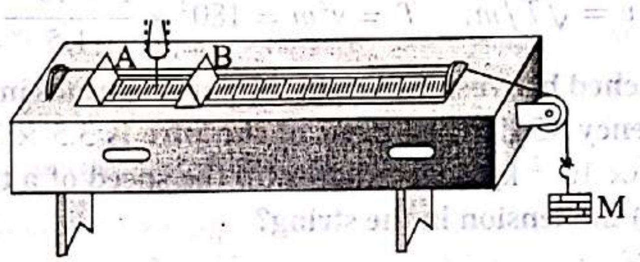


Fig. 21

It is a hollow wooden box over which a wire is stretched. One end of the wire is tied to a peg and the other end passing over a pulley carries a weight hanger. Under the wire are placed two bridges A and B. The distance between the two bridges can be adjusted.

A suitable weight is placed on the hanger. A tuning fork is excited and its stem is placed on the sound box between the bridges. The distance between the bridges is adjusted so that a paper rider placed on the wire vibrates vigorously. This happens when the wire between the bridges vibrates in unison with the tuning fork.

Examples

XIII.12. The transverse displacement of a string clamped at its two ends is given by,

$$y(x,t) = 0.06 \sin \frac{2\pi x}{3} \cos(120\pi t)$$

where x and y are in metre and t in second. The length of the string is 1.5 m and its mass is 3×10^{-2}kg. Answer the following questions.

1. **Does the function represent a travelling wave?**
2. **Interpret the wave as the superposition of two waves travelling in opposite direction. What are the wavelength, frequency and speed of propagation of each wave?**
3. **Determine the tension in the string**

1. As the equation involves harmonic functions of x and t separately it represents a stationary wave.

2. The general form of a stationary wave is

$$y = 2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$$

Comparing this with the given equation, we get,

$$A = 0.03\,m \quad \frac{2\pi x}{\lambda} = \frac{2\pi x}{3} \quad \therefore \lambda = 3 \text{ m}$$

$$\frac{2\pi vt}{\lambda} = 120\pi t; \quad \therefore v = 60\lambda = 60 \times 3 = \mathbf{180\ ms^{-1}}$$

3. Frequency $\nu = \frac{v}{\lambda} = \frac{180}{3} = \mathbf{60\ Hz}$

$$v = \sqrt{T/m}; \quad T = v^2 m = 180^2 \times \frac{3 \times 10^{-2}}{1.5} = \mathbf{648\ N}$$

XIII.13. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string and (b) the tension in the string? [NCERT]

$$\nu = 45 \text{ Hz}; \quad M = 3.5 \times 10^{-2} \text{ kg}; \quad m = 4.0 \times 10^{-2} \text{ kgm}^{-1}$$

$$v = ? \quad T = ?$$

Since $m = M/l, \quad l = M/m = 3.5 \times 10^{-2}/4.0 \times 10^{-2} = 0.875$ m

(a) For fundamental frequency, $\lambda/2 = l \quad \lambda = 2l = 2 \times 0.875 = 1.75$ m

$$v = \nu\lambda = 45 \times 1.75 = \mathbf{78.75\ ms^{-1}},$$

(b) $v = \sqrt{T/m}; \quad T = v^2 m = 78 \cdot 75^2 \times 4.0 \times 10^{-2} = \mathbf{248\ N}$

XIII.14. A string 1 m long with mass 0.01 kg/m is under tension of 400 N. Find the fundamental frequency of transverse vibration of the string.

$$l = 1 \text{ m}; \quad m = 0.01 \text{ kgm}^{-1}; \quad T = 400 \text{ N}; \quad \nu = ?$$

$$\nu = \frac{1}{2l}\sqrt{T/m} = \frac{1}{2 \times 1}\sqrt{\frac{400}{0.01}} = 100 \text{ Hz}$$

XIII.15. Find the frequency of the note emitted by a string of length $10\sqrt{10}$ cm under a tension of 3.14 kg. Radius of the string = 0.5 mm and density = 9.8×10^3 kgm^{-3}

$$l = 10\sqrt{10} \times 10^{-2} \text{m}; \quad T = 3.14 \times 9.8 \text{ N}; \quad r = 0.5 \times 10^{-3} \text{ m}$$

$$m = \pi r^2 \rho = \pi \times (0.5 \times 10^{-3})^2 \times 9.8 \times 10^3$$

$$\nu = 1/2l\sqrt{T/m} = \frac{1}{2 \times 10\sqrt{10} \times 10^{-2}}\sqrt{\frac{3.14 \times 9.8}{3.14 \times (0.5 \times 10^{-3})^2 \times 9.8 \times 10^3}}$$

$$= \mathbf{100\ Hz}$$

Vibration of air columns

An air column inside a pipe can be set into vibration by holding an excited tuning fork at its mouth or by blowing air into it. Then a stationary wave is set up in the pipe by superposition of direct wave and the one reflected at the end of the pipe.

There are two types of pipes commonly used; (1) a pipe closed at one end (closed pipe) and (2) a pipe open at both ends (open pipe). When stationary wave is formed in a pipe, an antinode is always produced at the open end and a node at the closed end because air column in contact with the closed end cannot move; whereas at the open end it is free to move to and fro with maximum amplitude.

The nature of the reflected wave will be different in the closed pipe and open pipe. In the case of the closed pipe the reflection takes place at the rigid wall; while in the case of the open pipe the reflection takes place at the yielding wall. In the case of a closed pipe the incident wave is almost completely reflected. For an open pipe the air outside is rarer than the air inside the tube. When the wave falls on the surface of the rarer medium outside the pipe, it is partially reflected and partially refracted. So, in the case of an open pipe intensity of the reflected wave is less than that of the incident wave. As a result of this, interference between incident wave and reflected wave is not perfect.

The air column inside the tube can vibrate with different frequencies. The lowest frequency is called the *fundamental frequency* and higher frequencies are called the *overtones*. Integral multiples of the fundamental are called *harmonics*.

Owing to the difficulty of representing the longitudinal stationary waves diagramatically, it is customary to represent them by transverse wave curves.

1. Closed pipe

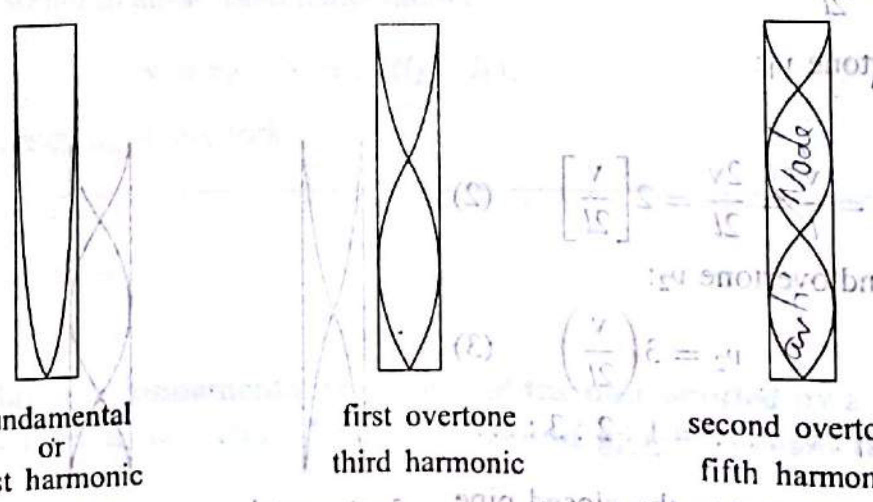


Fig. 22

Consider a tube of length l closed at one end.

When vibrations are produced by blowing air into the pipe at the open end or by holding a tuning fork of suitable frequency, stationary waves are produced in the tube. In its simplest mode of vibration, a stationary wave is produced in the tube with a node at the closed end and an antinode at the open end. This is the fundamental mode of vibration and

End Correction (Edge Effect)

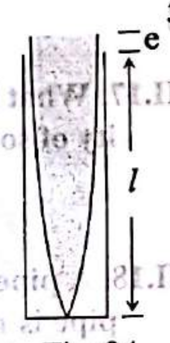


Fig. 24

Experimental results show that antinode at the open end of the tube does not coincide exactly with the open end of the tube; but projects slightly outside it by an amount 'e' which is called the **end correction**.

If 'd' is the diameter of the tube, the end correction,

$$e = 0.3\,d$$

*To determine velocity of sound in air–Resonance column experiment

The resonance column consists of a long tube held vertically in a tall jar containing water. The length of air column can be adjusted by raising or lowering the tube. To perform the experiment a vibrating tuning fork is held above the mouth of the resonance tube and the length of air column is adjusted till resonance is obtained. If l_1 is the length of air column and λ the wavelength of sound,

$$(l_1 + e) = \frac{\lambda}{4}; \qquad (1)$$

where e = end correction.
If l_2 is the second length of resonance with the same tuning fork,

$$l_2 + e = \frac{3\lambda}{4} \qquad (2)$$

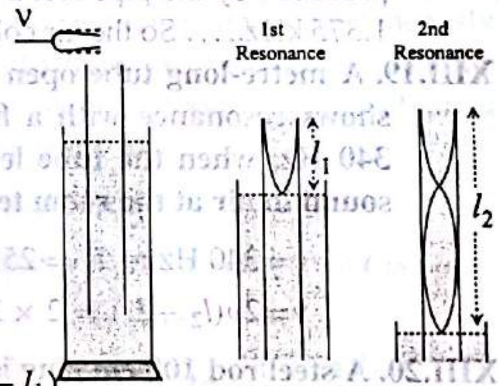


Fig. 25

Subtracting (2) − (1); $l_2 - l_1 = \frac{\lambda}{2}$; $\lambda = 2(l_2 - l_1)$.

The velocity of sound in air at room temperature,

$$v = \nu\lambda \;\therefore\; v = 2\nu(l_2 - l_1),$$

where ν = frequency of tuning fork.

Examples

XIII.16. Calculate the fundamental frequency of the note emitted by a pipe of length 17 cm closed at one end. (Velocity of sound in air at room temperature = 348 ms^{-1})

$$l = 17\text{ cm} = 17 \times 10^{-2}\text{ m}; \quad v = 348\text{ ms}^{-1}$$

$$\nu = \frac{(2n+1)v}{4l}; \quad \text{where } n = 0, 1, 2, \ldots$$

For the fundamental, $n = 0$;

$$\therefore \quad \nu = \frac{v}{4l} = \frac{348}{4 \times 17 \times 10^{-2}} = \mathbf{511.8\ Hz.}$$

XIII.17. What is the fundamental frequency of an open pipe of length 32.6 cm? Velocity of sound in air = 334 ms^{-1}

$$\nu = \frac{v}{2l} = \frac{334}{2 \times 32.6 \times 10^{-2}} = \mathbf{512.27\ Hz}$$

XIII.18. A pipe 30.0 cm long is open at both ends. Which of the harmonic modes of the pipe is resonantly excited by a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? (velocity of sound in air = 330 ms^{-1}) **[NCERT]**

(a) Fundamental frequency of the vibration of air column inside the open pipe, $\nu = v/2l = 330/(2 \times 0.30) = 550$ Hz.

The harmonics are, 550×1 Hz, $550 \times 2 = 1100$ Hz $= 1.1$ kHz, $550 \times 3 = 1650 = 1.65$ kHz.

So the **second harmonic** mode is resonantly excited by 1.1 kHz source.

(b) For the closed pipe, $\nu = v/4l = 330/4 \times 0.3 = 275$ Hz.

Since a closed pipe can produce only odd harmonics, the harmonics that can be produced by the pipe are, 275 Hz, $275 \times 3 = 825 = 0.825$ Hz, $275 \times 5 = 1375 = 1.375$ kHz,.... So the air column **does not resonate** with 1.1 kHz.

XIII.19. A metre-long tube open at one end with a movable piston at the other end, shows resonance with a fixed frequency source, a tuning fork of frequency 340 Hz, when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the room temperature. **[NCERT]**

$\nu = 340$ Hz; $l_1 = 25.5$ cm; $l_2 = 79.3$ cm; $v = ?$

$v = 2\nu(l_2 - l_1) = 2 \times 340(79.3 - 25.5) = 26584$ cms^{-1} $=$ **365.84 ms^{-1}**

XIII.20. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibration of the rod is 2.53 kHz. What is the speed of sound in steel? **[NCERT]**

$l = 100$ cm $= 1$ m; $\nu = 2.53 \times 10^3$ Hz; $v = ?$

$\lambda/2 = l \quad \therefore \lambda = 2l = 2$ m

$v = \nu\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3$ ms^{-1} $=$ **5.06 kms^{-1}**

BEATS

When two sound waves of nearly equal frequencies travelling in a medium along the same direction superimpose, the intensity of the resultant sound at a particular position rises and falls regularly with time. *This phenomenon of regular variation in the intensity of sound with time at a particular position when two sound waves of nearly equal frequencies superimpose on each other is called beats.*

The number of beats heard per second is called beat frequency. It is equal to the difference in frequencies.

If ν_1 and ν_2 are the frequencies of the sound waves, beat frequency $= \nu_1 \sim \nu_2$

Graphical representation of beats

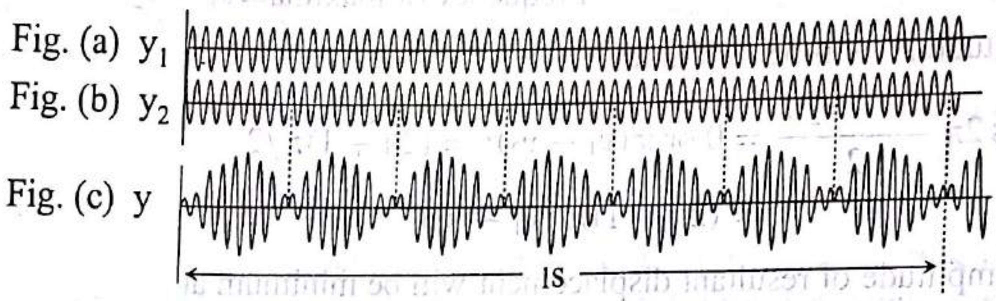


Fig. 26

Figures (a) and (b) represent two harmonic waves of frequencies 49 Hz and 56 Hz. Figure (c) is the resultant wave. It shows that the amplitude increases and decreases alternately. It is clear that the beat frequency is seven which is equal to the difference in frequencies of the waves (a) and (b).

DOPPLER EFFECT IN SOUND

It is a matter of common experience that when a train travelling at a high speed with its whistle blowing approaches an observer on the platform, the pitch of the whistle appears to rise and then falls at the moment the train passes him. When a source of sound and a listener are at rest or moving in the same direction with the same speed, the listener hears a sound of the same frequency as the frequency of vibration of the source. But there is an apparent change in frequency when there is relative motion between the source and the listener; the pitch rising when the source and the listener approach each other and falling when they recede away from each other.

Doppler effect is applicable to all types of wave motions. Doppler had acutally put forward this principle in the year 1842 in respect of light in his attempt to explain the colour of stars.

The *phenomenon of the apparent change in the frequency of sound produced by a source as heard by a listener whenever there is relative motion between the source and the listener is called Doppler effect.*

General expression for apparent frequency

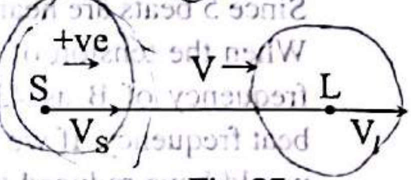


Fig. 27

Consider a source S producing sound of frequency ν. Let V be the velocity of sound in the medium and λ the wave-length of sound when the source and the listener are at rest. Then the frequency of sound heard by the listener is

$$\nu = V/\lambda \quad \therefore \lambda = V/\nu \quad (\because V = \nu\lambda)$$

Let the source and the listener be moving with velocities V_s and V_l in the direction of propagation of sound from source to listener. The direction S to L is taken positive.

The relative velocity of sound wave with respect to the source $= V - V_s$

$$\text{Apparent wavelength of sound} \quad \lambda' = \frac{V - V_s}{\nu}$$

Since the listener is moving with a velocity V_l, the relative velocity of sound with respect to the listener $= V' = V - V_l$

Apparent frequency of sound as heard by the listener is given by,

$$\nu' = \frac{\text{Relative velocity of sound with respect to the listener}}{\text{Apparent wavelength}} = \frac{V'}{\lambda'}$$

$$\nu' = \left[\frac{V - V_l}{V - V_s}\right] \times \nu$$

Note:

Velocities in the direction in which the sound travels in order to reach the listener are taken positive; while those in the opposite direction are taken negative.

Case (1) Listener at rest and source in motion

(a) *When the source moves towards the stationary listener,* V_s is positive and $V_l = 0$

$$\nu' = \left(\frac{V}{V - V_s}\right) \times \nu$$

(b) *When the source moves away from the stationary listener,* V_s is negative and $V_l = 0$

$$\therefore \nu' = \left(\frac{V}{V + V_s}\right) \times \nu$$

Case (2) Listener in motion and source at rest

(a) *When the listener moves towards the stationary source,* V_l is negative and $V_s = 0$

$$\therefore \nu' = \left(\frac{V + V_l}{V}\right) \times \nu$$

(b) *When the listener moves away from the stationary source,* V_l is positive and $V_s = 0$

$$\therefore \nu' = \left(\frac{V - V_l}{V}\right) \times \nu$$

Case (3) When both the source and the listener are in motion

(a) *When the source and the listener move towards each other,* V_s is positive and V_l is negative

$$\therefore \nu' = \left(\frac{V + V_l}{V - V_s}\right) \times \nu$$

(b) *When the source and the listener move away from each other.* V_s is negative and V_l is positive

$$\therefore \nu' = \left(\frac{V - V_l}{V + V_s}\right) \times \nu$$

(c) *When the source moves away from the listener and the listener moves towards the source,* V_s is negative and V_l is negative

$$\therefore \nu' = \left(\frac{V + V_l}{V + V_s}\right) \times \nu$$

(d) *When the listener moves away from the source and the source moves towards the listener,* V_l is positive and V_s is positive

$$\therefore \nu' = \left(\frac{V - V_l}{V - V_s}\right) \times \nu$$

Effect of motion of medium

When a wind is blowing with a velocity W in the direction of propagation of sound, the resultant velocity of sound will be $(V + W)$.

$$\therefore \nu' = \left(\frac{V + W - V_l}{V + W - V_s}\right) \times \nu$$

If the wind is blowing in a direction opposite the direction of propagation of sound, the resultant velocity of sound will be $(V - W)$.

Limitation of Doppler effect

Doppler effect is applicable as long as the relative velocity between the source and the listener is less than the velocity of sound. The principle is not applicable if the source moves towards the listener with supersonic velocity.

Doppler effect in sound is asymmetric

Let the listener be at rest and the source be moving with a velocity v' towards the listener. Then the apparent frequency,

$$\nu' = \frac{V \times \nu}{V - v'} \quad (1)$$

If the source is at rest and the listener is approaching the source with the same velocity v', the apparent frequency,

$$\nu'' = \frac{(V + v')\nu}{V} \quad (2)$$

These two frequencies, ν' and ν'', are not equal

It means that the observed frequencies differ when the source or listener approaches the other with the same speed. Hence Doppler effect in sound is *asymmetric*.

Doppler effect in light

Doppler effect is observed in light also. When a source of light, such as a star, approaches earth or recedes from it, Doppler effect is observed. When a star recedes from earth with a very high velocity ($> 20 \times 10^3$km/s) the frequency of light decreases or wavelength increases and hence its spectral lines are shifted towards the red end by about 200 Å. This is called **red shift**. When the star approaches earth the spectral lines are shifted towards the violet end. This indicates an increase in frequency. Most stars are found to show red shift.

The Doppler effect in light is symmetric

Applications of Doppler effect

1. *To estimate the speed of a submarine (Sonar)*

   Ultrasonic waves are transmitted from the ship and their reflected waves obtained by reflection from the submarine are observed. By finding the difference in frequency of the transmitted wave and the reflected wave, we can calculate the velocity of the submarine.

2. *To estimate the speed of aeroplane, automobile etc.*

   Short radio waves are emitted from an observation centre. These waves are reflected from the plane (or automobile) and received by the centre. By measuring the change in frequency of the transmitted and the reflected waves, the velocity and the direction of motion of the plane (or the automobile) can be calculated.

3. *To track artificial satellites*

   Doppler effect provides a convenient method for tracking an earth satellite. The earth satellite emits radio signals of constant frequency ν. The apparent frequency ν' of the signal as received by the tracking station on the earth is noted. From this the velocity of the earth satellite can be obtained.

4. *To estimate the velocity and rotation of the sun*

   By the study of Doppler shift from the light received from the western and eastern edges of the sun, it has been found that the shift is due to a velocity of 2 kms^{-1}. Since no such shift is observed from light received from north and south edges, it is concluded that sun rotates about north–south axis with a velocity 2 kms^{-1}.

Examples

XIII.26. A railway engine blowing a whistle of frequency 636 Hz approaches a person standing on a railway platform with a velocity 108 km/hr. Calculate the apparent frequency of the whistle as heard by the person. (Velocity of sound = 348 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu$$

$V = 348$ ms^{-1}, $V_l = 0$; $V_s = 108$ km/h $= 30$ ms^{-1}; $\nu = 636$ Hz; $\nu' = ?$

$$\nu = \left(\frac{348}{348 - 30}\right) \times 636 = \mathbf{696\ Hz}$$

XIII.27. A car sounding its horn of frequency 740 Hz is moving away with a velocity 72 km/hr from a person standing by the side of the road. Find the apparent pitch of the horn as heard by the person. (The velocity of sound = 350 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu$$

$V = 350 \text{ ms}^{-1}$; $V_l = 0$; $V_s = -72 \text{ km/h} = -20 \text{ ms}^{-1}$; $\nu = 740$ Hz; $\nu' = ?$

$$\nu' = \left(\frac{350}{350 + 20}\right) \times 740 = \mathbf{700\ Hz}$$

XIII.28. A boy stands on the side of a railway track. He blows a whistle of frequency 700 Hz. A train passes at 72 km/hr. What is the frequency of the whistle appearing to a man sitting in the train as the train (a) approaches the boy (b) moves away from the boy. (Velocity of sound = 350 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu$$

(a) $V = 350 \text{ ms}^{-1}$; $V_l = -72 \text{ km/h} = -20 \text{ ms}^{-1}$; $V_s = 0$; $\nu = 700$ Hz; $\nu' = ?$

$$\nu' = \left(\frac{350 + 20}{350}\right)700 = \mathbf{740\ Hz}$$

(b) $V = 350 \text{ ms}^{-1}$; $V_l = 72 \text{ km/h} = 20 \text{ ms}^{-1}$; $V_s = 0$; $\nu = 700$ Hz; $\nu' = ?$

$$\nu' = \left(\frac{350 - 20}{350}\right)700 = \mathbf{660\ Hz}$$

XIII.29. A police jeep is approaching a stationary observer at a speed of 90 km/hr sounding siren. The pitch of the siren heard by the observer is 480 Hz. Calculate the true pitch of the siren. (Velocity of sound = 350 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu \quad \therefore \nu = \left(\frac{V - V_s}{V - V_l}\right) \times \nu'$$

$V = 350 \text{ ms}^{-1}$, $V_s = 90 \text{ km/h} = 25 \text{ ms}^{-1}$; $V_l = 0$; $\nu' = 480$; $\nu = ?$

$$\nu = \left(\frac{350 - 25}{350}\right) \times 480 = \mathbf{445.7\ Hz}$$

XIII.30. The pitch of the whistle of an engine appears to drop to 5/6 th of the true value when it moves away from stationary observer. Calculate the speed of the engine. (Velocity of sound = 340 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right) \times \nu \quad ; \quad \frac{\nu'}{\nu} = \frac{V - V_l}{V - V_s}$$

$\frac{\nu'}{\nu} = 5/6$; $V = 350 \text{ ms}^{-1}$; $V_l = 0$; V_s is negative

$$\frac{5}{6} = \frac{340}{340 + V_s}; \quad 1700 + 5V_s = 2040 \quad \therefore V_s = \mathbf{68\ ms^{-1}}$$

XIII.31. Two trains travelling at 72 km/hr and 180 km/hr are crossing each other while the second train is whistling. If the frequency of the note is 800 Hz., find the apparent frequency of the note as heard by an observer in the first train (a) before

the trains cross each other (b) after they cross each other. (Velocity of sound in air = 350 ms^{-1})

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu$$

(*a*) $V = 350\ \text{ms}^{-1}$; $V_l = -72\ \text{km/h} = -20\ \text{m s}^{-1}$;

$V_s = 180\ \text{km/h} = 50\ \text{m s}^{-1}$; $\nu = 800\ \text{Hz}$; $\nu' = ?$

$$\nu' = \left(\frac{350 + 20}{350 - 50}\right) \times 800 = \mathbf{986.7\ Hz}$$

(*b*) $V = 350\ \text{ms}^{-1}$; $V_l = 72\ \text{km/h} = 20\ \text{ms}^{-1}$

$V_s = -180\ \text{km/h} = -50\ ms^{-1}$; $\nu = 800\ \text{Hz}$; $\nu' = ?$

$$\nu' = \left(\frac{350 - 20}{350 + 50}\right) \times 800 = \mathbf{660\ Hz}$$

XIII.32. A rocket is moving at a speed of 220 ms^{-1} towards a stationary target. While moving it emits a sound of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. Calculate (a) the frequency of sound wave as detected by a detector attached to the target and (b) the frequency of the echo as detected by a detector attached to the rocket. (Velocity of sound = 330 ms^{-1}) [NCERT]

(a) $V_s = 220\ \text{ms}^{-1}$; $\nu = 1000\ \text{Hz}$; $V = 330\ \text{ms}^{-1}$; $v_l = 0$; $\nu' = ?$

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu = \left(\frac{330 - 0}{330 - 220}\right) \times 1000 = 3000\ \text{Hz}$$

(b) Here target is the source giving sound of frequency 3000 Hz and the rocket is the listener.

$V_s = 0$; $V_l = -220\ \text{ms}^{-1}$; $\nu = 3000$; $\nu' = ?$

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right)\nu = \left(\frac{330 + 220}{330}\right) \times 3000 = 5000\ \text{Hz}$$

XIII.33. A train standing in a station-yard blows a whistle of frequency 400 Hz in still air. A wind starts blowing in the direction from the yard to the station with a speed of 10 ms^{-1}. (a) What are the frequency, wavelength and speed of sound for an observer standing on the station's platform? (b) Is the situation exactly equivalent to the case when the air is still and the observer runs towards the yard at a speed of 10 m s^{-1}? [NCERT]
[Velocity of sound in still air = 340 m s^{-1}]

(a) $\nu = 400\ \text{Hz}$; $W = 10\ \text{m s}^{-1}$; $V = 340\ \text{m s}^{-1}$ $\nu' = ?$

$$\nu' = \left(\frac{V + W - V_l}{V + W - V_s}\right) \times \nu = \nu$$

$(\because\ V_l = 0 \text{ and } V_s = 0)$

$\therefore\ \nu' = \nu = \mathbf{400\ Hz}$

Speed of sound $= \nu + W = 340 + 10 = \mathbf{350\ ms^{-1}}$

Wavelength, $\lambda = \dfrac{V}{\nu} = \dfrac{350}{400} = \mathbf{0.875\ m}$

Thus we see that the frequency does not change; but the speed and wavelength of sound change.

(b) No, because when the observer runs towards the source, the frequency also changes (increases)

XIII.34. A police man on duty detects a fall of 10 % in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is 330 m s^{-1}, calculate the speed of the car.

Let ν' and ν'' be the apparent frequencies of the horn before and after the car crosses the police man.

$$\nu' = \left(\frac{V - V_l}{V - V_s}\right) \times \nu = \left(\frac{V}{V - V_s}\right) \times \nu; \quad \nu'' = \left(\frac{V}{V + V_s}\right)\nu.$$

$$\therefore \nu'/\nu'' = (V + V_s)/(V - V_s); \quad (100/90) = (330 + V_s)/(330 - V_s)$$

$$3300 - 10V_s = 2970 + 9V_s; \quad V_s = \mathbf{17.36\ ms^{-1}} = \mathbf{62.5\ km/h}$$

XIII.35. A sonar system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the sonar with a speed of 360 kmh^{-1}. What is the frequency of the sound reflected by the submarine? Take the velocity of sound in water to be 1450 ms^{-1}. [NCERT]

For the incident sound, *take sonar as the source and the enemy submarine as the listener*:

$$\nu = 40.0\text{ kHz}; \quad v_s = 0; \quad v_l = -360\text{ km/}h = -100\text{ m/s}; \quad v = 1450\text{ m/s}; \ \nu' = ?$$

$$\nu' = \left(\frac{v - v_l}{v - v_s}\right) \times \nu = \left(\frac{1450 + 100}{1450 - 0}\right) \times 40 = \mathbf{42.76\ kHz}$$

For the reflected sound, *consider the submarine as the source and sonar as the listener*:

$$v_s = 100\text{ m/s}; \quad v_l = 0; \ v = 1450\text{ m/s}; \quad \nu' = 42.76\text{ kHz}; \ \nu'' = ?$$

$$\nu'' = \left(\frac{v - v_l}{v - v_s}\right) \times \nu' = \left(\frac{1450 - 0}{1450 - 100}\right) \times 42.76 = \mathbf{42.76\ kHz}$$

XIII.36. The wavelength of yellow sodium line (5896 Å) emitted by a star is red-shifted to 6010 Å. What is the component of the star's recessional velocity along the line of sight?

Since the wavelength of light increases due to Doppler effect *i.e.*, since frequency decreases, the portion of the star from which the light is observed moves away from the observer. $\therefore$ V_s is negative

$$V_l = 0; \quad \lambda = 5896\text{ Å} = 5896 \times 10^{-10}\text{ m};$$

$$\lambda' = 6010\text{ Å} = 6010 \times 10^{-10}\text{ m}; \ V = 3 \times 10^8\text{ m s}^{-1}; \ V_s = ?$$

$$\nu' = \frac{(V - V_l)}{V - V_s} \times \nu$$

$$\frac{V}{\lambda'} = \left(\frac{V}{V + V_s}\right) \times \frac{V}{\lambda} \quad \because \nu = \frac{V}{\lambda}$$

$$\lambda' = \left(\frac{V + V_s}{V}\right) \times \lambda$$

$$6010 \times 10^{-10} = \frac{(3 \times 10^8 + V_s)}{3 \times 10^8} \times 5896 \times 10^{-10}$$

$$3 \times 10^8 + V_s = \frac{6010 \times 3 \times 10^8}{5896} = 3.058 \times 10^8$$

$$V_s = 0.058 \times 10^8 = \mathbf{5.8 \times 10^6\ ms^{-1}}$$

IMPORTANT POINTS

1. $V = \nu\lambda;\ \nu = 1/T$
2. $V = \sqrt{E/P}$; *In solid,* $V = \sqrt{Y/P}$; *In fluid,* $V = \sqrt{B/P}$
3. *The velocity V of sound in a gas does not depend on the pressure of the gas.*
4. $V = \sqrt{\gamma RT/M};\ V_1/V_2 = \sqrt{T_1/T_2};\ V_0/V_t = \sqrt{273/(273 + t)};$
   $V_t = V_0\sqrt{(273 + t)/273}$
   Or, $V_t = (V_0 + 0.6 \times t)$ *m/s*
5. *Displacement y of a particle at a distance x at a time t due to a progressive wave propagating along the positive direction of the x-axis is*

$$y = A\sin[2\pi/\lambda(Vt - x) + \phi] \text{ or } y = A\sin(\omega t - kx + \phi)$$

   If there is no initial phase, $\phi = 0$, *then*

$$y = A\sin[2\pi/\lambda(vt - x)] \text{ or } y = A\sin(\omega t - kx)$$

   where $k = 2\pi/\lambda;\ \omega = 2\pi V/\lambda = 2\pi/T;\ V = \omega/k$
6. *Velocity of transverse vibration of a stretched string is* $V = \sqrt{T/m}$; m = *linear density = mass / length* $= \pi r^2 d$
   $T = Mg$, *if a mass is suspended at the end of the string.*
   Fundamental frequency, $\nu = (1/2l)\sqrt{T/m}$.
7. *For stationary waves in a stretched string,*

$$\nu = V/2l; \quad \nu_1 = 2V/2l; \quad \nu_2 = 3V/2l, \ldots$$

   For the vibration of air column,
   (a) In a closed tube, $\nu = V/4l,\ \nu_1 = 3V/4l;\ \nu_2 = 5V/4l, \ldots$
   (b) In an open tube, $\nu = V/2l,\ \nu_1 = 2V/2l;\ \nu_2 = 3V/2l, \ldots$
   End correction, $e = 0.3\ d$
   For resonance column, $v_t = 2n(l_2 - l_1)$
8. *Beat frequency* $= \nu_1 - \nu_2$ *or* $\nu_2 - \nu_1$
   For a string νl = *a constant. As* l *increases* ν *decreases and vice versa*
9. *Doppler frequency,* $\nu' = [(V - V_l)/(V - V_s)]\,\nu$
   The direction from the source to the listener is taken as positive.