CHAPTER V

WORK, ENERGY AND POWER

WORK

In our daily life 'work' implies an activity resulting in muscular or mental exertion. However in physics the term 'work' is used in a specific sense which involves the displacement of a particle under the action of a force.

Work is done by a force if the point of application of the force is displaced. Work is measured as the product of the component of the force in the direction of displacement and magnitude of the displacement.

If a force F moves its point of application through a distance S in its direction, the work done is given by,

$$W = F \times S$$

In many situations the displacement of a body may not be in the direction of force. In such cases the component of the force in the direction of displacement is effective in causing the displacement.

Let a force F make an angle θ with the direction of displacement S. Its component in the direction of displacement is $F\cos\theta$. Then the work done by the force is given by,

$$W = F\cos\theta \times S = FS\cos\theta$$

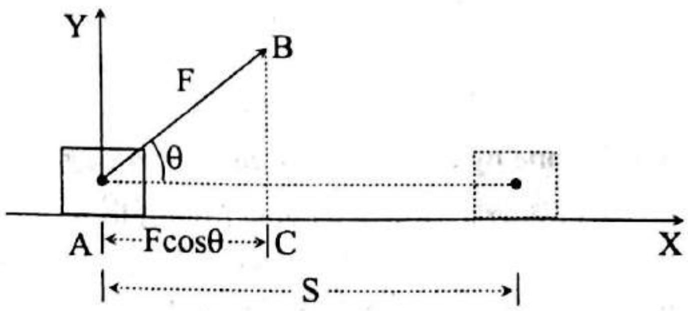

Fig. 3

Work is scalar quantity. It can be considered as the scalar product of the vectors $\vec{F}$ and $\vec{S}$.

$$\text{Thus, } W = \vec{F} \cdot \vec{S}$$

Unit of work: joule (J). It is the work done when a force of 1 N displaces its point of application through 1 m in its direction.

Dimensions of work: Work = Force × displacement = $MLT^{-2} \times L = ML^2T^{-2}$

Positive and negative work

Work can be *positive* or *negative* according as θ is acute or obtuse. Positive work means that the force or its component is in the *same direction* as displacement, while negative work means that the force or its component is *opposite* to the direction of displacement.

For example:-

1. When a person lifts a body from the ground, the work done by the lifting force is positive; while work done by the force of gravity is negative.

2. When a body is pulled along a rough surface the work done by the pulling force is positive; while the work done by the frictional force is negative.

If the displacement is perpendicular to the force, $\theta = 90°$ and the work,

$$W = \vec{F} \cdot \vec{S} = FS\cos\theta = 0.$$

Hence work done by the force is zero.

Eg: When a body moves along a frictionless horizontal surface its weight and normal reaction are perpendicular to the direction of motion. Hence these forces do no work.

Similarly when a body moves round a circle with uniform speed, the centripetal force which is always normal to the path of the particle, does no work, though it is responsible for keeping the body in the circular path.

Work done by a constant force – Graphical method

If the force F is constant, the force-displacement graph AB is a straight line parallel to the displacement axis. Let OA represent F and OC represent S, then work done is given by,

$$W = F \times S = OA \times OC = \text{area of the rectangle OABC.}$$

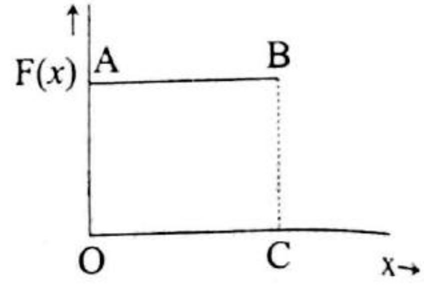

Fig. 4

Work done by a variable force

Consider a particle moving from A to B (fig. 5) under the action of a variable force. At any instant during its motion, let F be the force acting on it. To move the body through a small distance dx, work done is given by

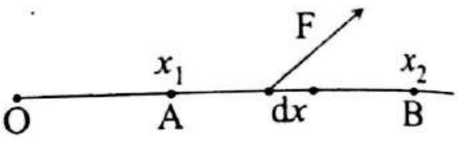

Fig. 5

$$dW = \vec{F} \cdot \vec{dx}$$

To move the body from A to B, the total work done,

$$W = \int_{x_1}^{x_2} \vec{F} \cdot \vec{dx}$$

Work done by a variable force — Graphical Method

Consider a variable force acting on a body in the X-direction. The magnitude of the force is changing. The magnitude of the force is assumed to be a function of x.

In the figure, $F(x)$ is plotted as function of x. AB represents the force-displacement graph between x_1 and x_2. Let the total displacement be divided into a number of equal intervals Δx. The interval Δx may be supposed to be so small that the force $F(x)$ may be considered approximately constant. Then the work done in moving the body by Δx at given point $x = F(x) \times \Delta x =$ area of strip $abcd$.

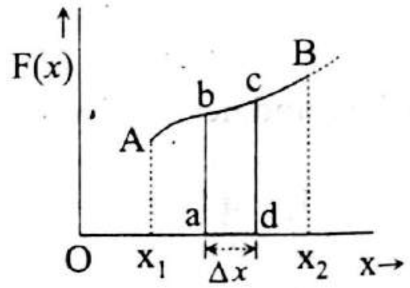

Fig. 6

The total work done is equal to the sum of all such terms corresponding to intervals between x_1 and x_2. This is written as,

$$W = \sum_{x_1}^{x_2} F(x) \times \Delta x = \text{area } x_1 \text{ AB } x_2$$

Thus the area enclosed by the graph AB, the X-axis and the two ordinates at A and B represents the work done by the force, when the body is displaced from x_1 to x_2.

Work done in lifting a body

Let a body of mass m be lifted vertically through a small height h from the surface of the earth. Work has to be done against the force of gravity.

Force applied to lift the body, $F = mg$

Displacement in the direction of the force, $S = h$

$\therefore$ Work done in lifting the body, $W = F \times S =$ **mgh**

Conservative and non-conservative forces

Conservative force

A force is said to be conservative, if work done by the force in moving a body is independent of the path traversed by the body; but depends only on the initial and final positions.

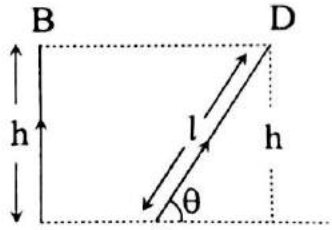

Fig. 7

For example gravitational force is a conservative force. Let a body of mass m be lifted vertically through a height h along AB, then work done $= mg \times h = mgh$. The same body is lifted along a smooth inclined plane of length l and inclination θ.

Downward force on the mass along the plane $= mg \sin\theta$

$$\text{Work done} = mg\sin\theta \times l = mg \times l\sin\theta = mgh$$

Another property of conservative force is that the *total work done by a conservative force on a particle is zero when it moves round any closed path and returns to the initial position.*

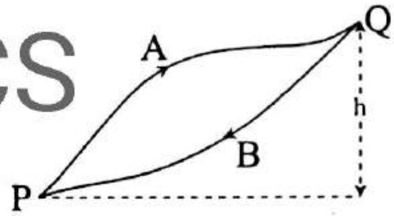

Fig. 8

Consider a body of mass m lifted through a vertical height h along a path PAQ. The work done by the gravitational force is given by, $W_1 = -mgh$; where the negative sign shows that the displacement h is opposite to the direction of the gravitational force, mg.

When the body is taken back from Q to P along QBP, the work done by the gravitational force is, $W_2 = mgh$. Hence the total work done by the gravitational force when the body is taken along the closed path $PAQBP$ is $W = -mgh + mgh = 0$.

Electrostatic force is another example for conservative force.

Non-conservative force

If the work done by a force on a particle moving between two points depends on the path taken, the force is called non-conservative force. The total work done by a non-conservative force along a closed path is not zero.

Friction is a typical example of non-conservative force. Work done by frictional force depends on the path; the longer the path between two points, the greater is the work done.

POWER

Power is the time rate at which work is done. It is the work done in unit time. If W is the work done in a time t, the average power, $P = W/t$.

Unit of power is joule/sec or **watt** (W)

When work is done at the rate of one joule/second, the power is said to be one watt.

Practical units of power (i) $kW = 1000\ W$ (ii) Horse power $(H.P) = 746$ W

Dimensions of power: $ML^2T^{-2}/T = \mathbf{ML^2T^{-3}}$

Power and velocity

If a constant force F displaces a body through a distance S in time t.

$$\text{Work done} = F \times S$$

Power $= F \times S/t = Fv$, where v is the velocity of the body.

Examples

V.1. A small pebble of mass 1 g falling from a cliff of height 1 km hits the ground with a speed of 50 ms^{-1}. What is the work done by the unknown resistive force? **[NCERT]**

$$\text{Kinetic energy acquired} = \frac{1}{2}mv^2 = 10^{-3} \times 50^2 = 1.25 \text{ J}$$

$$\text{Work done by gravitational force} = mgh = 10^{-3} \times 10 \times 100 = 10 \text{ J}$$

$$\text{Work done by resistive force} = 1.25 - 10 = \mathbf{-8.75 \text{ J}}$$

V.2. A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance by which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force which is 50 N. Calculate the work done by the two forces over 20 m. **[NCERT]**

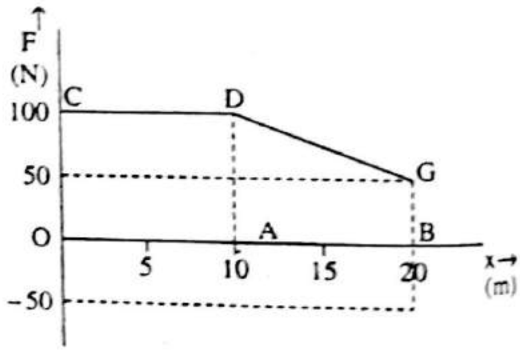

Fig. 9

Work done by the woman

$= $ area of the fig OCDGBO

$= $ area of (rectangle OCDA + trapezium ABGD)

$= 100 \times 10 + \frac{1}{2}(100 + 50)10$

$= \mathbf{1750 \text{ J}}$

Work done by the frictional force $= F \times S = -50 \times 20 = \mathbf{-1000 \text{ J}}$

V.3. Calculate the work done in lifting a mass of 5 kg vertically through 8 m.

Force on the mass = weight of the body $= 5 \times 9.8$ N

Vertical lift $= 8$ m

Work done = Force × vertical lift $= 5 \times 9.8 \times 8 = \mathbf{392 \text{ J}}$

V.4. A man pushes a roller with a force of 50 N through a distance of 20 m. Calculate the work done if the handle of the roller is inclined at an angle of 60° with the ground.

$$W = F \times S\cos\theta = 50 \times 20 \times \cos 60 = \mathbf{500\ J}$$

V.5. A body constrained to move along the Z-axis of a co-ordinate system is subjected to a constant force $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})N$. What is the work done by this force in moving the body over a distance of 4 m along Z-axis. [NCERT]

$$F = (-\hat{i} + 2\hat{j} + 3\hat{k});\quad S = 4\,\hat{k};\quad W = ?$$

$$W = \vec{F}\cdot\vec{S} = (-\hat{i} + 2\hat{j} + 3\,\hat{k})\cdot 4\,\hat{k} = \mathbf{12\ J}$$

V.6. A pump on the ground floor of a building can pump up water to fill a tank of volume $30\,m^3$ in 30 minutes. If the tank is 40 m above the ground and the efficiency of the pump is 40%, how much electric power is consumed by the pump? [NCERT]

Mass of water lifted, $m = 30 \times 1000 = 30{,}000$ kg

$$\text{Power consumed} = \frac{mgh}{t} = \frac{30\,000 \times 9.8 \times 40}{30 \times 60} = 6533\ \text{W}$$

This is 40% of the actual power of the engine

$$\text{Actual power} = \frac{6533 \times 100}{40} = \mathbf{1.63 \times 10^4}\ \text{W} = 16.3\ \text{kW}$$

$\therefore$ Electic power consumed by the pump = **16.3 kW**

V.7. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration due to viscous resistance of the air until at half its original height, it attains maximum terminal speed and it moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first half and second half of its journey? [NCERT]

$$\text{Mass of drop} = \frac{4}{3}\pi r^3 d = \frac{4}{3} \times \pi(2 \times 10^{-3})^3 \times 1000 = 3.35 \times 10^{-5}\ \text{kg}$$

Distance moved in each half of the journey = 250 m

Work done by gravitational force = mgh

$$= 3.35 \times 10^{-5} \times 9.8 \times 250 = \mathbf{0.082\ J}$$

This is the same for both halves of the journey.

V.8. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to road is 200 N and is directly opposed to motion. (a) How much work does the road do on the cycle? (b) How much work does the cycle do on the road? [NCERT]

(a) Work is done by the frictional force, $F = 200$ N, which is acting on the cycle in a direction opposite to the direction of displacement. $S = 10$ m; $\theta = 180°$.

$$W = FS\cos\theta = 200 \times 10 \times \cos 180 = -2000\ \text{J} = -2\text{kJ}$$

(b) According to Newton's third law, an equal and opposite force acts on the road due to the cycle. But, since the displacement of the road is zero, the work done by the cycle on the road is zero.

V.9. A particle of mass 0.5 kg travels along a straight line with a velocity $v = \alpha x^{3/2}$ where $\alpha = 5m^{-1/2}s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m? [NCERT]

$$v = \alpha x^{3/2} \quad \therefore a = (dv/dt) = \alpha \times (3/2) \times x^{1/2} \times (dx/dt)$$

ie., $$a = (3/2)\alpha v x^{1/2} = (3/2)\alpha \times \alpha x^{3/2} \times x^{1/2} = (3/2)\alpha^2 x^2$$

$$\therefore W = \int_0^2 F dx = \int_0^2 ma dx = \int_0^2 (3/2)m\alpha^2 x^2 dx = (1/2)m\alpha^2 \left[x^3\right]_o^2$$

$$= (1/2) \times 0.5 \times 5^2 \times 8 = \mathbf{50\ J}$$

V.10. An elevator of total mass 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N opposes the motion. Determine the minimum power delivered to the elevator ($g = 10$ ms^{-2}) [NCERT]

$m = 1800$ kg; $g = 10$ ms^{-2}; $v = 2$ ms^{-1}; $F_r = 4000$ N; $P = ?$

Force applied by the motor.

$$F = mg + F_r = 1800 \times 10 + 4000 = 22,000 \text{ N}$$

$$P = F \times v = 22,000 \times 2 = 44,000 \text{ W} = \mathbf{44\ kW}$$

ENERGY

Energy is defined as the capacity to do work. It is measured in the same unit as work.

Like work, energy is a scalar quantity. Its unit is the same as that for work, *i.e.*, joule (J).

There are different forms of energy such as mechanical energy, light energy, heat energy, electrical energy, sound energy, chemical energy, mass energy etc.

Mechanical energy may be classified into (a) kinetic energy and (b) potential energy.

(a) Kinetic energy

It is the energy possessed by a body by virtue of its motion.

Like work, kinetic energy is a scalar quantity; unlike work, kinetic energy is never negative.

Expression for the kinetic energy of a moving body

Consider a body of mass m moving with a velocity v. Its kinetic energy is equal to the work done by an external agency in increasing its velocity from zero to v.

Let a constant force F act on the body at rest and move it through a distance S, thereby changing its velocity to v. Let a be the constant acceleration produced. Then

$$v^2 = u^2 + 2aS \ ; \ v^2 = 2aS \ ; \ a = v^2/2S$$

$$\text{Force, } F = ma = mv^2/2S$$

$$\text{Work done, } W = F \times S = (mv^2/2S) \times S = (1/2)mv^2$$

Kinetic energy of the body = **(1/2) mv²**

Regarding kinetic energy it is worth noting that:-

1. KE is always a positive scalar quantity.
2. The KE depends on the frame of reference. eg: KE of a man sitting in a train is zero with respect to the train but has a positive value in the frame of the earth.
3. KE (K) of a body is related its momentum p as $K = p^2/2m$, where m is the mass of the body.

WORK-ENERGY PRINCIPLE

Work done by a force in displacing a body measures the change in kinetic energy of the body.

Let a constant force F acting on a body of mass m change its velocity from u to v in travelling a distance S. Then

Work done $= F \times S$

Change in KE $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}m \times 2aS = maS = F \times S.$

Work-energy theorem for a variable force

Consider a body of mass m moving along a straight line. The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt}(\frac{1}{2}mv^2) = m\frac{dv}{dt}v = Fv = F\frac{dx}{dt}.$$ Thus

$$dK = Fdx$$

Integrating from initial position x_1 to final position x_2 we have

$$\int_{K_1}^{K_2} dK = \int_{x_1}^{x_2} Fdx$$

$$K_2 - K_1 = \int_{x_1}^{x_2} Fdx$$

Thus the change in kinetic energy is equal to the work done.

(b) Potential energy

Potential energy of a body is defined as the energy possessed by the body by virtue of its position or state of strain.

For example a body situated at a height above the ground possesses potential energy with respect to earth because of its position. A compressed spring possesses potential energy because of its state of strain.

Potential energy can be positive as well as negative.

Gravitational potential energy

It is the energy possessed by a body by virtue of its position above the surface of the earth.

Consider a body of mass m situated at a height h above the ground. Its potential energy with respect to the surface of the earth is equal to the negative work done by the gravitational force in lifting the body from the ground level to that height.

Work done in lifting the body $= mg \times h = mgh$

$\therefore$ Potential energy, $V = \mathbf{mgh}$

The notion of potential energy is applicable only to the class of forces where work done against the force gets stored up as energy. When external constraints are removed it manifests itself as kinetic energy. Mathematically, force can be written as the negative space rate of potential energy

$$F(x) = -\frac{dV}{dx} \quad \text{or} \quad -dV = F(x)dx$$

Potential energy of a spring

- One end of an elastic spring is fixed and it is stretched or compressed by applying a variable force at the other end. We assume the spring to be of negligible mass. The elongation or compression of the spring is one dimensional along the X-axis.

When the spring is not in a strained state, the position of the free end of the spring is taken as $x = 0$. If the spring is compressed or stretched by a small distance x, the spring will exert a restoring force trying to bring it back to equilibrium position. The spring force, $F = -kx$, where k is a constant called *spring constant*. The negative sign shows that the spring force is against the direction of displacement. If we want to stretch a spring we have to apply a force opposite to the restoring force.

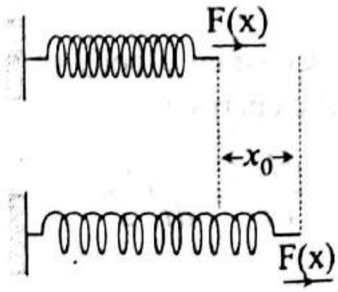

Fig. 10

$$F_{ext} = +kx.$$

Work done to stretch the spring through an additional distance $dx = kx \times dx$.

Hence work done in stretching the spring from 0 to x_m.

$$= \int_0^{x_m} kx\,dx = k[x^2/2]_0^{x_m} = (1/2)kx_m^2$$

This is stored as potential energy of the spring.

Potential energy of a spring, $\mathbf{V}_x = \mathbf{(1/2)kx^2}$, where x is any displacement.

Graphical treatment

Since the spring is stretched by a varying force, area under the graph OC represents the work done.

Work done $=$ Area of triangle OBC $= \frac{1}{2}x \times kx = \frac{1}{2}kx^2$

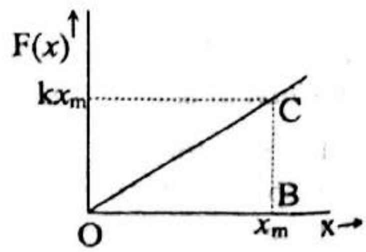

Fig. 11

Conservation of mechanical energy

The total mechanical energy of a system is conserved, if the forces doing work on it are conservative.

Suppose that a body undergoes displacement Δx under the action of a conservative force F. Then by work energy theorem we have,

$$\Delta K = F(x)\Delta x. \quad (1)$$

The potential energy function $V(x)$ can be defined such that

$$-\Delta V = F(x)\Delta x \quad (2)$$

From equations (1) and (2),

$$\Delta K + \Delta V = 0$$

$$\Delta(K + V) = 0$$

This means that $(K + V)$, the sum of the potential and kinetic energies of the body is a constant.

The quantity $K + V(x)$ is called total mechanical energy of the system.

Individually kinetic energy K and potential energy $V(x)$ may vary from point to point. But the sum remains a constant.

Illustration

1. Freely falling body

Consider a body of mass m situated at a height h above the ground.

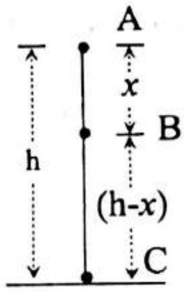

Fig. 12

Since the body is at rest, the energy is completely potential. As it falls down, its height decreases and its velocity increases. Hence its potential energy decreases and kinetic energy increases. It can be shown that the total energy at any point is a constant.

Let us consider three positions of the freely falling body; (i) the position A, the initial position, (ii) the position B, an intermediate position when the body has fallen through a distance x and (iii) the position C just before it strikes the ground. It can be shown that the total energy at A = total energy at B = total energy at C.

Total energy at the position A

$$PE = mgh \text{ and } KE = 0$$

Total energy, TE $= PE + KE = mgh + 0 = mgh$

Total energy at the position B

The velocity of the body at B is $v = \sqrt{2gx}$.$(\because u = 0; S = x;\ a = g)$

$$PE = mg(h - x) \text{ and } KE = \frac{1}{2}\text{m} \times 2gx = mgx$$

Total energy $= PE + KE = mg(h - x) + mgx = mgh$

Total energy at C

Velocity just when it hits the ground $v = \sqrt{2gh}$

$$PE = 0; \; KE = \frac{1}{2} \text{m} \times 2gh = mgh$$

$$\text{Total energy } = PE + KE = 0 + mgh = mgh$$

Thus the total energy is constant at every point along its path.

The variation of energy with position is shown graphically.

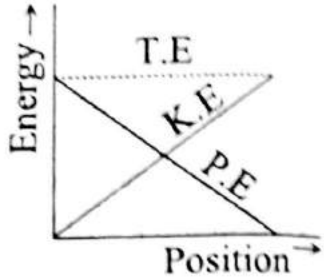

Fig. 13

2. Vibration of a simple pendulum

OA is the normal position of the pendulum. When the bob is displaced to B through a vertical height h its $PE = mgh$, where m is the mass of the bob.

On releasing the bob it moves towards A. PE of the bob is being converted into KE At A its energy is completely kinetic. Then it moves to the other side. The kinetic energy is being converted into potential energy. At C, h above the reference level, the energy is completely potential. Thus it keeps on oscillating.

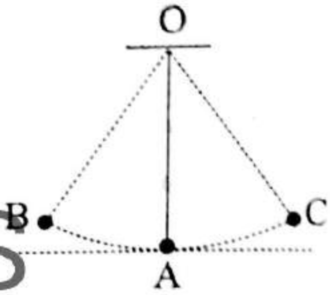

Fig. 14

Graphical representation of energy with position

Potential energy is maximum at the extreme ends and zero at the mean position. KE is zero at the extreme ends and maximum at the mean position.

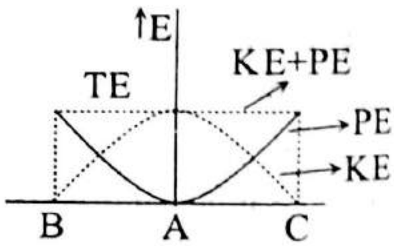

Fig. 15

3. Oscillating spring

Consider an elastic spring, fixed at one end and carrying a block of mass m at the other. The spring is kept horizontally and the mass can move on a smooth horizontal surface.

If we stretch the spring by pulling the block from its equilibrium position $x = 0$ to the position $x = x$ on one side of the equilibrium position and let go, the speed of the block increases from zero to maximum as the spring moves from $x = x$ to $x = 0$. Its PE is completely converted into KE. The loss of PE is equal to the gain in KE. The block overshoots the equilibrium position and the speed decreases until the block reaches the point $x = -x$ on the other side of the equilibrium position. At this position PE is maximum and $KE = 0$. Thus the block keeps on oscillating.

The energy-position graph of the oscillating spring is similar to that of the oscillating pendulum.

Various forms of energy. The law of conservation of energy

Energy can manifest itself in a number of forms.

1. Internal energy

This is the energy possessed by a body because of the motion of molecules constituting it. The molecules possess kinetic energy due to vibrational motion. They also possess potential energy due to inter-molecular force of attraction. The sum of the kinetic energy and potential energy of the molecules of a body is the internal energy. When the internal energy of a molecule increases its temperature increases. Thus temperature is a measure of the internal energy of the body.

2. Thermal energy

It is the energy possessed by hot bodies. In a steam engine the heat energy associated with steam can perform work in moving the wheels of the engine.

3. Chemical energy

A chemical compound has less energy than its separate parts, the difference being in the specific arrangement and motion of electrons and nuclei in the compound. This difference in energy is called chemical energy or energy of chemical binding. In a chemical reaction energy can be absorbed or released. For example, when coal is burned chemical energy is converted into heat energy.

4. Electrical energy

Since electric charge and current attract or repel each other, they exert force on each other. Thus work has to be done for relative motion. The energy associated with this work is called electrical or electromagnetic energy.

5. Mass-energy

According to Einstein, matter and energy are inter-convertible. This is called *mass-energy equivalence*. The energy E associated with a mass m is given by, $E = mc^2$, where c is the speed of light in vacuum. $c = 3 \times 10^8$ ms^{-1}.

6. Nuclear energy

Protons and neutrons attract each other and combine to form nuclei. The energy associated with this is called nuclear energy. Because of attraction between nucleons, a helium nucleus has less energy than its separated constituents (2 protons and 2 neutrons). Hence when the constituent particles are fused to form a helium nucleus there is a loss of mass which is converted into energy. This is the principle of production of nuclear energy by fusion. In *nuclear fission*, a less stable heavy nucleus like uranium breaks up into two or more stable nuclei and a few neutrons. In this process also a certain mass is converted into energy.

Transformation of energy

It is the phenomenon of the change of energy from one form to another. Eg:

1. In electric bulb electrical energy is converted into light and heat.

2. In electric fan, electric motor etc electrical energy is converted into mechanical energy.
3. In heat engines, heat energy is converted into mechanical energy.

The principle of conservation of energy

We have seen that the total mechanical energy of a system is conserved if forces doing work on it are conservative. If some of the forces involved are non conservative part of the mechanical energy may get transformed into other forms, such as heat, light and sound. However the total energy of an isolated system does not change as long as we account all forms of energy.

Energy may be transformed from one form to another, but the total energy of an isolated system remains constant. Hence energy can neither be created or destroyed.

Since the universe as a whole may be viewed as an isolated system, the total energy of this universe is a constant. The principle of conservation of energy cannot be proved. However no violation of this principle has been observed.

Examples

V.11. A shot travelling at the rate of $100\,\text{ms}^{-1}$ is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 cm thick?

$v_1 = 100\,\text{ms}^{-1}$; $S_1 = 4\,\text{cm}$; $S_2 = 9\,\text{cm}$; $v_2 = ?$

Work done = Change in KE

Let F be the resistance of the plank.

$$FS = \frac{1}{2}mv^2;\ FS_1 = \frac{1}{2}mv_1^2;\ FS_2 = \frac{1}{2}mv_2^2$$

$$\frac{S_2}{S_1} = \frac{v_1^2}{v_2^2};\qquad \frac{v_1}{v_2} = \sqrt{\frac{S_2}{S_1}} = \sqrt{9/4} = 3/2$$

$$v_2 = \frac{3}{2}v_1 = \frac{3}{2} \times 100 = \mathbf{150\,ms^{-1}}$$

(*This problem can be solved using the equations of motion also*)

V.12. A car of mass 1000 kg moving with a speed 18.0 km/h on a horizontal road collides with a horizontally mounted spring of spring constant $6.25 \times 10^3\ N/m$. What is the maximum compression of the spring? [NCERT]

$m = 10^3$ kg; $v = 18$ km/h $= 5$ m/s; $k = 6.25 \times 10^3$; $x = ?$

$(1/2)mv^2 = (1/2)kx^2$; $x = \sqrt{mv^2/k} = \sqrt{10^3 \times 5^2/6.25 \times 10^3} = \mathbf{2\ m}$

V.13. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or proton? Obtain the ratio of their speeds.
($m_e = 9.11 \times 10^{-31}\ kg$; $m_p = 1.67 \times 10^{-27}\ kg$; $1\ eV = 1.6 \times 10^{-19}\ J$) [NCERT]

$$K_e = 10\ keV;\ K_p = 100\ keV;\ m_e = 9.11 \times 10^{-31}\ kg;\ m_p = 1.67 \times 10^{-27}\ kg$$

$$(K_e/K_p) = (1/2)m_e v_e^2/(1/2)m_p v_p^2 = 10/100 = 1/10$$

$$\therefore v_e/v_p = \sqrt{(m_p/m_e) \times 1/10} = (1.67 \times 10^{-27}/9.11 \times 10^{-31}) \times (1/10) = \mathbf{13.54}$$

Hence, $v_e > v_p$. The electron is faster.

V.14. A body of mass 5 kg initially at rest is subjected to a force of 20 N. What is the kinetic energy acquired at the end of 10 seconds?

Acceleration $a = F/m = 20/5 = 4\,\mathrm{ms^{-2}}$

$$v(t) = v(0) + at \; ; \; v(t) = 0 + 4 \times 10 = 40\,\mathrm{ms^{-1}}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 40^2 = \mathbf{4000\,J = 4\,kJ}$$

V.15. A bullet of mass 0.012 kg and horizontal speed $70\,\mathrm{ms^{-1}}$ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of a thin wire. Calculate the height to which the block rises. Also estimate the amount of heat produced. [NCERT]

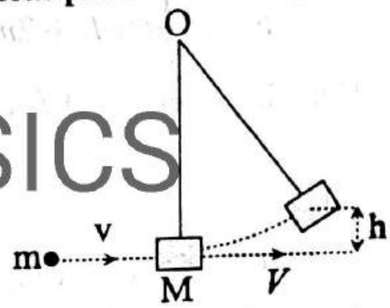

Fig. 16

Let m be the mass of the bullet and M that of the block. Let v be the velocity of the bullet and V that of the block.

By law of conservation of momentum,

$$mv = (M + m)V \quad ; V = mv/(M + m)$$

$$V = \frac{0.012 \times 70}{0.012 + 0.4} = 2.04\,\mathrm{ms^{-1}}$$

Let h be the vertical height reached by the block. Then,

KE of the block = PE of the block at the extreme position

$$\frac{1}{2}(M + m)V^2 = (M + m)gh$$

$$h = \frac{V^2}{2g} = \frac{2.04^2}{2 \times 9.8} = \mathbf{0.212\,m}$$

Heat produced = KE of bullet before collision − KE of the (block + bullet) after collision

$$= \frac{1}{2}mv^2 - \frac{1}{2}(M + m)V^2$$

$$= \frac{1}{2} \times 0.012 \times 70^2 - \frac{1}{2} \times 0.412 \times 2.04^2 = \mathbf{28.5\,J}$$

V.16. A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity of v_o at the lowest point A, such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the top most point C. Obtain an expression for (i) v_o (ii) the speeds at B and C (iii) the ratio of KE at B and C. [NCERT]

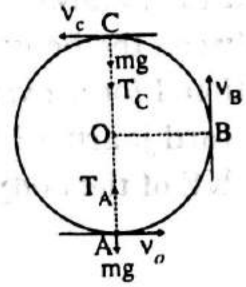

Fig. 17

At A

$$T_A - mg = \frac{mv_o^2}{L} \tag{1}$$

Kinetic energy at A

$$E_A = \frac{1}{2}m v_o^2$$

At C

The string slackens. Hence $T_C = 0$

$$\therefore \quad mg = \frac{m v_c^2}{L} \tag{2}$$

Energy at C

$$E_C = \frac{1}{2}m v_C^2 + mg \times 2L, \quad \text{where } mg \times 2L \text{ is increase in PE} \tag{3}$$

From equation (2) and (3)

$$E_C = \frac{1}{2}mgL + 2mgL = \frac{5}{2}mgL \tag{4}$$

Equating energies at A and C

$$\frac{1}{2}m v_o^2 = \frac{5}{2}mgL$$

$$\text{or} \quad \mathbf{v_o = \sqrt{5gL}} \tag{5}$$

From equation (2)

$$v_C = \sqrt{gL} \tag{6}$$

At B

$$E_B = \frac{1}{2}m v_B^2 + mgL \tag{7}$$

From equation (4) and (7)

$$\frac{1}{2}m v_B^2 + mgL = \frac{5}{2}mgL$$

$$\mathbf{v_B = \sqrt{3gL}}$$

Ratio of kinetic energies

$$\frac{K_B}{K_C} = \frac{1/2 m v_B^2}{1/2 m v_c^2} = 3$$

V.17. A body of 0.3 kg is taken up an inclined plane of length 10 m and height 5 m, and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the (a) work done by the gravitational force over the round trip? (b) work done by the applied force over the upward journey? (c) work done by the frictional force over the round trip? and (d) KE of the body at the end of the trip?

$$\text{Angle of inclination, } \theta = \sin^{-1} 5/10 = 30°$$

(a) Since gravitational force is conservative, work done along a closed path is zero.

(b) Applied force during upward journey $= F = mg\sin\theta + \mu mg\cos\theta$

$$\text{Work done } = F \times S = mg(\sin\theta + \mu\cos\theta) \times S$$

$$= 0.3 \times 9.8[0.5 + 0.15 \times 0.866]10 = \mathbf{18.22\ J}$$

(c) Work done by frictional force during upward and downward journey

$$= 2(\mu mg\cos\theta \times S) = 2 \times 0.15 \times 0.3 \times 9.8 \times 0.866 \times 10$$
$$= 2 \times 3.82 = \mathbf{7.64\ J}$$

(d) Force acting on the body as it slides down $= mg\sin\theta - \mu mg\cos\theta$

Work done on moving down $= (mg\sin\theta - \mu mg\cos\theta)S$

KE at the bottom $=$ Work done $= mg(\sin\theta - \mu\cos\theta)S$

$$= 0.38 \times 9.8(0.5 - 0.15 \times 0.866) \times 10$$
$$= \mathbf{10.88\ J}$$

V.18. A toy rocket of mass 0.1 kg has fuel of mass 0.02 kg which burns out in 3 seconds. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust on the rocket? What is the energy content per unit mass of the fuel? (Ignore the small mass variation of the rocket due to fuel burning)

$m = 0.1$ kg; $u = 0$; $v = 20\,\text{ms}^{-1}$; $t = 3s$; $a = ?$; $F = ?$

$v = u + at$; $20 = a \times 3$; $a = \dfrac{20}{3}\,\text{ms}^{-2}$

$F = ma = 0.1 \times \dfrac{20}{3} = \dfrac{2}{3}$ N; Thrust of the rocket $=$ **(2/3) N**

$S = ut + \dfrac{1}{2}at^2 = \dfrac{1}{2} \times \dfrac{20}{3} \times 3^2 = 30$ m

Work done, $FS = \dfrac{2}{3} \times 30 = 20$ J

Energy content for the whole mass of the fuel $= 20$ J

Energy content per unit mass of the fuel $= \dfrac{20}{0.02} = \mathbf{1000\ Jkg^{-1}}$

V.19. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with uniform speed of 7 ms^{-1}. It hits the floor of the elevator and does not rebound. What is the heat produced by the impact? Will the answer be different if the elevator was stationary? Height of the elevator $= 3$ m [NCERT]

Distance travelled by the bolt $= 3$ m

Work done $= mgh = 0.3 \times 9.8 \times 3 = 8.82$ J

Heat produced $=$ **8.82 J**

The answer will be the same even if the elevator was at rest.

V.20. The bob of a pendulum is released from horizontal position A. If the length of the pendulum is 1.5 m, what is the speed with which the bob reaches the lowermost point B, given that it dissipates 5% of its initial energy against air resistance. **[NCERT]**

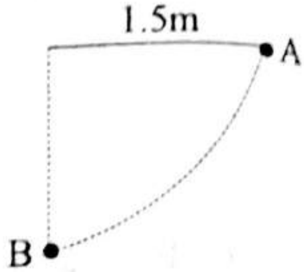

Fig. 18

When the bob is at the highest point, PE $= mgh$

Energy lost $= 5\%$

Energy of the bob when it is at B $= \dfrac{95}{100} \times mgh$

If v is the velocity of the hole at B,

$$(1/2)\,mv^2 = (95/100) \times mgh$$

$$v^2 = \frac{2 \times 95 \times gh}{100};\ v = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}} = \mathbf{5.28\,ms^{-1}}$$

V.21. A body of mass 1 kg initially at rest is dropped from a height of 2 m on to a vertical spring having force constant 490 Nm^{-1}. Calculate the maximum distance through which the spring will be compressed.

Loss in gravitational potential energy of the body
= Gain in potential energy of the spring

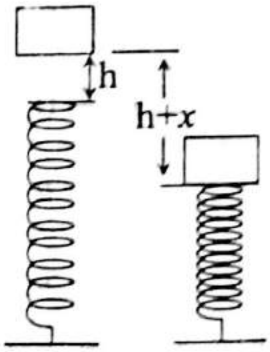

Fig. 19

$$mg(h + x) = \frac{1}{2}kx^2$$

$$m = 1\,kg; \quad h = 2\,m; \quad k = 490\,Nm^{-1}$$

$$1 \times 9.8(2 + x) = \frac{1}{2} \times 490 \times x^2$$

$$100x^2 - 4x - 8 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 3200}}{200} = \frac{4 \pm \sqrt{3216}}{200} = \mathbf{0.303\,m}$$

V.22. A 1 kg block collides with a horizontal weightless spring of force constant 2 Nm^{-1}. The block compresses the spring by 4 m from rest position. Assuming that the coefficient of kinetic friction between the block and the horizontal surface is 0.25, what was the speed of the block at the instant of collision?

Let v be the velocity of the block at the instant of collision. Then by law of conservation of energy,

Initial KE of block = PE of spring + work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx$$

$$i.e., \frac{1}{2} \times 1 \times v^2 = \frac{1}{2} \times 2 \times 4^2 + 0.25 \times 1 \times 9.8 \times 4$$

$$\frac{1}{2}v^2 = 16 + 9.8 = 25.8 \therefore v = \sqrt{2 \times 25.8} = \mathbf{7.183\,ms^{-1}}$$

COLLISIONS

If two objects in motion strike against each other or come close to each other such that the motion of one of them or both of them changes abruptly, a collision is said to have taken place.

The force involved in a collision may be large, but it acts only for a very short time. We come across many examples of collision daily. The coins of a carrom game colliding with one another or collision between two automobiles in a road accident etc are examples of collision by physical contact.

But in the case of charged bodies they may not actually come into contact with each other, but they will affect each others motion when one is within the field of the other. This is collision without physical contact.

Types of collisions

1. Elastic collision

Those collisions in which both momentum and kinetic energy of the system are conserved are called elastic collisions. The collision between subatomic particles is generally elastic. The collision between two steel or glass balls is nearly elastic.

In elastic collisions, the forces involved during interaction are of conservative nature.

2. Inelastic collision

Those collisions in which momentum of the system is conserved but kinetic energy is not conserved are called inelastic collisions.

Most of the collisions in every day life are inelastic.

Elastic collision in one dimension

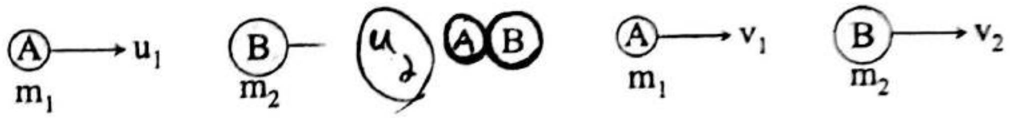

Fig. 20

Consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 along a straight line in the same direction. Let A be moving faster than B. After some time, A collides with B. After collision, let them move with velocities v_1 and v_2 in the same direction.

By law of conservation of momentum,

Total momentum after collision=Total momentum before collision

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad (1)$$

For perfectly elastic collision,

Total KE after collision = Total KE before collision

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \quad (2)$$

From eqn (1),

$$m_1(v_1 - u_1) = m_2(u_2 - v_2) \quad (3)$$

From eqn (2), $m_1(v_1^2 - u_1^2) = m_2(u_2^2 - v_2^2)$

$$i.e., m_1(v_1 + u_1)(v_1 - u_1) = m_2(u_2 + v_2)(u_2 - v_2) \quad (4)$$

Dividing (4) by (3) we get,

$$v_1 + u_1 = u_2 + v_2 \quad (5)$$

$$v_1 - v_2 = -(u_1 - u_2) \quad (6)$$

The relative velocity after collision is numerically equal to the relative velocity before collision.

To find the velocities after collision

From equation (5), $v_2 = u_1 - u_2 + v_1$
Substituting this value of v_2 in equation (3)

$$m_1(v_1 - u_1) = m_2(u_2 - u_1 + u_2 - v_1)$$

$$m_1v_1 - m_1u_1 = 2m_2u_2 - m_2u_1 - m_2v_1$$

$$m_1v_1 + m_2v_1 = m_1u_1 - m_2u_1 + 2m_2u_2$$

$$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad (6)$$

Similarly from eqns (5) and (3) we get,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \quad (7)$$

Special cases

1. Let $m_1 = m_2 = m$. Equations (6) and (7) give,
   $v_1 = u_2$ and $v_2 = u_1$
   Thus in one dimensional elastic collision between two bodies of equal mass, the bodies merely exchange their velocities.
2. If a body suffers a one dimensional elastic collision with another body of the same mass at rest, the first body is stopped dead, but the second begins to move with the velocity of the first.
3. If the body B is at rest before collision and $m_1 \neq m_2$, $u_2 = 0$,then from eqn: (6) we get, $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times u_1$ and from eqn: (7) we get, $v_2 = \frac{2m_1u_1}{m_1 + m_2}$

Inelastic collision in one dimension

For inelastic collision, the ratio of the relative velocity after collision bears a constant ratio to the relative velocity before collision and is opposite in sign.

i.e., $\frac{v_2 - v_1}{u_2 - u_1} = -e$, a constant . This constant is called coefficient of restitution.

$$\therefore v_1 - v_2 = -e(u_1 - u_2)$$

If $e = 1$, the collision is perfectly elastic.

If $e = 0$ the collision is perfectly inelastic.

Generally e lies between 0 and 1.

Normal impact on a fixed plane

Consider a body moving with a velocity u hitting directly against a fixed plane. Let v be the velocity of rebounding and e the coefficient of restitution.

$$v_1 - v_2 = -e(u_1 - u_2)$$

Here $u_1 = u$, $u_2 = 0$, $v_1 = -v$ and $v_2 = 0$

$$\therefore -v - 0 = -e(u - 0); \; \mathbf{v = eu}$$

Hence velocity of reflection is e times velocity of incidence.

Collision in two dimensions

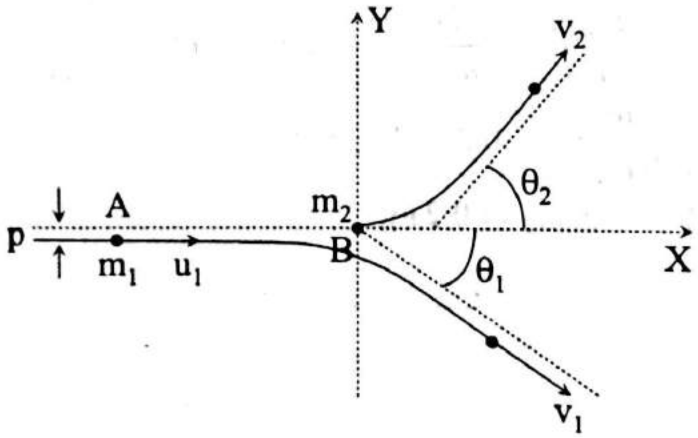

Fig. 21

Consider a particle A of mass m_1 moving with velocity u_1 in the X direction. Let it collide with a particle B of mass m_2 at rest. After collision, let A move with velocity v_1 in a direction making an angle θ_1 with the X-axis and B with a velocity v_2 in a direction making an angle θ_2 with the X-axis.

Applying the law of conservation of momentum in the X and Y directions

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \cdots\cdots\cdots \; X\text{-direction}$$

$$0 = m_2 v_2 \sin\theta_2 - m_1 v_1 \sin\theta_1 \cdots\cdots\cdots \; Y \text{ direction}$$

Assuming the collision to be perfectly elastic, conservation of kinetic energy yields,

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

i.e., $m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2$

Note: The distance p between the initial line of motion of A and a line parallel to it through the center of the target particle B is called the *impact parameter*. $p = 0$ corresponds to a head-on collision.

Examples

V.23. Two ball bearings of mass m each moving in opposite directions with equal speeds v collide head on with each other. Predict the outcome of the collision assuming it to be perfectly elastic.

$$m_1 = m_2 = m;\ u_1 = +v \text{ and } u_2 = -v$$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2} = 0 + \frac{2m \times -v}{2m} = -\mathbf{v}$$

$$v_2 = \frac{(m_2 - m_1)u_2}{m_1 + m_2} + \frac{2m_1u_1}{m_1 + m_2} = \frac{2\,\text{mv}}{2\,\text{m}} + 0 = \mathbf{v}$$

The two balls bounce back with equal speeds

V.24. A ball of mass 0.1 kg makes an elastic head on collision with a ball of unknown mass initially at rest. If the 0.1 kg ball rebounds at one-third of its original speed. what is the mass of the other ball?

$$m_1 = 0.1\,\text{kg};\ m_2 =?;\ u_2 = 0;\ u_1 = u;\ v_1 = -u/3$$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2};\ i.e.,\ -u/3 = \frac{(0.1 - m_2)u}{0.1 + m_2}$$

$$-\frac{1}{3} = \frac{0.1 - m_2}{0.1 + m_2};\quad -0.3 + 3m_2 = 0.1 + m_2$$

$$2m_2 = 0.4 \qquad \therefore m_2 = \mathbf{0.2\,kg}$$

IMPORTANT POINTS

Work *Work done by a constant force* $W = \vec{F} \cdot \vec{S} = FS\cos\theta$. *Dimension* ML^2T^{-2}.

Work done by a variable force $W = \int_{x_1}^{x_2} F \cdot dx$ = *area under* $F - S$ *diagram.*

Power *is rate of work done.* $P = W/t$. *Dimension* ML^2T^{-3}

Energy *is the ability to perform work. It has the same unit and dimensions as work.*

Potential energy *of a body is the energy possessed by virtue of its location or state of strain. In a conservative force field*

$$V_i - V_f = \int_{x_1}^{x_2} F \cdot dx$$

Force is negative potential gradient.

A stretched spring or a compressed one possesses potential energy

$$V = \int F \cdot dx = \frac{1}{2}Kx^2$$

Kinetic energy is the energy possessed by virtue of velocity

$$KE \text{ of a moving body} = \frac{1}{2}mv^2$$

Principle of conservation of mechanical energy. *The total mechanical energy remains constant in a conservative force field.*

Collision. *In perfectly elastic collision momentum and kinetic energy are both conserved. In an inelastic collision momentum is conserved. But kinetic energy is not.*