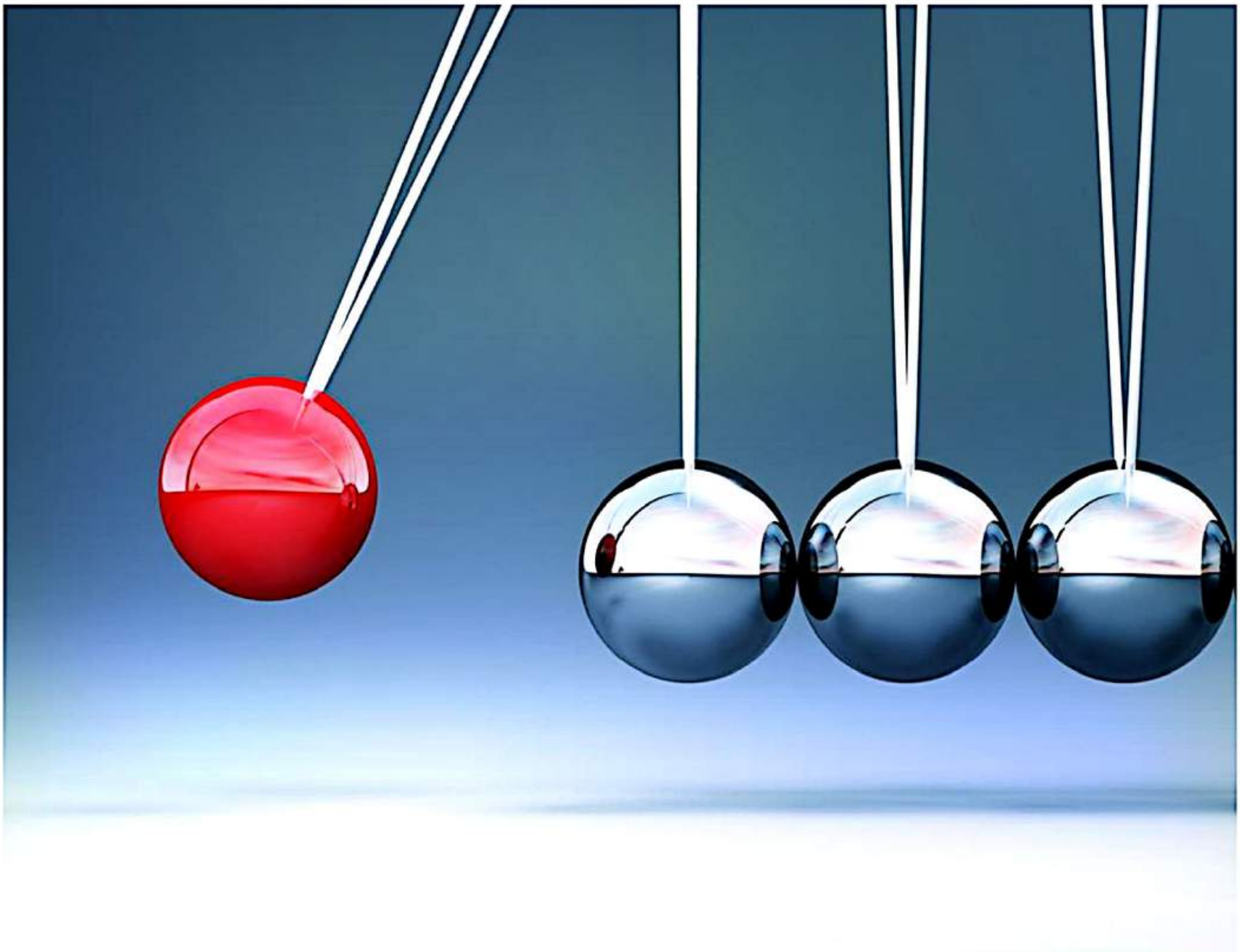
Chapter–5

Work, Energy and Power



CBSE CLASS XI NOTES

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Guidelines to NCERT Exercises

6.1. *The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.*

(i) *Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.*

(ii) *Work done by gravitational force in the above case.*

(iii) *Work done by friction on a body sliding down an inclined plane.*

(iv) *Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.*

(v) *Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.*

Ans.

(i) Work done is *positive*, because the bucket moves in the direction of the force applied by the man.

(ii) Work done by gravitational force is *negative* because the bucket moves upwards while the gravitational force acts downwards.

(iii) Work done is *negative*, because force of friction acts on the body in the opposite direction of its motion.

(iv) Work done is *positive*, because the applied force acts in the direction of motion of the body.

(v) Work done is *negative*, because the resistive force of air acts in a direction opposite to the direction of motion of the vibrating pendulum.

6.2. *A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1. Compute the*

(i) *work done by the applied force in 10 s,*

(ii) *work done by the friction in 10 s,*

(iii) *work done by the net force on the body in 10 s, and*

(iv) *change in kinetic energy of the body in 10 s. Interpret your results.*

Ans. Here $m = 2\text{ kg}$, $u = 0$, $F = 7\text{ N}$,

$$\mu_k = 0.1, \quad t = 10\text{ s}$$

Force of friction,

$$f_k = \mu_k R = \mu_k mg = 0.1 \times 2 \times 9.8 = 1.96\text{ N}$$

Net force with which the body moves,

$$F' = F - f_k = 7 - 1.96 = 5.04\text{ N}$$

Acceleration,

$$a = \frac{F'}{m} = \frac{5.04}{2} = 2.52\text{ ms}^{-2}$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times (10)^2$$

$$= 126\text{ m}$$

(i) Work done by the applied force,

$$W_1 = Fs = 7 \times 126 = \mathbf{882\ J}.$$

(ii) Work done by the friction,

$$W_2 = -f_k \times s = -1.96 \times 126 = \mathbf{-246.9\ J}.$$

(iii) Work done by the net force,

$$W_3 = F's = 5.04 \times 126 = \mathbf{635\ J}.$$

(iv) Final velocity acquired by the body after 10 s,

$$v = u + at = 0 + 2.52 \times 10 = 25.2\text{ ms}^{-1}$$

Change in K.E. of the body

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times (25.2)^2 - 0 = \mathbf{635\ J}$$

Thus the change in K.E. of the body is equal to the work done by the net force on the body.

6.3. *Given below (Fig. 6.46) are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found*

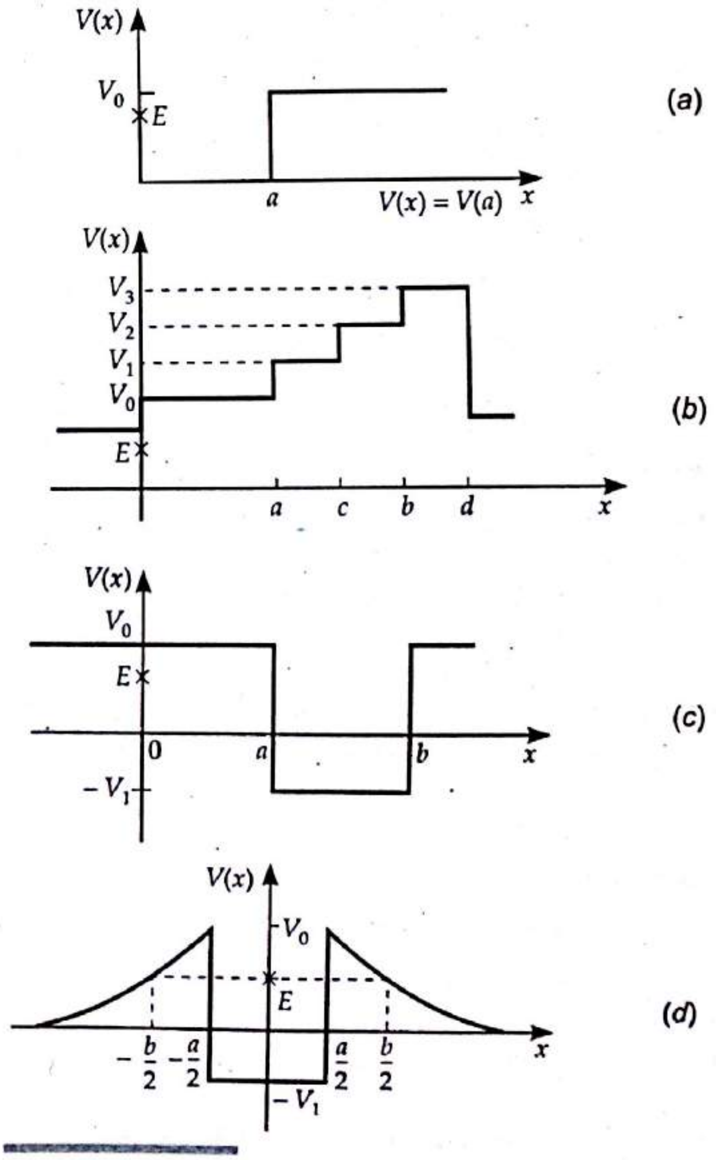

Fig. 6.46

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for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

Ans. Total energy, $E = \text{K.E.} + \text{P.E.}$

$\therefore \quad \text{K.E.} = E - \text{P.E.}$

The particle can exist in such a region in which its K.E. is positive.

(*a*) For $x > a$, P.E. $(V_0) > E$

$\therefore$ K.E. is negative. The particle cannot exist in the region $x > a$. Here $E_{min} = 0$.

(*b*) In every region of the graph, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be found in any region. Here $E_{min} = -V_1$.

(*c*) For $x < a$ and $x > b$, P.E. $(V_0) > E$

$\therefore$ K.E. is negative. The particle cannot be found in the region $x < a$ and $x > b$. Here $E_{min} = -V_1$.

(*d*) For $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be present in these regions. Here $E_{min} = -V_1$.

6.4. *The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\ Nm^{-1}$, the graph of $V(x)$ versus x is shown in Fig. 6.47. Show that a particle of total energy 1 J moving under this potential must "turn back" when it reaches $x = \pm 2$ m.*

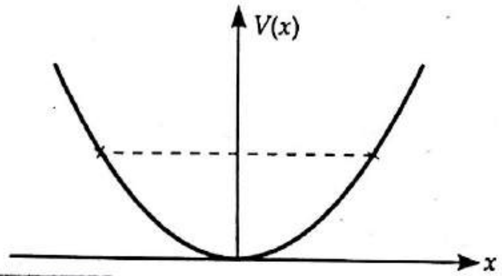

Fig. 6.47

Ans. At any instant, the energy of the oscillator is partly kinetic and partly potential. Its total energy is

$$E = K + V$$

or

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

An oscillating particle turns back at the point where its instantaneous velocity is zero *i.e.*, the particle will turn back at such a point x where $v = 0$.

$$\therefore \quad E = 0 + \frac{1}{2}kx^2$$

But $E = 1$ J, $k = 0.5\ \text{Nm}^{-1}$

$$\therefore \quad 1 = \frac{1}{2} \times 0.5 \times x^2 \quad \text{or} \quad x^2 = 4$$

or $\quad$ $x = \pm$ **2 m.**

6.5. *Answer the following :*

(*a*) *The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere or both ?*

[Delhi 12]

(*b*) *Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?*

(*c*) *An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?*

(*d*) *In Fig. 6.48 (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6.48 (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?*

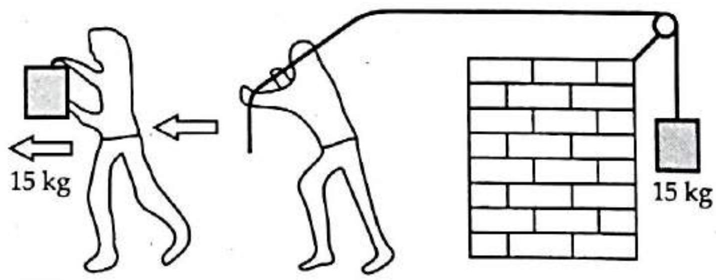

Fig. 6.48

Ans. (*a*) Heat energy required for the burning of the casing of a rocket in flight is obtained from the rocket itself. It is obtained at the expense of the mass of the rocket and its kinetic and potential energies.

(*b*) The gravitational force acting on the comet is a conservative force. The work done by a conservative over any path is equal to the negative of the change in P.E. Over a complete orbit of any shape, there is no change in P.E. of the comet. Hence no work is done by the gravitational force on the comet.

(*c*) As the satellite comes closer to the earth, its potential energy decreases. As the sum of kinetic and potential energy remains constant, the kinetic energy and velocity of the satellite increase. But the total energy of the satellite goes on decreasing due to the loss of energy against friction.

(*d*) In **case (*i*)**, no work is done against gravity because the displacement of 2 m (horizontal) and the weight of 15 kg (acting vertically downwards) are perpendicular to each other. Work is done only against friction.

In **case (*ii*)**, work has to be done against gravity ($= mg\,h = 15 \times 9.8 \times 2 = 294$ J) in addition to the work to be done against friction while moving a distance of 2 m. Thus the *work done in case (ii) is greater* than that in case (*i*).

6.6. *Underline the correct alternative :*

(*a*) *When a conservative force does positive work on a body, the potential energy of the body increases / decreases / remains unaltered.*

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(*b*) *Work done by a body against friction always results in a loss of its kinetic/potential energy.*

(*c*) *The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.*

(*d*) *In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.* **[Central Schools 08]**

Ans.

(*a*) The work done by a conservative force is equal to the negative of the potential energy. *When the work done is positive, the potential energy decreases.*

(*b*) Friction always opposes motion. A body does work against friction at the expense of its kinetic energy. *Work done by a body against friction results in a loss of its kinetic energy.*

(*c*) Internal forces in a many-particle system cancel out in pairs and so they cannot change the net momentum of the system. Only the external forces can produce change in momentum. *The rate of change of momentum of a many-particle system is proportional to the external force on the system.*

(*d*) In an elastic collision, the kinetic energy of the system decreases after the collision but the total energy of the system and its total linear momentum do not change after the inelastic collision.

6.7. *State if each of the following statements is true or false. Give reasons for your answer.*

(*a*) *In an elastic collision of two bodies, the momentum and energy of each body is conserved.*

(*b*) *Total energy of a system is always conserved, no matter what internal and external forces on the body are present.*

(*c*) *Work done in the motion of a body over a closed loop is zero for every force in nature.*

(*d*) *In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.*

Ans.

(*a*) *False.* Total momentum and total energy of the entire system are conserved and not of individual bodies.

(*b*) *False.* The external forces acting on a body may change its energy.

(*c*) *False.* In case of a non-conservative force like friction, the work in the motion of a body over a closed loop is not zero.

(*d*) *True.* In an elastic collision, a part of the initial K.E. of the system always changes into some other form of energy.

6.8. *Answer carefully, with reasons :*

(*a*) *In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact) ?* **[Delhi 12]**

(*b*) *Is the total linear momentum conserved during the short time of an elastic collision of two balls ?*

(*c*) *What are the answers to (a) and (b) for an inelastic collision ?*

(*d*) *If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy.)*

Ans.

(*a*) During the short time of collision when the balls are in contact, the kinetic energy of the balls gets converted into potential energy. In an elastic collision, though the kinetic energy before collision is equal to the kinetic energy after the collision but kinetic energy is not conserved during the short time of collision.

(*b*) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(*c*) In an inelastic collision, the total K.E. is not conserved during collision as well as even after the collision. But the total linear momentum of the two balls is conserved.

(*d*) The collision is elastic because the forces involved are conservative.

6.9. *A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to*

(*i*) $t^{1/2}$ (*ii*) t (*iii*) $t^{3/2}$ (*iv*) t^2

Ans. Instantaneous velocity, $v = 0 + at = at$

Power, $P = Fv = mav = ma \times at = ma^2t$

As m and a are constant, so $P \propto t$

∴ *Alternative (ii) is correct.*

6.10. *A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to*

(*i*) $t^{1/2}$ (*ii*) t (*iii*) $t^{3/2}$ (*iv*) t^2

Ans. By work-energy theorem,

$$W = P \times t = \frac{1}{2} mv^2$$

or $$v^2 = \frac{2\,Pt}{m}$$

∴ $$v = \frac{ds}{dt} = \left(\frac{2\,Pt}{m}\right)^{1/2}$$

On integration,

$$s = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$$

∴ $$s \propto t^{3/2}$$

Hence, *alternative (iii) is correct.*

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6.11. *A body constrained to move along the Z-axis of a co-ordinate system is subject to a constant force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the X-, Y-, and Z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the Z-axis ?*

Ans. Here, $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

As the body moves a distance of 4 m along Z-axis, so

$$\vec{s} = 4\hat{k} \text{ m.}$$

$$\therefore \quad W = \vec{F}\cdot\vec{s} = (-\hat{i} + 2\hat{j} + 3\hat{k})\cdot(4\hat{k})$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k})\cdot(0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= -1\times 0 + 2\times 0 + 3\times 4 = \mathbf{12\ J.}$$

6.12. *An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds.*

(Electron mass $= 9.11 \times 10^{-31}$ kg, proton mass $= 1.67 \times 10^{-27}$ kg, 1 eV $= 1.60 \times 10^{-19}$ J).

[Delhi 08]

Ans. K.E. of the electron $= \frac{1}{2} m_e v_e^2 = 10$ keV

K.E. of the proton $= \frac{1}{2} m_p v_p^2 = 100$ keV

$$\therefore \quad \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2} = \frac{10 \text{ keV}}{100 \text{ keV}} = \frac{1}{10}$$

or
$$\frac{9.11 \times 10^{-31} \times v_e^2}{1.67 \times 10^{-27} \times v_p^2} = \frac{1}{10}$$

or
$$\frac{v_e^2}{v_p^2} = \frac{1670}{9.11} = 183.3$$

or
$$\frac{v_e}{v_p} = \mathbf{13.53}$$

Thus the electron moves faster than the proton.

6.13. *A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1} ?*

Ans. Whether the rain drop falls with decreasing acceleration or with uniform speed, the work done by gravitational force on the drop remains same.

Here $r = 2$ mm $= 2 \times 10^{-3}$ m

Distance moved in each half journey,

$$h = \frac{500}{2} = 250 \text{ m}$$

Density of water,

$$\rho = 10^3 \text{ kgm}^{-3}$$

Mass of rain drop,

$$m = \text{Volume} \times \text{density} = \frac{4}{3}\pi r^3 \rho$$

$$= \frac{4}{3}\pi \times (2 \times 10^{-3})^3 \times 10^3 = \frac{32\pi}{3} \times 10^{-6} \text{ kg}$$

Work done by the gravitational force on the rain drop in each journey,

$$W = F \times s = mg \times h$$

$$= \frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250 = \mathbf{0.082\ J.}$$

For entire journey,

Work done by gravitational force + Work done by resistive force = Gain in K.E.

$$2 \times 0.082 + W_r = \frac{1}{2} mv^2$$

or
$$W_r = \frac{1}{2} \times \frac{32\pi \times 10^{-6} \times (10)^2}{3} - 0.164$$

$$= 0.0017 - 0.164 = \mathbf{-0.1623\ J.}$$

6.14. *A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?*

Ans. Momentum is always conserved, whether the collision is elastic or inelastic.

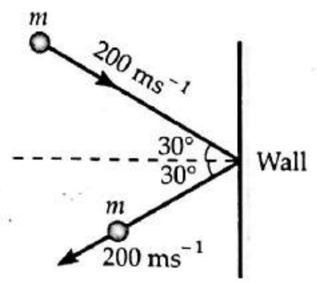

Fig. 6.49

As the wall is heavy, the molecule rebounding with its own speed does not produce any velocity in the wall. Let m be the mass of the molecule and M that of the wall.

K.E. before collision,

$$K_i = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

K.E. after collision,

$$K_f = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

$$\therefore \quad K_f = K_i$$

As the K.E. is conserved, the collision is elastic.

6.15. *A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30% how much electric power is consumed by the pump ?*

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Ans. Mass of water = Volume × density

$$= 30 \times 1000 = 3 \times 10^4 \text{ kg}$$

$$\therefore \text{ Output power} = \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$$

$$= \frac{3 \times 10^4 \times 9.8 \times 40}{15 \times 60} = \frac{39200}{3} \text{ W}$$

As $\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100$

$$\therefore \text{ Input power} = \frac{\text{Output power}}{\text{Efficiency}} \times 100$$

$$= \frac{39200}{3 \times 30} \times 100 = 43.6 \times 10^3 \text{ W}$$

$$= \mathbf{43.6 \text{ kW}}.$$

6.16. *Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v. If the collision is elastic, which of the situations shown in Fig 6.50, is a possible result after collision ?*

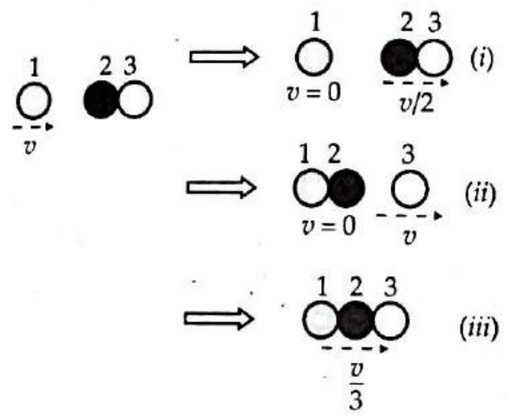

Fig. 6.50

Ans. The system consists of three identical ball bearings marked as 1, 2 and 3. Let m be the mass of each ball bearing.

Total kinetic energy of the system before collision

$$= \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv^2$$

Case *(i)* K.E. of the system after collision

$$= 0 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

Case *(ii)* K.E. of the system after collision

$$= 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

Case *(iii)* K.E. of the system after collision

$$= \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \left(\frac{1}{6}\right)mv^2$$

Because in an elastic collision, the kinetic energy of the system remains unchanged. Hence, *case (ii) is the only possible result of the collision.*

6.17. *The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6.51. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.*

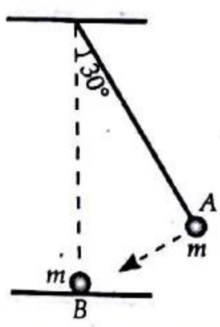

Fig. 6.51

Ans. When the bob A hits bob B on the table, it transfers its entire K.E. to the bob B because the collision is elastic. The bob A comes to rest at the location of B while the bob B begins to move with the velocity of A.

6.18. *The bob of a pendulum is released from a horizontal position A as shown. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5% of its initial energy against air resistance ?*

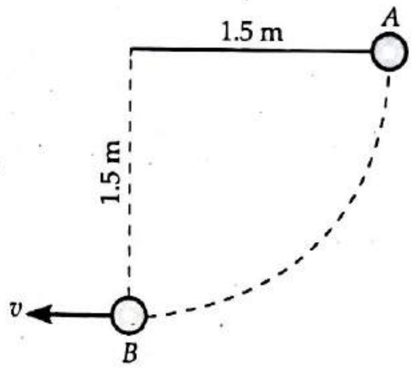

Fig. 6.52

Ans. Here $h = 1.5$ m, $v = ?$

P.E. of the bob at $A = mgh$

K.E. of the bob at $B = \frac{1}{2}mv^2$

As 5% of the P.E. is dissipated against air resistance, so

$$\frac{1}{2}mv^2 = 95\% \text{ of } mgh$$

or $$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

or $$v = \sqrt{\frac{2 \times 95 \times gh}{100}} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}}$$

$$= \sqrt{27.93} = \mathbf{5.3 \text{ ms}^{-1}}.$$

6.19. *A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s⁻¹. What is the speed of the trolley after the entire sand bag is empty ?*

Ans. As the trolley carrying the sandbag is moving with uniform speed of 27 km/h, so no external force is acting on the trolley + sandbag system. When the sand leaks out, it does not cause any external force to act on the system. Hence the speed of the trolley remains unchanged even after the sandbag becomes empty.

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6.20. *A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m.*

Ans. Velocity, $v = ax^{3/2}$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{3}{2}ax^{1/2}\frac{dx}{dt} = \frac{3}{2}ax^{1/2}.v$$

$$= \frac{3}{2}ax^{1/2}.ax^{3/2} = \frac{3}{2}a^2x^2.$$

Force, $F = m \times \text{acceleration} = \frac{3}{2}ma^2x^2$

Work done,

$$W = \int_0^2 F\,dx = \frac{3}{2}\int_0^2 ma^2x^2\,dx = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{3 \times 0.5 \times (5)^2}{2 \times 3}[2^3 - 0^3] = \mathbf{50\ J.}$$

6.21. *The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, $v = 36$ km/h and the density of air is $1.2\ kg\ m^{-3}$. What is the electrical power produced ?*

Ans. (*a*) Volume of the air passing through the windmill in time t

= Area of circle × distance covered by wind in time t

$= A \times vt = Avt$

Mass of the air passing through the windmill in time t,

$$m = \text{Density} \times \text{volume} = \rho Avt.$$

(*b*) Kinetic energy of the air is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\rho\,A\,v\,t \times v^2 = \frac{1}{2}\rho Av^3t.$$

(*c*) K.E. of air converted into electrical energy in time t

$$K' = 25\%\text{ of }K = \frac{25}{100} \times \frac{1}{2}\rho Av^3t = \frac{1}{8}\rho\,Av^3t$$

Electrical power produced

$$= \frac{K'}{t} = \frac{1}{8}\rho\,a\,v^3 = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$$

$$[\because v = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}]$$

$= 4.5 \times 10^3$ W = **4.5 kW.**

6.22. *A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?*

Ans. (*a*) Here $m = 10$ kg, $h = 0.5$ m, $n = 1000$, $g = 9.8\ \text{ms}^{-2}$

Work done against the gravitational force,

$$W = n \times mgh = 1000 \times 10 \times 9.8 \times 0.5$$

$$= \mathbf{49{,}000\ J.}$$

(*b*) Mechanical energy supplied by 1 kg of fat

$$= 20\%\text{ of }3.8 \times 10^7\ \text{J}$$

$$= \frac{20 \times 3.8 \times 10^7}{100} = 76 \times 10^5\ \text{J}$$

∴ Fat consumed for 76×10^5 J of energy = 1 kg.

Fat consumed for 49,000 J of energy

$$= \frac{1 \times 49{,}000}{76 \times 10^5} = \mathbf{6.45 \times 10^{-3}\ kg.}$$

6.23. *A large family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (b) Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15 m with a permission to cover upto 70%.*

[Delhi 03]

Ans. (*a*) Let the area needed to supply 8 kW = A m²

Energy incident per unit area = 200 W

Energy incident on area $A = 200 \times A$ W

Energy converted into useful electrical energy

$$= 20\%\text{ of }200 \times A = 40A\ \text{W}$$

But 40 A W = 8 kW = 8000 W

or $$A = \frac{8000}{40} = \mathbf{200\ m^2.}$$

(*b*) Area of the roof of the given house,

$$A' = 70\%\text{ of }20\ \text{m} \times \mathbf{15\ m}$$

$$= \frac{70 \times 20 \times 15}{100} = 210\ \text{m}^2$$

$$\text{Required ratio} = \frac{A}{A'} = \frac{200}{210} = \mathbf{20 : 21.}$$

6.24. *A bullet of mass 0.012 kg and horizontal speed $70\ ms^{-1}$ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block.*

[Chandigarh 07]

Ans. Mass of bullet, $m = 0.012$ kg

Speed of bullet, $v = 70\ \text{ms}^{-1}$

Mass of block, $M = 0.4$ kg

If V is the velocity of the combination after collision, then from the law of conservation of momentum,

$$mv = (M + m)V$$

or $$V = \frac{mv}{M + m} = \frac{0.012 \times 70}{0.4 + 0.012}$$

$$= \frac{0.84}{0.412} = 2.04\ \text{ms}^{-1}$$

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Let h be the height through which the block rises. Then from the conservation of energy,

P.E. of the combination = K.E. of the combination

$$(M + m) gh = \frac{1}{2}(M + m) V^2$$

or $$h = \frac{V^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212 \text{ m}$$

Amount of heat produced

= Loss in K.E. of the bullet

= Initial K.E. of the bullet

– K.E. of the combination

$$= \frac{1}{2} mv^2 - \frac{1}{2}(M + m) V^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

$$= 29.4 - 0.86 = \mathbf{28.54 \text{ J}}.$$

6.25. *Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.53). Will the stones reach the bottom at the same time ? Will they reach there with the same speed ? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$ and $h = 10$ m, what are the speeds and times taken by the two stones ?*

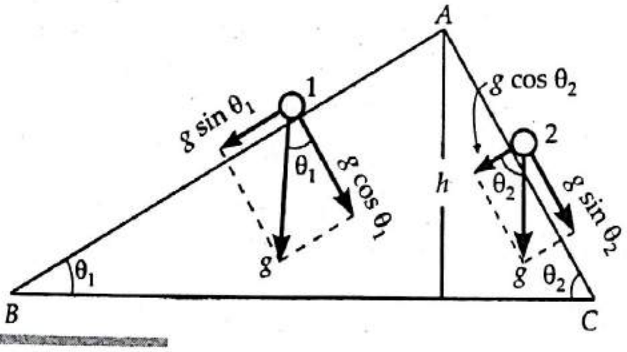

Fig. 6.53

Ans. Let a_1 be the acceleration of the stone 1 down the inclined track AB. Then

$$a_1 = g \sin \theta_1.$$

If the stone 1 takes time t_1 to slide down the track AB, then

$$AB = 0 + \frac{1}{2} a_1 t_1^2 \qquad \left[s = ut + \frac{1}{2} at^2\right]$$

$$\frac{h}{\sin \theta_1} = \frac{1}{2} g \sin \theta_1 \, t_1^2$$

or $$t_1^2 = \frac{2h}{g \sin^2 \theta_1}$$

or $$t_1 = \frac{1}{\sin \theta_1} \sqrt{\frac{2h}{g}}$$

Similarly, for stone 2, we can write

$$t_2 = \frac{1}{\sin \theta_2} \sqrt{\frac{2h}{g}}$$

For both the stones, h is same.

As $\theta_1 < \theta_2 \quad \therefore \sin \theta_1 < \sin \theta_2$

Consequently, $t_1 > t_2$

Thus, the stone 2 on the steeper plane AC reaches the bottom earlier than stone 1.

As both the stones are initially at the same height h, so

P.E. at A = K.E. at B or C

$$mgh = \frac{1}{2} mv^2$$

$$v = \sqrt{2gh}$$

i.e., both the stones will reach the bottom with the same speed.

Given : $\theta_1 = 30°$, $\theta_2 = 60°$, $h = 10$ m, $g = 10$ ms^{-2}

$$\therefore \quad t_1 = \frac{1}{\sin \theta_1} \sqrt{\frac{2h}{g}}$$

$$= \frac{1}{\sin 30°} \sqrt{\frac{2 \times 10}{10}} = 2\sqrt{2} \text{ s.}$$

$$t_2 = \frac{1}{\sin \theta_2} \sqrt{\frac{2h}{g}} = \frac{1}{\sin 60°} \sqrt{\frac{2 \times 10}{10}} = 2\sqrt{\frac{2}{3}} \text{ s.}$$

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = \sqrt{2} \times 10$$

$$= 1.414 \times 10 = \mathbf{14.14 \text{ ms}^{-1}}.$$

6.26. *A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 Nm^{-1} as shown in Fig. 6.54(a). The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that spring has negligible mass and the pulley is frictionless.*

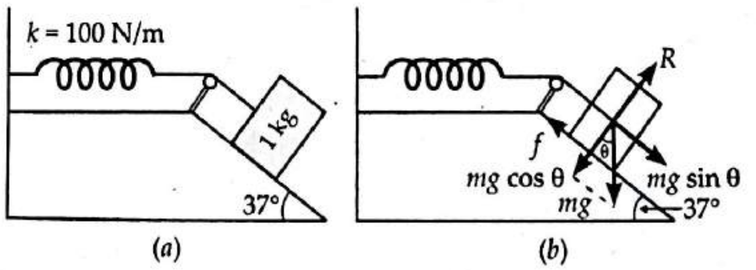

Fig. 6.54

Ans. Here $m = 1$ kg, $k = 100$ Nm^{-1}, $g = 10$ ms^{-2}

Clearly, from Fig. 6.54(b), we have

$$R = mg \cos \theta \; ; \quad f = \mu R = \mu \, mg \cos \theta$$

Net force on the block down the incline

$$= mg \sin \theta - f = mg \sin \theta - \mu \, mg \cos \theta$$

$$= mg (\sin \theta - \mu \cos \theta)$$

Distance moved, $x = 10$ cm $= 0.1$ m

In equilibrium

Work done = P.E. of stretched spring

$$mg (\sin \theta - \mu \cos \theta) x = \frac{1}{2} kx^2$$

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or $2\,mg\,(\sin\theta - \mu\cos\theta) = kx$

or $2 \times 1 \times 10\,(\sin 37° - \mu\cos 37°) = 100 \times 0.1$

or $20\,(0.601 - \mu \times 0.798) = 10$

$\therefore$ $\mu = \mathbf{0.126}.$

6.27. *A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does not rebound. What is the heat produced by the impact ? Would your answer be different, if the elevator were stationary ?*

Ans. As the elevator is moving down with a uniform speed ($a = 0$), so the value of g remains the same.

Here $m = 0.3$ kg, $h = 3$ m, $g = 9.8\ ms^{-2}$

P.E. lost by the bolt $= mgh = 0.3 \times 9.8 \times 3 = 8.82$ J

As the bolt does not rebound, the energy is converted into heat.

$\therefore$ Heat produced = **8.82 J**

Even if the elevator were stationary, the same amount of heat would have produced because the value of g is same in all inertial frames of reference.

6.28. *A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?*

Ans. The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms^{-1} with respect to the trolley's new velocity.

Total initial momentum,

$$p_i = (m_1 + m_2)\,u_1 = (20 + 200) \times \frac{36 \times 5}{18} = 2200\ \text{kg ms}^{-1}$$

Let new velocity of the trolley $= v_2$

Child's velocity relative to the trolley in opposite direction = 4 ms^{-1}

$\therefore$ Child's actual velocity (relative to ground)

$$= v_2 - 4$$

Total final momentum,

$$p_f = m_1 v_1 + m_2 v_2 = 20\,(v_2 - 4) + 200\,v_2 = 220\,v_2 - 80$$

By conservation of linear momentum,

$$p_f = p_i$$

$$220\,v_2 - 80 = 2200$$

$$\therefore \quad v_2 = \frac{2280}{220} = \mathbf{10.36\ ms^{-1}}$$

Time taken by the child to cover length of the trolley

$$= \frac{10\ \text{m}}{4\ \text{ms}^{-1}} = 2.5\ \text{s}$$

Distance covered by the trolley in 2.5 s

$$= 10.36 \times 2.5 = \mathbf{25.9\ m.}$$

6.29. *Which of the following potential energy curves in Fig. 6.55 cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.*

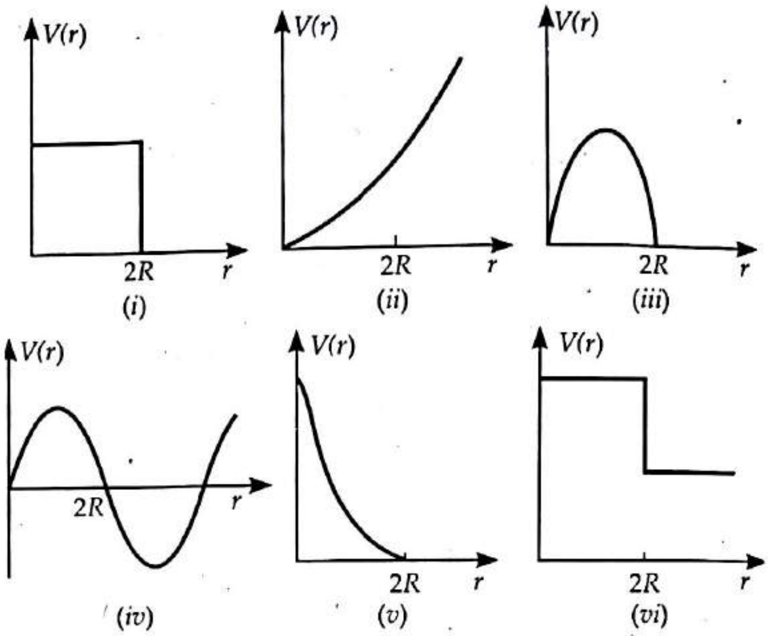

Fig. 6.55

Ans. During the short time of collision, the kinetic energy gets converted into potential energy. But the P.E. of a system of two masses varies inversely as the distance between them, *i.e.*, $V \propto 1/r$. Hence all the potential energy curves except the one shown in Fig. 6.55(v) cannot describe an elastic collision.

6.30. *Consider the decay of a free neutron at rest :*

$$n \rightarrow p + e^-$$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of neutron or a nucleus (Fig. 6.56).

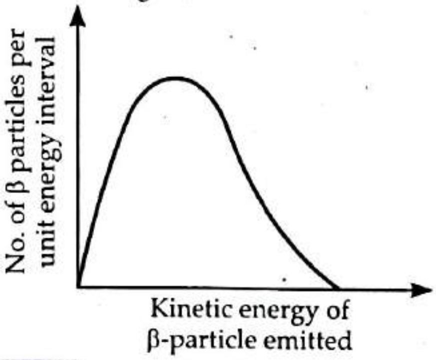

Fig. 6.56

Ans. If the decay of the neutron (inside the nucleus) into proton and electron is according to the given reaction, then the energy released in the decay must be carried by the electrons coming out of the nucleus. By mass-energy conservation, these electrons must have a definite value of energy. However, the given graph shows that the emitted electron can have any value of energy between zero and the maximum value. Hence the given decay mode cannot account for the observed continuous energy spectrum in the β-decay.

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