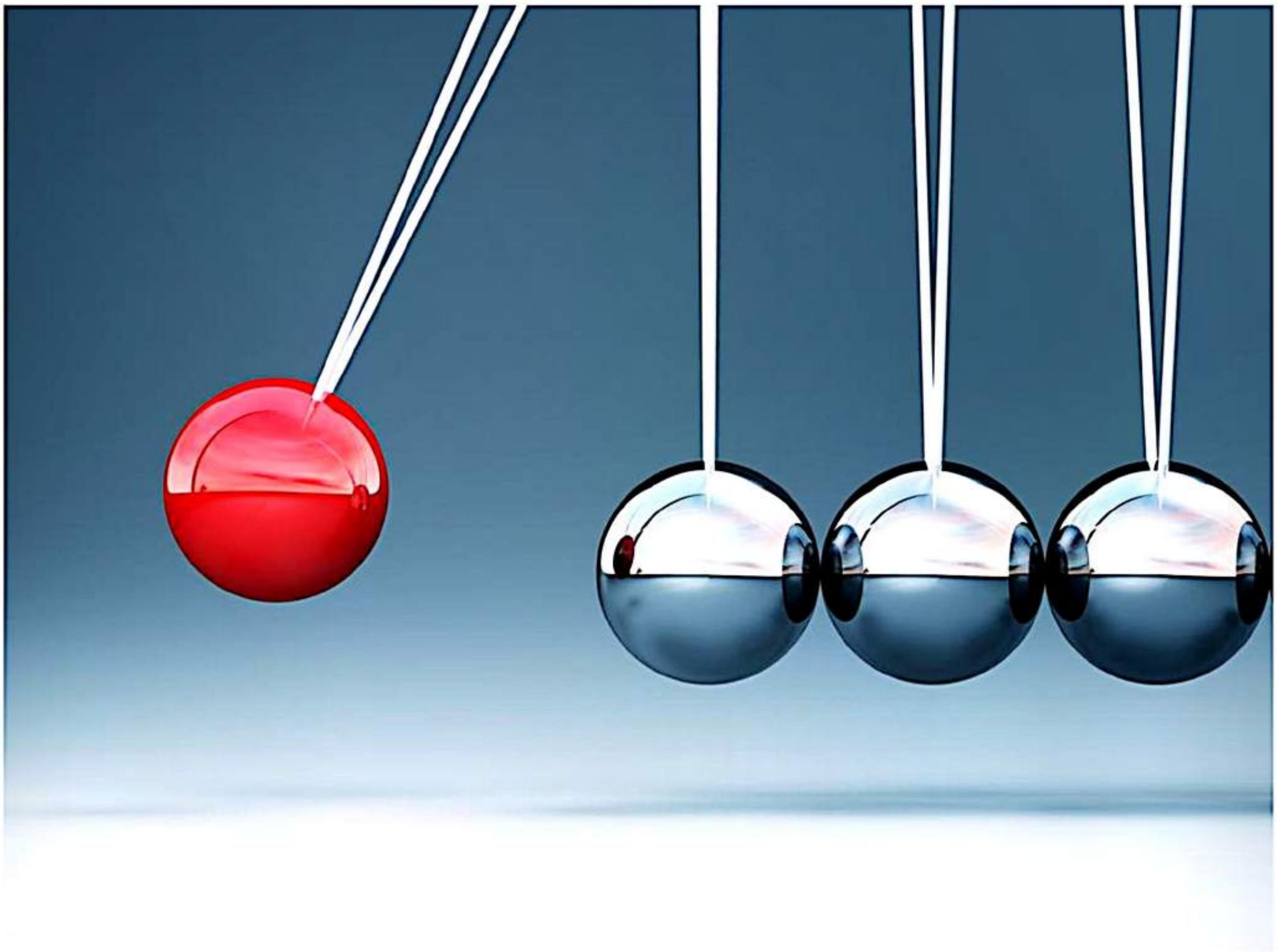
Chapter–5

Work, Energy and Power



CBSE CLASS XI NOTES

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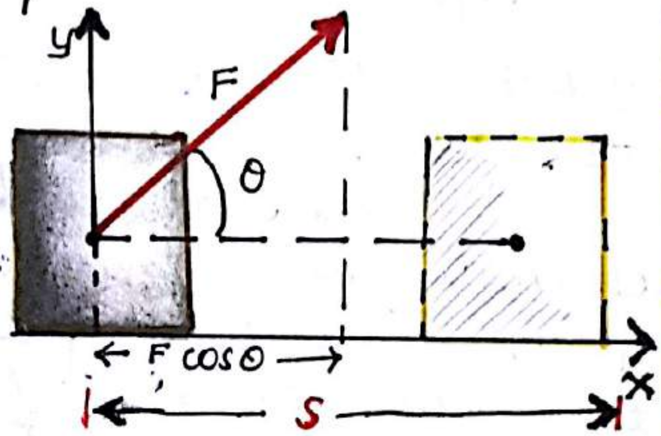
WORK

Work is said to be done if the force acting on a body displaces the body in the direction of force $W = F \cdot S$



$W = FS\cos\theta$

* Work is a scalar Quantity its unit is joule.
* Dimension is ML^2T^{-2}

Types of work

(a) positive work - A body falls freely under gravitational pull.

$W = FS\cos\theta$ $\theta = 0^\circ$

$\boxed{W = FS}$

positive work

(b) Negative work.

A body is made to slide over a rough surface. work done by frictional force it is negative, $\theta = 180^\circ$.

$W = FS\cos\theta$

$W = FS\cos 180^\circ$

$\boxed{W = -FS}$

(c) zero work

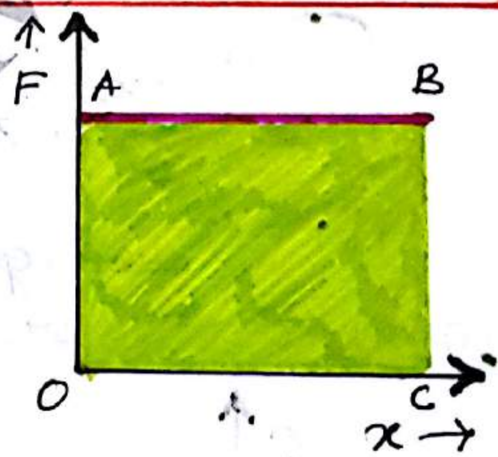
A man walking on an horizontal road, carrying a load $\theta = 90^\circ$

$W = FS\cos\theta$

$W = FS\cos 90^\circ$

$\boxed{W = 0}$

Work done by a constant force - Graphical method.



If force is constant force displacement graph is represented as above.

OA - represents force F and OC - represents displacement 'x'

work done

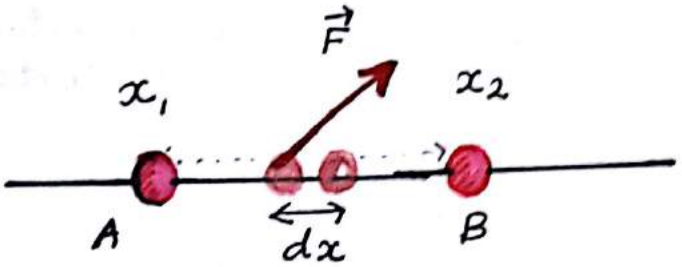
$= F \times x = OA \times OC$

$=$ Area of ▭gle OABC.

* Work done by a variable force.

consider a body subjected to va-

-riable forces when it moves from A to B. At any instant t.

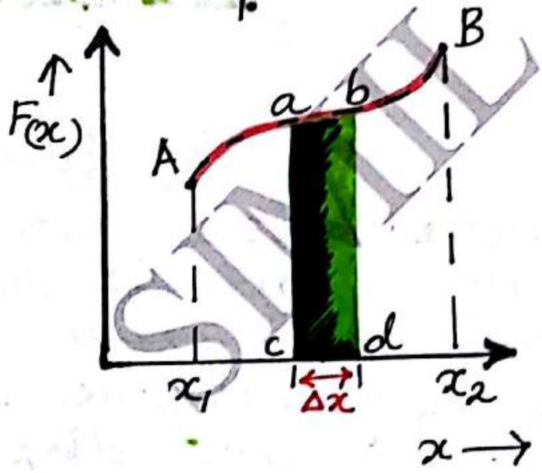


Let 'F' be the force acting on it, displaces the body through dx.

work done = $F \times dx$

$$\boxed{\begin{aligned} dW &= F\,dx \\ W &= \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x} \end{aligned}}$$

graphical representation



work done dW in displacing a body 'Δx' distance by a force $F(x)$ can be written as

$\Delta W = F(x)\,\Delta x$

= area of strip abcd.

Total work done

$$W = \sum_{x_1}^{x_2} F(x)\,\Delta x = \text{Area } x_1 AB x_2$$

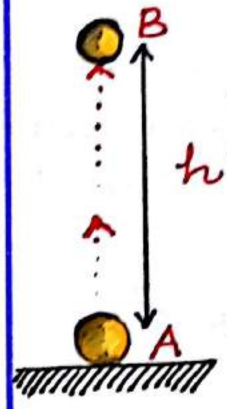
Potential Energy P.E

It is the energy possessed by a body by virtue of its position.

eg; P.E of water in a tank which is kept at a height.

work done in lifting a body.

consider a body of mass 'm' at a point A on ground. In order to move the body from A to B, we have to do some work. This work is stored as P.E in the body.



work done = $F \times S$

$= F \times h$

$$\boxed{P.E = mgh}$$

conservative and non conservative force.

conservative force	non-conservative force.
① Amount of work done in moving a body depends only	① It depends upon the path taken when body moves from

on initial and final positions of object. It is Independent of the path taken.	one point to another point.
(2) when a body moves around any closed path work done is '0'.	(2) work done is not zero.
eg; Gravitational force	eg. frictional force.

POWER..

The time rate of doing work is called power.

$$\boxed{P = \frac{W}{t}} = \frac{\text{work done}}{\text{time}}$$

work done = F.S

$$\therefore \boxed{P = \frac{W}{t} = \frac{F.S}{t} = F.\frac{S}{t}}$$

$$\boxed{P = F.V} \quad \because \text{ since } V = \frac{S}{t}$$

1 kW = 1000W

1 Horse power (HP) = 746W

* unit watt (W)
* Dimension ML^2T^{-3}

ENERGY

Energy is defined as the capacity to do work.

→ scalar Quantity

→ same unit of work i.e; joule (J)

→ (a) kinetic Energy.

→ (b) Potential Energy.

Kinetic Energy (K.E)

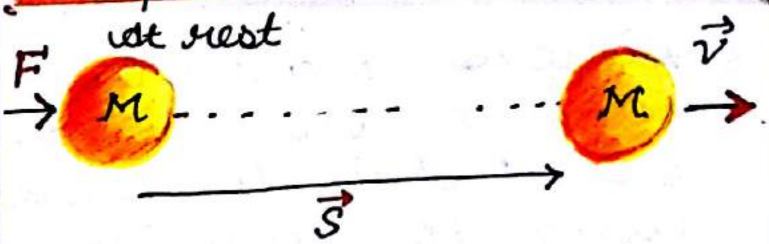
It is the Energy possessed by a body by virtue of its motion.

eg; A bullet fired from a gun can pearce a target due to its K.E

→ K.E is always +ve & scalar Quantity

$$\rightarrow \boxed{K.E = \frac{1}{2}mv^2} \quad \boxed{K.E = \frac{P^2}{2m}}$$

Proof



consider a body of mass 'M' moving

with velocity 'v'. which is initially at rest.

Let a constant force 'F' act on the body at rest and move it through a distance 'S'. thereby changing its velocity to V. Let 'a' be the constant acceleration.

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2as$$

$$v^2 = 2as$$

$$a = \frac{v^2}{2s} \quad \cdots ①$$

$$F = ma = \frac{mv^2}{2s} \quad \cdots ②$$

$$W = F \times S = \frac{mv^2}{2s} \times s$$

$$W = \frac{1}{2} mv^2$$

$$W = K.E$$

$\therefore$ $\boxed{K.E \text{ of a body} = \frac{1}{2} mv^2}$

LONG ANSWER QUESTION

① Discuss Elastic collision in one dimension. Obtain the expressions for velocities of two bodies after such a collisions?

② state and explain law of conservation of energy. Illustrate the law in the case of

(a) freely falling body

(b) an oscillating pendulum.

③ Derive an Expression for P.E of a spring. Represent it graphically?

Short Answer Questions

(1) A light body and heavy body have equal K.E? which one has larger momentum?

(2) what is work-energy theorem?

(3) Distinguish b/w elastic and Inelastic collision

(4) A bullet is fired from a rifle. If the rifle recoils freely, determine wether the kinetic energy of the rifle is greater, equal or less compared to that of bullet.

state and prove work-energy principle?

principle:- Work done by a force in displacing a body measures the change in K.E of the body.

proof:-

let a constant force 'F' acting on a body of mass 'm' change its velocity from u to v in travelling a distance S. then

work done = F × S

change in K.E = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

» $K.E = \frac{1}{2}m(v^2 - u^2)$

» $K.E = \frac{1}{2}m \times 2aS$

» $K.E = maS$

» $K.E = F \times S$

» $K.E = W$

hence proved.

POTENTIAL ENERGY

potential energy of a body is defined as the energy possessed by the body by virtue of its position.

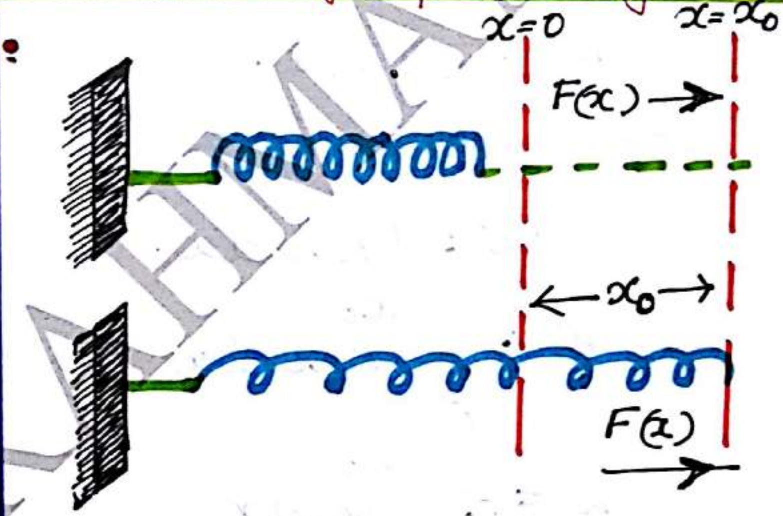
P.E → +ve, → -ve.

Gravitational P.E

Energy possessed by virtue of its position above the surface of earth

$P.E = mgh$

Derive an expression for P.E of a spring represent it graphically.



consider an elastic spring fixed at one end. It can be streched or compressed at the other end by applying force

when the spring is not in a strained state, the position of the free end of the spring is taken as x=0.

If the string is streched by a small distance 'x' by an external force

$F = kx.$

The restoring force $F = -kx$ acts is a direction opposite to external force

$$F = kx$$

work done in stretching the string from '0' to 'x_0'

$$dW = F \cdot dx$$

$$dw = kx\, dx.$$

Total work done

$$\int dw = \int kx \cdot dx.$$

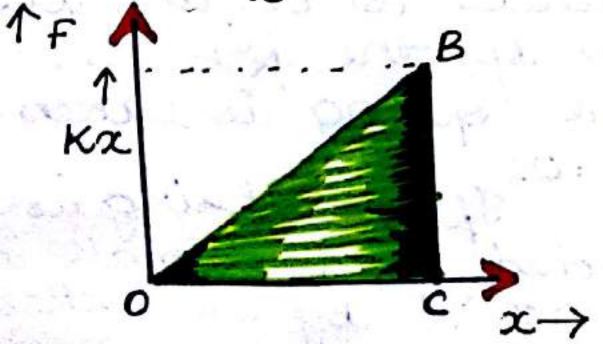
$$W = \int_0^{x_0} kx \cdot dx$$

$$W = k \int_0^{x_0} x \cdot dx$$

$$W = k \left[\frac{x^2}{2} \right]_0^{x_0}$$

$$W = k \left[\frac{x_0^2}{2} - 0 \right]$$

$$W = k\frac{x_0^2}{2} = \frac{1}{2} k x_0^2.$$



potential energy of the spring

$$V_x = \frac{1}{2} kx^2.$$

from graph

$$V_x = \text{area of OBC}$$

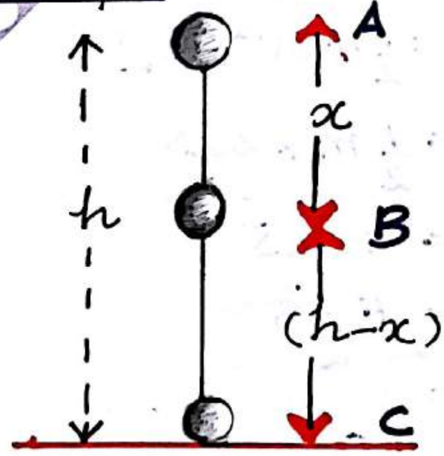
$$= \frac{1}{2} x \cdot kx = \frac{1}{2} kx^2$$

* state and prove the law of conservation of energy?

<u>statement</u>

"Energy can neither be created nor be distroyed."

<u>proof</u>



consider a body of mass 'm' situated at a height 'h' above the ground.

<u>Total energy @ position A</u>

as the body is at rest at A

$$P \cdot E = mgh$$

$$K \cdot E = 0$$

$$\boxed{T \cdot E = PE + K \cdot E}$$

$$T \cdot E = mgh + 0$$

$$T \cdot E = mgh$$

Total energy @ the position B

In the free fall let the body cross any point B with a velocity V.

where $AB = x$

$$V^2 = u^2 + 2as$$

$$V^2 = 0^2 + 2gx$$

$$V^2 = 2gx$$

$$K \cdot E = \frac{1}{2} mV^2 = \frac{1}{2} (2gx)$$

$$K \cdot E = mgx$$

$$P \cdot E = mg(h - x)$$

$$P \cdot E = mgh - mgx$$

$\therefore$ total energy

$$T \cdot E = P \cdot E + K \cdot E$$

$$T \cdot E = mgh - mgx + mgx$$

$$\boxed{T \cdot E = mgh}$$

Total Energy at C.

let the body allowed to fall freely under gravity, when it strikes the ground its velocity is V.

$$V^2 = u^2 + 2as$$

$$V^2 = 0^2 + 2gh$$

$$V^2 = 2gh$$

$$\therefore K \cdot E = \frac{1}{2} mV^2$$

$$K \cdot E = \frac{1}{2} m (2gh)$$

$$K \cdot E = mgh$$

$\therefore$ total energy

$$T \cdot E = K \cdot E + P \cdot E$$

$$T \cdot E = mgh + 0$$

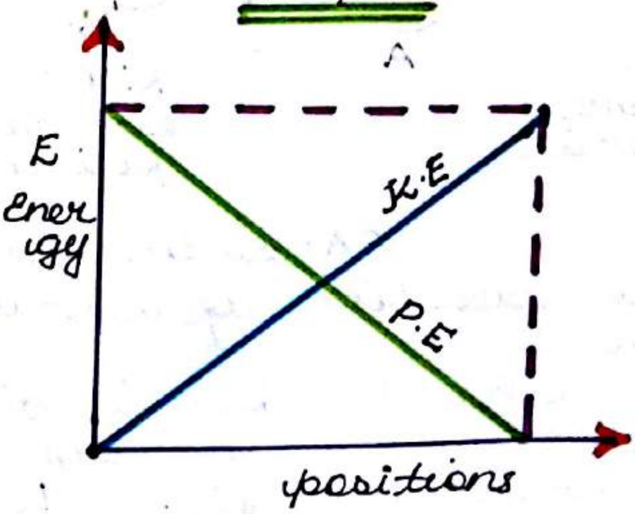
$$\boxed{T \cdot E = mgh}$$

$\therefore$ total energy

$$T \cdot E @ A = T \cdot E @ B = T \cdot E @ C = mgh$$

$\therefore$ Total energy remain constant at all positions.

graphical representation.



T.E during free fall is constant.

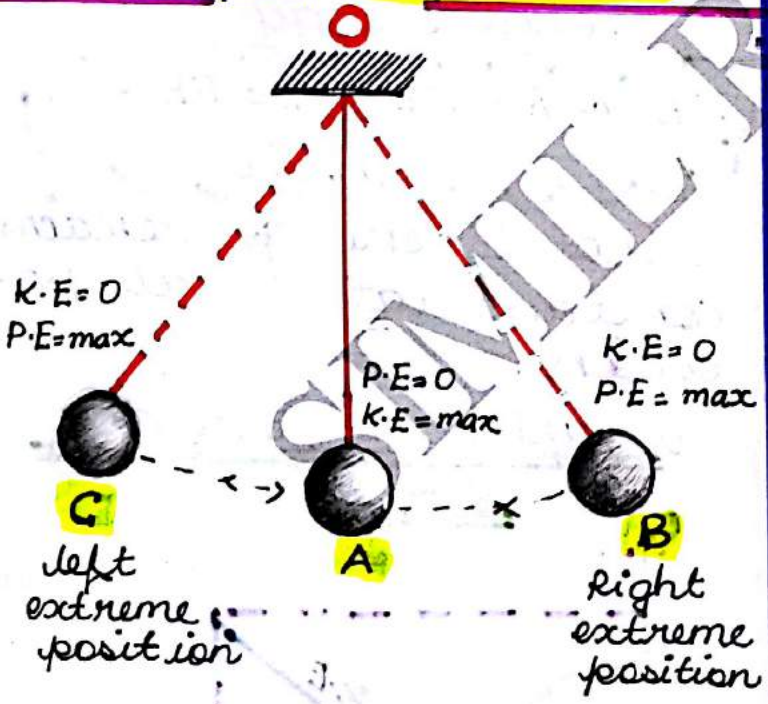
At A - Energy is entirely P.E

At B - energy is partially K.E & P.E

At C - Energy is entirely kinetic

It shows that energy can neither be created nor be destroyed. But one form of energy can be transformed to another.

* Vibrations of a Simple Pendulum.



OA: Is the normal position of pendulum

When the bob is displaced to B through a vertical height h its $P.E = mgh$

where 'm' is the mass of the bob.

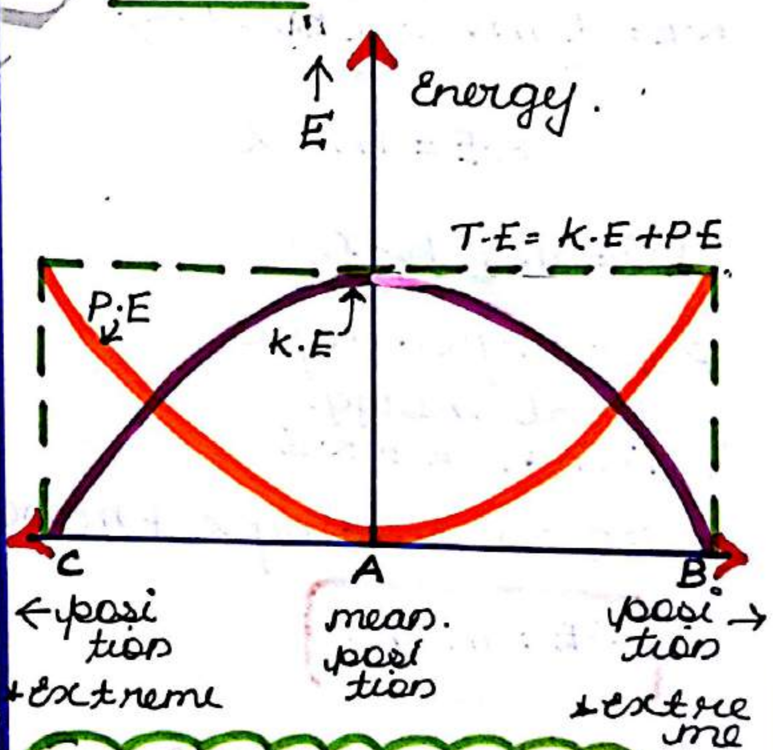
On releasing the bob it move towards A. P.E of the bob is converted into K.E at A.

When it moves to the other side. The K.E is being converted into P.E.

* At extreme positions B and C K.E = 0, P.E = max

Thus it keeps on oscillating

graphical representation.



Various forms of Energy

(1) Thermal energy:- energy possessed by hot bodies.

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(2) Electrical Energy:-

since electric charges attract or repel, they exert force on each other. Thus work is done. The energy associated with this work is called electrical energy.

(3) Nuclear Energy:

Protons and neutrons attract each other and combine to form nuclei. The energy associated with this is called nuclear energy.

(4) Internal energy.

The sum of K.E and P.E of molecules of a body is Internal energy.

COLLISIONS

If two objects in motion strike against each other or come close to each other such that the motion of one of them or both of them changes, a collision is said to have taken place.

Types of collision

(1) Elastic collision.
(2) Inelastic collision.

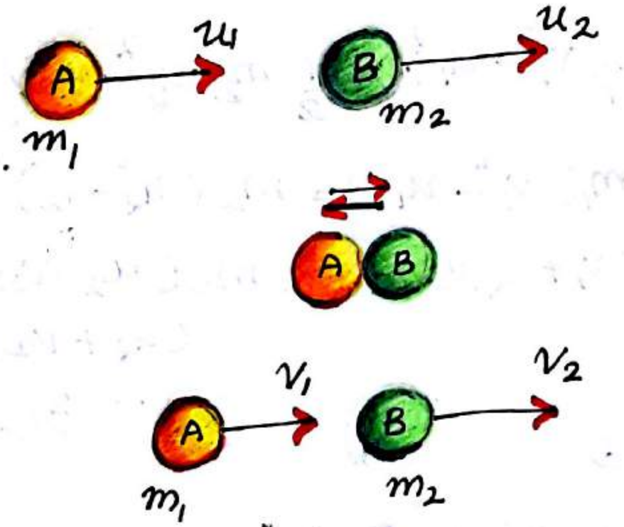
Elastic collision

Those collisions in which both momentum and K.E of a system are conserved are called elastic collisions.

Inelastic collision.

Those collisions in which momentum of the system is conserved but K.E is not conserved is called In elastic collisions.

Elastic collion in one Dimension.



consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 along a straight line in the same direction. let A be moving faster than B. After sometimes 'A' collides with B.

$v_1, v_2 \rightarrow$ velocities after collision.

by law of conservation of momentum

total $\vec{P}$ after collision = total $\vec{P}$ before collision

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m_1 (v_1 - u_1) = m_2 (u_2 - v_2) \quad \ldots (1)$$

Total K.E after collision = Total K.E before collision

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$\frac{1}{2} m_1 (v_1^2 - u_1^2) = \frac{1}{2} m_2 (u_2^2 - v_2^2)$$

$$m_1 (v_1^2 - u_1^2) = m_2 (u_2^2 - v_2^2)$$

$$m_1 (v_1 + u_1)(v_1 - u_1) = m_2 (u_2 - v_2)(u_2 + v_2) \quad \ldots (2)$$

(2) ÷ (1)

$$\Rightarrow v_1 + u_1 = u_2 + v_2 \quad \ldots (3)$$

$$\therefore v_1 - v_2 = u_2 - u_1$$

$$v_1 - v_2 = -(u_1 - u_2)$$

The relative velocity after collision is numerically equal to relative velocity before collision.

To find velocities after collision

$$v_2 = u_1 - u_2 + v_1 \quad \ldots (4)$$

(4) in (1) $\Rightarrow$

$$m_1 (v_1 - u_1) = m_2 (u_2 - u_1 + u_2 - v_1)$$

$$m_1 v_1 - m_1 u_1 = 2 m_2 u_2 - m_2 u_1 - m_2 v_1$$

$$m_1 v_1 + m_2 v_1 = m_1 u_1 - m_2 u_1 + 2 m_2 u_2$$

$$(m_1 + m_2) v_1 = (m_1 - m_2) u_1 + 2 m_2 u_2$$

$$\boxed{\therefore v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2 m_2}{(m_1 + m_2)} u_2}$$

similarly from eqtns (3) and (1) we have

$$\boxed{v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2 m_1}{m_1 + m_2}\right) u_1}$$

Simil...